$\boxed{\begin{matrix} 5.5 \\ 1/2 \end{matrix}}$

The Substitution Rule

Ex 1: $\int 2x\sqrt{1+x^2}\,dx$ can we guess?

The Substitution Rule.

If $u = g(x)$ is a differentiable fct whose range is an interval I and f is cont on I, then $\int f(g(x))\,g'(x)\,dx = \int f(u)\,du$.

$\square$ proof.

$$\frac{d}{dx}\left[F(g(x))\right] = F'(g(x))\cdot g'(x)$$

$$\Rightarrow \int F'(g(x))\,g'(x)\,dx = F(g(x)) + c$$

$$= F(u) + c$$

$$= \int F'(u)\,du.$$

Ex 2: $\int e^{\sin\theta}\cos\theta\,d\theta$

Ex 3: $\int (2-x)^6\,dx$

5.5
2/2

Ex4: $\int \frac{1}{(1+2x)^2}\,dx$

Ex 5: $\int x \sec^2(x^2)\,dx$

Ex6: $\int \tan^4(3x)\cdot\sec^2(3x)\,dx$

Ex7: $\int \frac{1}{(1+t^2)\arctan(t)}\,dt.$

Substitution and definite Integrals.

Ex8: $\int_0^4 (x-2)^{17}\,dx$

Ex9: $\int_0^{\sqrt{\pi}} x\sin(x^2)\,dx$

Ex10: $\int_{-\pi/6}^{\pi/6} \tan^3(\theta)\,d\theta$ (symmetry)

Ex11: $\int_{e}^{e^2} \frac{dx}{x\sqrt[3]{\ln(x)}}$

Ex 12: $\int_{1/6}^{1/4} \sec(\pi t)\tan(\pi t)\,dt.$

Integrals of symmetric fcts: Suppose f is cont. on $[-a,a]$

a) If f is even, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

b) If f is odd, then $\int_{-a}^{a} f(x)\,dx = 0.$