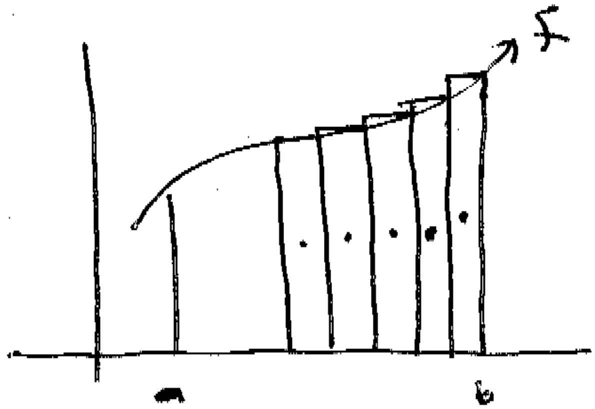
8.3
1/5

The weighted average.

Dusty's grade :) going into the final to get a 4.0.

HW
Test
Final.

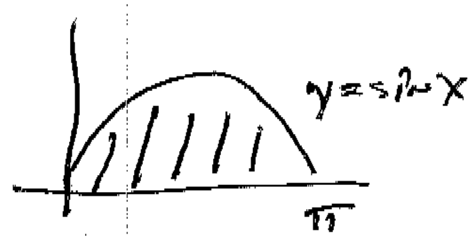
Find the center of mass of a plate w/ uniform density ρ per unit area.



Derive $\bar{x} = \frac{1}{A}\int_a^b x\, f(x)\,dx$

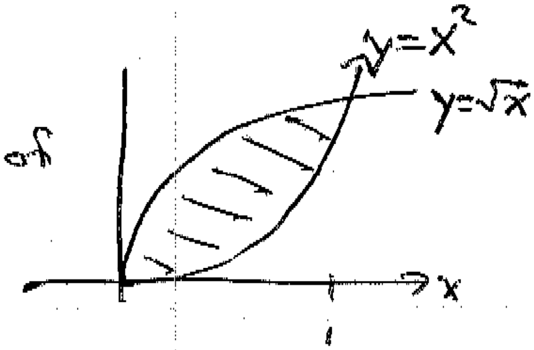
$\bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}[f(x)]^2\,dx$

Ex1: Find the centroid of



The moments.

Ex2: Find the centroid of



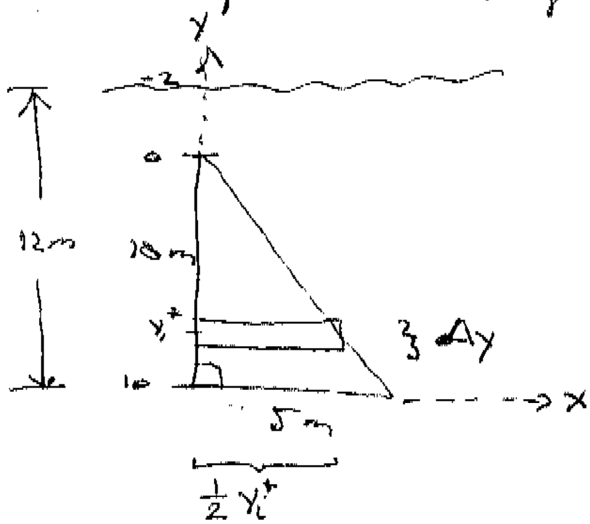
Hydrostatic Pressure

Pressure increases linearly due to the mass of the water above.

The important fact is that at any point in a liquid the pressure is the same in all directions.

$$\text{so} \quad P = \rho g d \quad (d \text{ depth})$$

Ex1: Find the hydrostatic force on the vertical plate submerged in the water



$$A_i = \tfrac{1}{2} y_i^* \Delta y$$

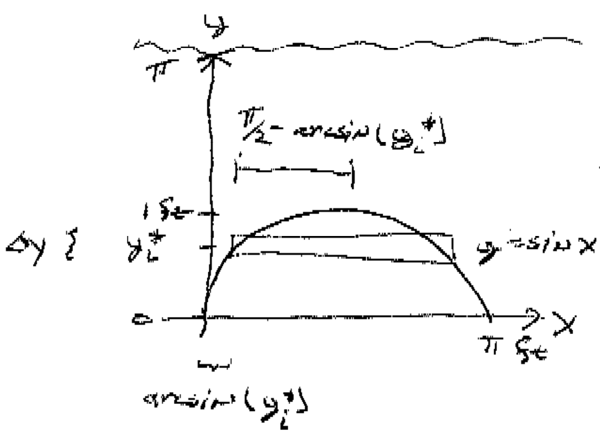
$$P_i = 1000(10)(2 + y_i^*)$$

$$F_i = 5000\, y_i^*(2 + y_i^*) \Delta y$$

$$F = \lim_{n\to\infty} \sum_{i=1}^{n} y_i^*(2 + y_i^*)\Delta y$$

$$= \int_0^{10} y(2+y)\,dy.$$

Ex2: Find the hydrostatic force on the plate.



$$A_i^* = 2\left(\tfrac{\pi}{2} - \arcsin(y_i^*)\right)\Delta y$$

$$P_i = 62.5(\pi - y_i^*)$$

$$F = \int_0^1 62.5(\pi - y)\, 2\left(\tfrac{\pi}{2} - \arcsin y\right) dy.$$

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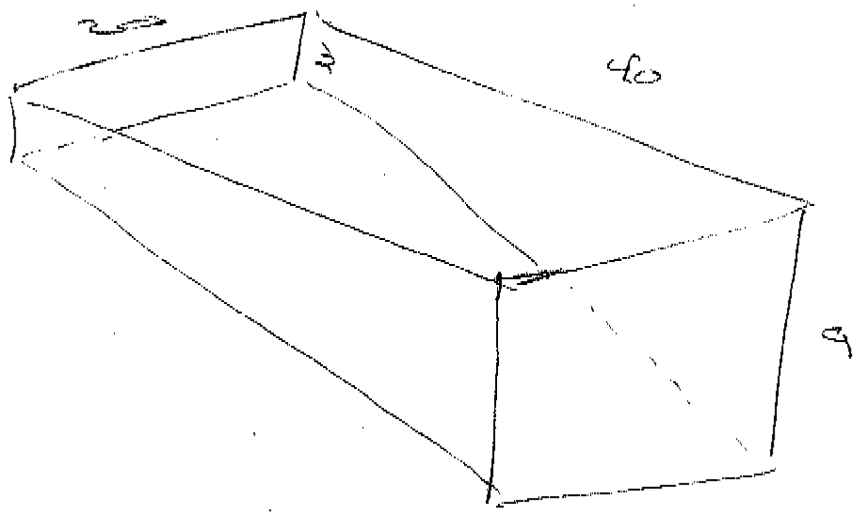
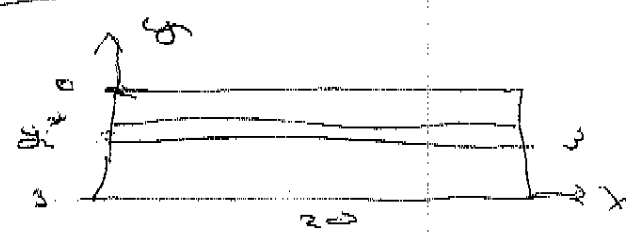
$\boxed{15}$ Swimming Pool.
Find the pressure on all ends.



a) shallow end



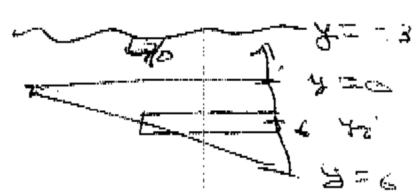
$A_i^* = 20\Delta y$

$P_i^* = 62.5\, y_i^*$

$\int_0^3 20(62.5)\, y\, dy = 20(62.5)\frac{9}{2} = 5625$ lbs.

b) $20(62.5)\, 81/2 = 50625$ lbs (deep end)

c) side $40(62.5)\, 9/2 +$



$A_i = (40 - \frac{40}{6} y_i)\Delta y$ 〃 11250

$P_i = 62.5(3 + y_i)$

$\int_0^6 62.5(3+y)(40 - \frac{40}{6}y)\,dy = 37500 = 48750$ lbs.

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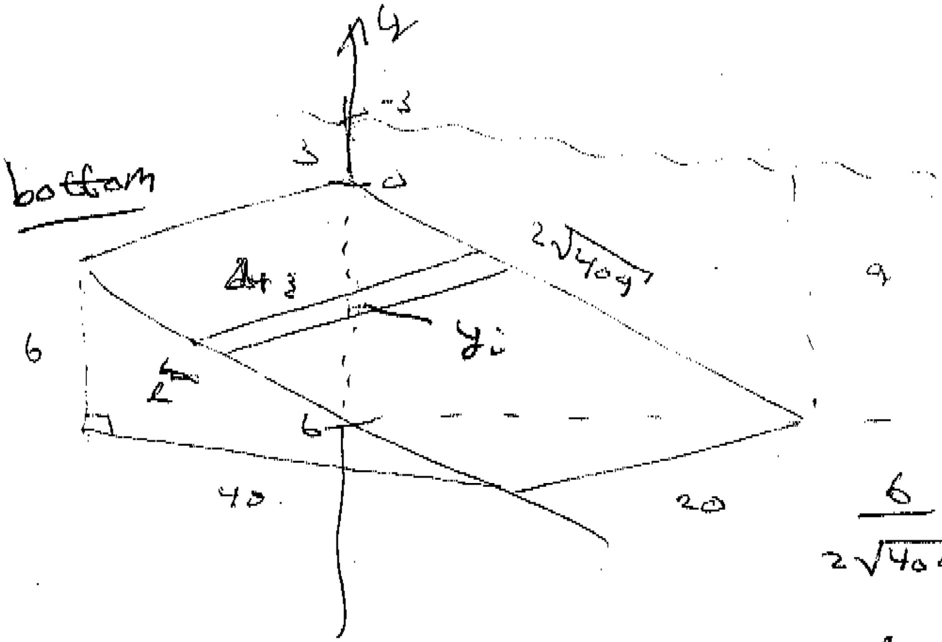
d) bottom



$$\frac{6}{2\sqrt{409}} = \frac{\Delta y}{l}$$

$$l = \frac{\sqrt{409}}{3}\Delta y$$

$$A_i = \frac{20}{3}\sqrt{409}\,\Delta y$$

$$P_i = 62.5(3+y_i)$$

$$F = \int_0^6 62.5(3+y)\left(\frac{20}{3}\right)\sqrt{409}\,dy$$

$$= 303356.2 \text{ lb}$$

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Recall $\overline{x} = \dfrac{\int_a^b x f(x)\,dx}{\int_a^b f(x)\,dx}$

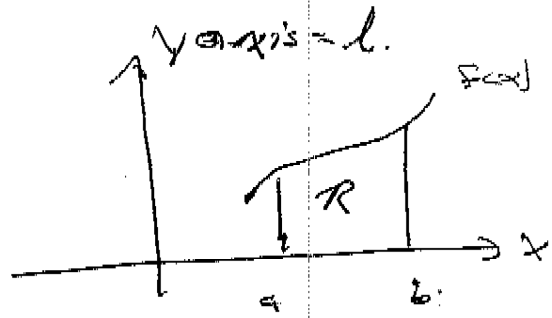
$$= \frac{1}{A}\int_a^b x f(x)\,dx$$

$$\Rightarrow \overline{x}A = \int_a^b x f(x)\,dx.$$

<u>Theorem of Pappus.</u>

Let $\mathcal{R}$ be a plane region that lies entirely on one side of a line l in the plane. If $\mathcal{R}$ is rotated about l, then the volume of the resulting solid is the product of the area A of $\mathcal{R}$ and the distance d traveled by the centroid of $\mathcal{R}$.

□ <u>proof</u> (special case).



using cylindrical shells

$$V = \int_a^b 2\pi x f(x)\,dx$$

$$= 2\pi \int_a^b x f(x)\,dx$$

$$= 2\pi \overline{x} A.$$

$$= Ad \quad ■$$

<u>EX3</u>: Find the volume of a sphere w/ radius r.