These are notes taken from two different review sessions (pages 1-2 at 10am and 3-4 at 8am). The purpose of posting these notes is to provide additional examples based on the needs and interests of students.

Notetaker's name Emma Miller Class CALC 125 Date 4/22/09

Review Day

Overview

5.1: MOTIVATION

* 5.2: Def. of Definite Integral - Riemann sums

5.3: FToC - know thm, derive thm, use thm

5.4: Indefinite Integrals (included elsewhere)

* 5.5: Substitution - w/ indef. + def. integral

6.1: Area btw curves $\rightarrow \int$ top fcn $- \int$ bottom fcn

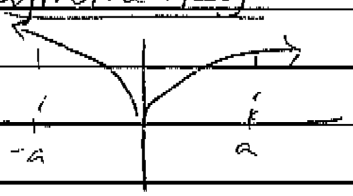
6.2: Volumes by slicing

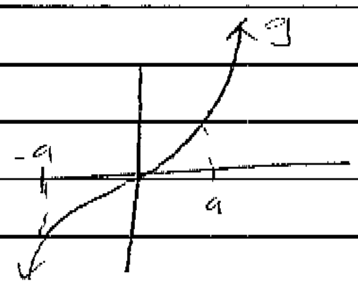
6.3: Volumes by shells

6.4: WORK = force * distance

6.5: Ave. Value of a function

Symmetry





$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ $\qquad$ $\int_{-a}^{a} g(x)\,dx = 0$

f is even fcn $f(-x)=f(x)$ $\qquad$ g is odd fn $g(-x) = -g(x)$

5.3 #29 FToC

$\int_{1}^{9} \frac{x-1}{\sqrt{x}}\,dx = \int_{1}^{9} \frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}\,dx$

$= \int x^{1/2} - x^{-1/2}\,dx$

$= \left[\frac{2}{3}x^{3/2} - 2x^{1/2}\right]_{1}^{9}$

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Riemann Sum / Def. of Definite Integral:

express $\int_4^7 (5x+2)\,dx$ as the limit of the Riemann sums (Right end pts)

$\Delta x = \frac{3}{N}$ $x_i = 4 + \frac{3i}{N}$ $f(x_i) = 5(4 + \frac{3i}{N}) + 2$

so $\int_4^7 (5x+2)\,dx = \lim_{N \to \infty} \sum_{i=1}^{N} \left[5\left(4 + \frac{3i}{N}\right) + 2\right]\frac{3}{N}$

Symmetry

$\int_{-\pi/2}^{\pi/2} \frac{x^2 \sin x}{1+x^6}\,dx = 0$ $f(-x) = \frac{(-x)^2 \sin(-x)}{1+(-x)^6}$

$= \frac{-x^2 \sin x}{1+x^6}$

$f(x) = \frac{x^2 \sin x}{1+x^6} \Rightarrow$ ODD! $\Rightarrow = -f(x)$

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Write $\lim\limits_{n\to\infty} \sum_{i=1}^{n} \left[3\ln\left(2+\frac{7i}{n}\right) - \sqrt{\left(2+\frac{7i}{n}\right)}\right]\frac{7}{n}$

as a definite integral on $[2,9]$

$\Delta x = \frac{7}{n}$ $a=2$, $b=9$, $x_i = 2+\frac{7i}{n}$

$$= \int_2^9 \left(3\ln(x) - \sqrt{x}\right)\,dx.$$

Ex:

FTC. $I = \int_{-3}^{11} (4x^2 - x]\,dx = \left[\frac{4}{3}x^3 - \frac{1}{2}x^2\right]_{-3}^{11} = \left(\frac{5324}{3} - \frac{121}{2}\right) - \left(-36 - \frac{9}{2}\right) = \frac{5264}{3}$

Method 2:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left[4\left(\frac{14i}{n}-3\right)^2 - \left(\frac{14i}{n}-3\right)\right]\frac{14}{n} =$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[4\left(\frac{196}{n^2}i^2 - \frac{84}{n}i + 9\right) - \frac{14i}{n} + 3\right]\frac{14}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[\frac{784}{n^2}i^2 - \frac{350}{n}i + 39\right]\frac{14}{n}$$

$$= \lim_{n\to\infty}\left(\frac{10976}{n^3}\underbrace{\sum_{i=1}^{n} i^2}_{\frac{n(n+1)(2n+1)}{6}} - \frac{4900}{n^2}\underbrace{\sum_{i=1}^{n} i}_{\frac{n(n+1)}{2}} + \frac{546}{n}\underbrace{\sum_{i=1}^{n} 1}_{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{10976}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{4900}{n^2}\cdot\frac{n(n+1)}{2} + \frac{546}{n}\cdot n\right)$$

$$= \frac{10976}{6}\cdot 2 - \frac{4900}{2} + 546$$

$$= \frac{5264}{3}$$

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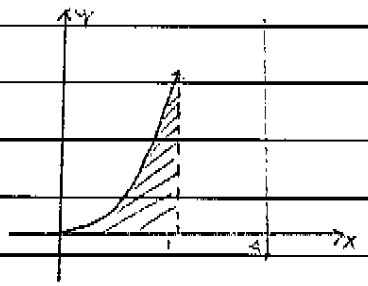
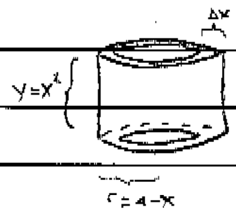
Review of volumes by slices and shells.



rotate about $x=4$

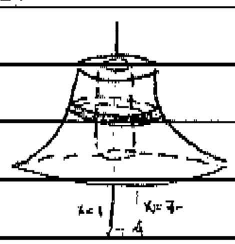
Shells



$$\Delta V = 2\pi(4-x)\cdot x^2\ \Delta x$$

$$V = \int_0^1 2\pi(4-x)x^2\ dx.$$

Slices:



$y = x^2$

$\sqrt{y} = x$

$$\text{Area} = (4-x)^3\pi$$

$$\Delta V = ((4-x)^2\pi - 9\pi)\ \Delta y$$

$$= \pi\ ((4-x)^2 - 9)\ \Delta y$$

$$\Rightarrow V \int_{y=0}^{y=1} \pi\ ((4-x)^2 - 9)\ dy$$

ex: 2

Express $\int_{-3}^{11} (4x^2-x)\ dx$ as the limit of Riemann sums

(Right end pts).

$\Delta x = \frac{14}{n}$

$x_i = -3 + \frac{14i}{n}$

$f(x_i) = 4\left(-3+\frac{14i}{n}\right)^2 - \left(-3+\frac{14i}{n}\right)$

$$I = \lim_{n\to\infty} \sum_{i=1}^{n} \left[4\left(-3+\frac{14i}{n}\right)^2 - \left(-3+\frac{14i}{n}\right)\right]\frac{14}{n}$$