9.4 : Models for Pop. Growth.

Ex1: Biologists stock a lake w/ 400 fish & est. the carrying capacity to be 10,000. The number of fish tripled in the 1st year.

(a) Assuming that the size of the fish pop. satisfies the logistic eqt, find a model $P(t)$ for the pop. after t yrs.

$$\frac{dP}{dt} = \underbrace{k}_{\substack{\text{initial} \\ \text{growth} \\ \text{rate } k}} P\left(1 - \frac{P}{\underbrace{10{,}000}_{\substack{\text{carrying} \\ \text{capacity} \\ K}}}\right)$$

$$\Rightarrow \quad \frac{dP}{dt} = \frac{k}{10{,}000} P(10{,}000 - P)$$

$$\Rightarrow \int \frac{dP}{P(10{,}000 - P)} = \int \frac{k}{10000}\, dt$$

partial fractions.

$$\frac{1}{P(10{,}000 - P)} = \frac{A}{P} + \frac{B}{10000 - P}$$

$$\Rightarrow \quad \begin{aligned} 1 &= 10000A \\ 0P &= -AP + BP \end{aligned} \quad \longrightarrow \quad A = \frac{1}{10{,}000} \;\&\; B = \frac{+1}{10000}$$

9.4
2/5

$$\Rightarrow \frac{1}{\cancel{10000}}\int\left(\frac{1}{P} + \frac{1}{10000-P}\right)dP = \frac{k}{\cancel{10000}}t + C_1$$

$$\Rightarrow \ln|P| \mp \ln|10000-P| = kt + C_2$$

$$\Rightarrow \ln\left|\frac{P}{10000-P}\right| = kt + C_2$$

$$\Rightarrow \ln\left|\frac{10000-P}{P}\right| = -kt + C_3$$

$$\Rightarrow \left|\frac{10000-P}{P}\right| = C_4 e^{-kt}$$

$$\Rightarrow \frac{10000-P}{P} = Ae^{-kt} \quad \left(\text{Note } A = \frac{10000-P_0}{P_0}\right)$$

$$\Rightarrow \frac{10000}{P} - 1 = Ae^{-kt} \qquad \text{so } A = 24$$

$$\Rightarrow \frac{10000}{P} = 1 + Ae^{-kt}$$

$$\Rightarrow P = \frac{10000}{1 + Ae^{-kt}}$$

$$\Rightarrow P(t) = \frac{10000}{1 + 24e^{-kt}}$$

$\Rightarrow$ solve for k given that $P(1) = 1200$.

(b) How long until the pop. hits 5000?

$$\frac{1}{2} = \frac{1}{1+24e^{-kt}} \Rightarrow 2 = 1 + 24e^{-kt}$$

$$\Rightarrow \frac{1}{24} = e^{-kt}$$

$$\Rightarrow t = -\frac{1}{k}\ln\left(\frac{1}{24}\right) =$$

9.4
3/5

Generalize from ex1 to the formula.

recall $P_0 = 400$ initial pop.

$K = 10000$ karrying kapacity

$k = 1.19$ krowth rate

$A = 24$

$$A = \frac{10000 - 400}{400} = 24.$$

k depends upon other info such as $P(1) = 1200$.

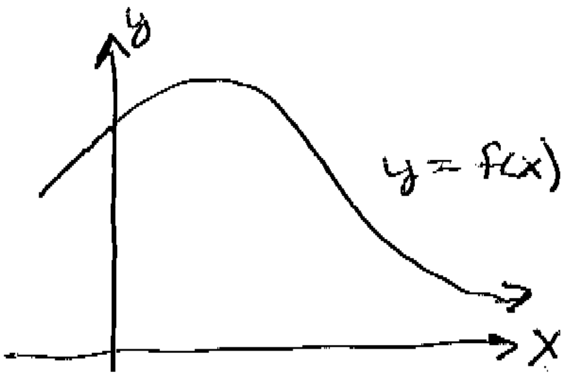
so, the solution $P(t) = \dfrac{K}{1 + Ae^{-kt}}$

is the solution to $\dfrac{dP}{dt} = kP\left(1 - \dfrac{P}{K}\right)$

9.4
4/5

slope fields

recall 9.1 # 12



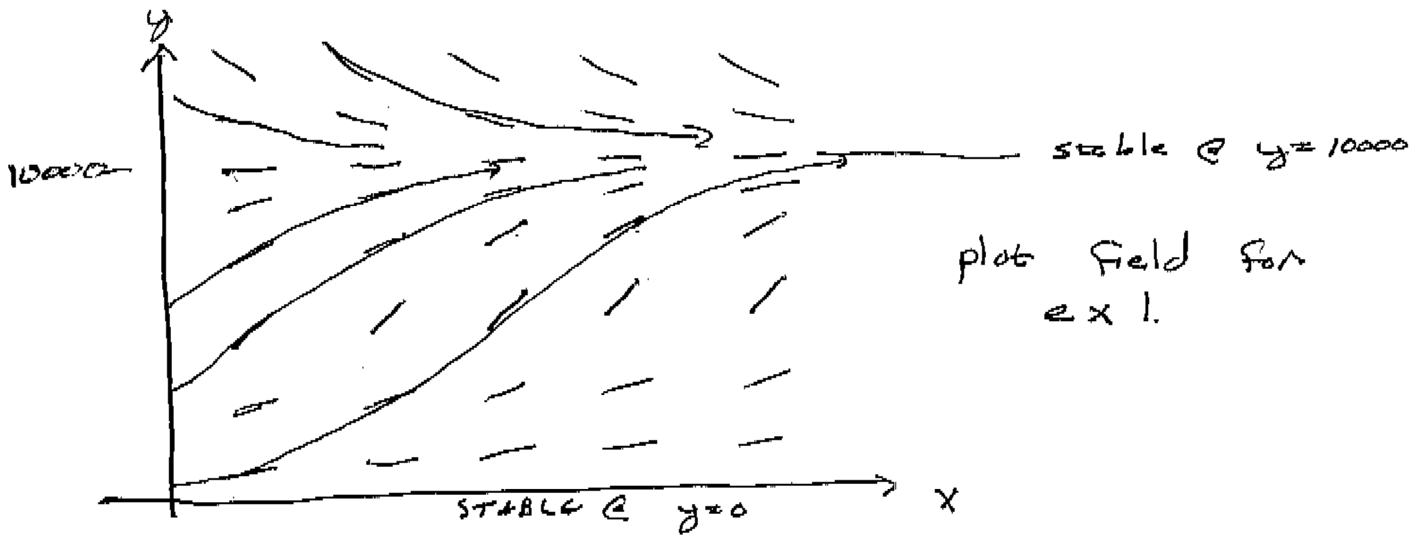
(A) $y' = 1 + xy$

(B) $y' = -2xy$

(C) $y' = 1 - 2xy$.

Notice, you choose (x, y) and you get out y' (the slope of the curve @ (x,y)).

$$(x, y) \longmapsto y'$$



plot field for ex 1.

9.4 HW
3/8

15 $\dfrac{dP}{dt} = \underbrace{0.08}_{\text{initial growth rate}} P\left(1 - \dfrac{P}{\underset{\text{carrying capacity}}{1000}}\right) - \underset{\text{constant rate @ which the pop. is harvested.}}{15}$

(a) (−15) represents the constant rate @ which fish are harvested.

(b) The pop. is stable when $\dfrac{dP}{dt} = 0$.

$$0 = 0.08P - \frac{0.08P^2}{1000} - 15$$

$$\Rightarrow 0 = 80P - 0.08P^2 - 15000$$

$$\Rightarrow 0 = 0.08P^2 - 80P + 15000$$

$$\Rightarrow P = \frac{80 \pm \sqrt{6400 - 4(0.08)(15000)}}{2(0.08)}$$

$$= 250 \text{ or } 750.$$