

Commands on the TI-83/4 calculator are a big part of this lesson. A few important commands are as follows: (1.) to edit lists: STAT → EDIT, (2.) to get y1: VARS → YVARS → FUNCTION → Y1, (3.) sequence of x\_i's: seq(x\_i, i, i\_0, i\_max, 1), (4.) sequence: LIST → OPS → SEQ, (5.) sum: LIST → MATH → SUM. The basic list structure used is below.

①

Notetaker's name \_\_\_\_\_ Class \_\_\_\_\_ Date May 18, 09

List 1: (L1): x\_i's .... use sequence  
 List 2: (L2): "Y1(L1)" .... make sure to enter y1  
 List 3: (L3): coefficients such as 1, 2, ..., 2, 1  
 List 4: (L4): "L2\*L3"  
 Home screen: (dx/factor)\*sum(L4)

Ch. 7.7

Numerical Integration.

We have 5 methods of numerical integration already

Riemann Sum

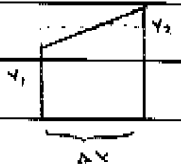
1) Right  $R_n = \Delta x (f(x_1) + f(x_2) + \dots + f(x_n))$

2) Left  $L_n = \Delta x (f(x_0) + f(x_1) + \dots + f(x_{n-1}))$

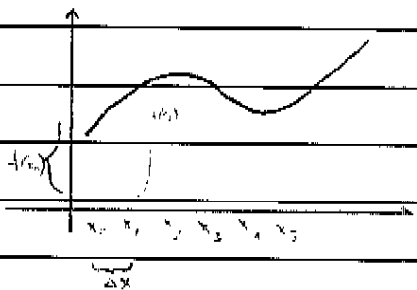
3) Midpoint

\* 4) Trapezoidal Rule

\* 5) Simpson's Rule



$$\text{Area} = \Delta x \left( \frac{y_1 + y_2}{2} \right) = \frac{\Delta x}{2} (y_1 + y_2)$$



$$T_n = \frac{\Delta x}{2} (f(x_0) + f(x_1)) + \frac{\Delta x}{2} (f(x_1) + f(x_2)) + \dots + \frac{\Delta x}{2} (f(x_{n-1}) + f(x_n))$$

$$= \frac{\Delta x}{2} (1f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + 1f(x_n))$$

Ex: 1

Use  $T_5$  to approx  $\int_0^2 e^{-x^2} dx$

$$T_5 = \frac{0.25}{2} (f(0) + 2f(0.25) + 2f(0.5) + \dots + 2f(1.75) + f(2)) = .8817$$

Simpson's Rule:

$$\int_a^b f(x) dx \approx \frac{\Delta x}{3} (1f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 4f(x_{n-1}) + 1f(x_n)) = .8821$$

Section 3.3

$$\int_1^2 e^{-x} dx = \left[ -\frac{1}{e^x} \right]_1^2 = \frac{1}{e} - \frac{1}{e^2} = 0.045733337 \quad \text{TRC}$$

numerical method  
 $f_{int}(x), x, 1, 3$

Trap  $T = \frac{2-1}{2} \left( \frac{1}{e} + 2f\left(\frac{1+2}{2}\right) + \frac{1}{e^2} \right) = 0.045733337$

$\Delta x = \frac{1}{3}$

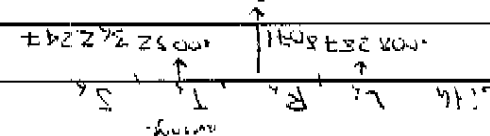
$x_i = 1 + \frac{i}{3}$

Simpson's Rule  $S = \frac{2-1}{6} \left( \frac{1}{e} + 4f\left(\frac{1+2}{2}\right) + 2f\left(\frac{1+2}{3}\right) + 2f\left(\frac{1+2}{3}\right) + \frac{1}{e^2} \right) = 0.045733337$

Left  $L_3 = \frac{2-1}{3} \left( \frac{1}{e} + 1f\left(\frac{1+2}{3}\right) + 1f\left(\frac{1+2}{3}\right) + \dots + f\left(\frac{1+2}{3}\right) + 0f\left(\frac{2}{3}\right) \right)$

Right  $R_3 = \frac{2-1}{3} \left( 0f\left(\frac{1}{3}\right) + 1f\left(\frac{1+2}{3}\right) + 1f\left(\frac{1+2}{3}\right) + \dots + f\left(\frac{1+2}{3}\right) + 1f\left(\frac{2}{3}\right) \right)$

approx  $\int_1^2 e^{-x} dx$



$L_3$  "X(L)"

0.0021846  
 0.0046434213

$L_1$  "L2 \* L3" \* "quit"

2nd stat math |S| (L3)

error =	exact answer - approx result
$E_T =$	exact answer - approx w/ trap rule

error bounds

$$|E_T| \leq K(b-a)^3$$

N = # of subintervals  
 $(b-a)^3$

$$|E_M| \leq \frac{K(b-a)^2}{24N^2}$$

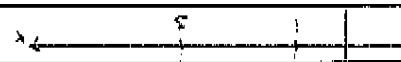
When  $|f''(x)| \leq K$  on  $[a, b]$

Find  $K$  in part example

$$f(x) = e^{-4x}$$

$$f'(x) = -4e^{-4x}$$

$$f''(x) = 16e^{-4x}$$



$$|E_T| = \frac{16e^{-4(2)}}{12 \cdot 4^2} = .005422$$

Error bound for Simpson's Rule.

$$|E_T| \leq \frac{K(b-a)^3}{180n^4}$$

where  $|f^{(4)}(x)| \leq K$  on  $[a, b]$

$$f^{(3)} = -64e^{-4x}$$

$$f^{(4)} = 256e^{-4x}$$

max on  $[1, 3]$  at  $x=1$

$$K = 256e^{-4}$$

$$|E_T| \leq \frac{256e^{-4}(2)^3}{180 \cdot 4^2} = .000643$$