9.4HW
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[1] $$\frac{dP}{dt} = 0.05P - 0.0005P^2$$

$$= 0.05P\left(1 - \frac{P}{100}\right)$$

$$k = 0.05 \;\&\; K = 100$$

$$P(t) = \frac{100}{1 + Ae^{-0.05t}} \;;\; A = \frac{100 - P_0}{P_0}$$

(a) the carrying capacity is 100 & $k = 0.05$

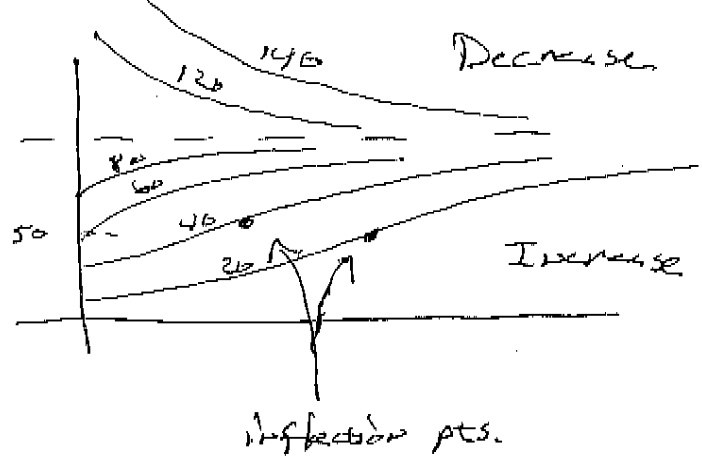
(b) direction field...

(i) slopes = 0 @ $P = 0$ & $P = 100$

(ii) The slopes are greatest @ $P = 50$

(iii) increasing on $0 < P < 100$ & decreasing on $P > 100$.

(c)



(d) equibrium solutions are @ $P = 0$ (unstable) & $P = 100$ (stable).

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[3] $\frac{dy}{dt} = ky\left(1 - \frac{y}{K}\right)$; $K = 8\times10^7$ kg & $k = 0.71/\text{yr}$.

(a) $y = \frac{K}{1 + Ae^{-kt}} = \frac{8\times10^7}{1 + 3e^{-.71t}}$; $y(0) = 2\times10^7$

$$A = \frac{8\times10^7 - 2\times10^7}{2\times10^7} = \frac{6\times10^7}{2\times10^7} = 3$$

After 1 yr: About 3.2×10^7 kg of biomass.

(b) solve $4\times10^7 = \frac{8\times10^7}{1 + 3e^{-.71t}}$

$$\Rightarrow 1 + 3e^{-.71t} = 2$$

$$\Rightarrow e^{-.71t} = \frac{1}{3}$$

$$\Rightarrow t = -\frac{1}{.71}\ln\left(\frac{1}{3}\right)$$

$$\approx 1.55 \text{ years.}$$

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$\boxed{4}$ Data set in text.

(a) $$y = \frac{K}{1+Ae^{-kt}}$$

$$\Rightarrow \frac{y}{K} = \frac{1}{1+Ae^{-kt}}$$

$$\Rightarrow \frac{K}{y} - 1 = Ae^{-kt}$$

In order to do exponential regression I must est. $K \approx 685$

$$= 37.78e^{-.44t} \text{ (from Excel)}$$

$$\Rightarrow \frac{685}{y} = 1 + 37.78e^{-.44t}$$

$$\Rightarrow y = \frac{685}{1+37.78e^{-.44t}}$$

(b) the initial growth rate is $k = 0.44$ or 44%.

(c) using Excel & estimating the carrying capacity @ 685...

exp model: $y = 35.58e^{0.2t}$

Log. model: $y = \frac{685}{1+37.78e^{-.44t}}$

(d) the logistic model is <u>much</u> better & we might improve it by adjusting K.

(e) I estimate $y(7) \approx 250$ yeast cells.

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5 world pop.

(a) Find k.

$$\max\ k_{max} = \frac{40-15 \text{ mil}}{5.3 \text{ bil}} = \frac{1}{212}$$

$$\min\ k_{min} = \frac{35-20 \text{ mil}}{5.3 \text{ bil}} = \frac{3}{1060}$$

$$K = 100,000,000,000$$

$$P_0 = 5,300,000,000$$

$$A = \frac{K-P_0}{P_0} \approx 17.87$$

$$\frac{100}{1+17.87e^{-\frac{3}{1060}t}} \le P \le \frac{100}{1+17.87e^{-t/212}}$$

(b) - (d)

Predictions

Year	K = 100 bil	K = 50 bil (A = 8.43)
2000	5.4 - 5.5 bil	5.4 - 5.5 bil
2100	7.1 - 8.6 bil	7.0 - 8.3 bil
2500	19.2 - 38.3 bil	16.7 - 28.4 bil

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$\boxed{7}$ the rumor mill

(a) $\frac{dy}{dt} = ky(P-y)$ where P is the total pop. size.

(b) $\int \frac{dy}{y(P-y)} = \int k\,dt$ $\quad \frac{1}{y(P-y)} = \frac{A}{y} + \frac{B}{P-y}$

$1 = AP \Rightarrow A = 1/P$

$0 \cdot y = -Ay + By \Rightarrow B = 1/P$

$\Rightarrow \frac{1}{P}\int \frac{1}{y} + \frac{1}{P-y}\,dP = kt + C$

$\Rightarrow \frac{1}{P}\left(\ln|y| - \ln|P-y|\right) = kt + C$

$\Rightarrow \ln\left|\frac{P-y}{y}\right| = -\frac{k}{P}t + C$ multiplied thru by (-1) to invert the argument of the log.

$\Rightarrow \left|\frac{P}{y} - 1\right| = e^{-k/P\,t + C}$

$\Rightarrow \frac{P}{y} - 1 = Ce^{-\frac{k}{P}t}$

$\Rightarrow y = \frac{P}{1 + Ce^{-\frac{k}{P}t}}$

$900 = \frac{1000}{1 + 11.5e^{-.1006t}}$

$\Rightarrow t = 7.6$
or 3:36 pm

(c) $P = 1000$, $y(0) = 80$; $y(4) = 500$

$80 = \frac{1000}{1 + C} \Rightarrow 1 + C = \frac{1000}{80} \Rightarrow C = \frac{1000}{80} - 1 = 11.5$

$500 = \frac{1000}{1 + 11.5e^{-\frac{k}{1000}(4)}} \Rightarrow \frac{\frac{1000}{500} - 1}{11.5} = e^{-k/1000\,(4)}$

$\Rightarrow k = -\frac{1000}{4}\ln\left(\frac{1}{11.5}\right)$

$\boxed{k = 610.6}$

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12 Logistic Curve Fitting.

using the TI 84

$L2$ = years since 1955

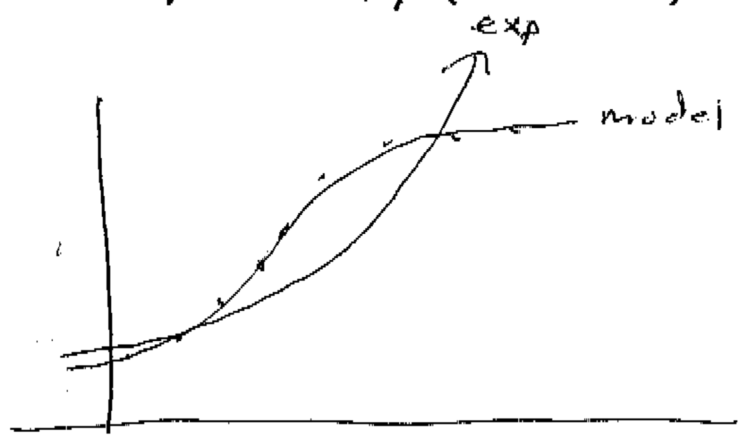
$L4$ = Pop − 29000

Logistic L_2, L_4, Y_1

Logistic model so $P(t) = \frac{11103}{1+12.3e^{-0.147t}} + 29000$

ExpReg L_2, L_4, Y_2

Exponential model $P(t) = 1094(1.0668)^t + 29000$



$\boxed{9.4\,HW \quad 7/8}$

$\boxed{14}$

(a) $\frac{dy}{dt} = ky^{1+c}$

$\Rightarrow \frac{dy}{y^{1+c}} = k\,dt$

$\Rightarrow -\frac{1}{c} y^{-c} = kt + A$

$\Rightarrow y^{-c} = -ckt + A$

$\Rightarrow y = (A - ckt)^{-1/c}$ $\quad$ $y(0) = y_0$

$\Rightarrow y(0) = A^{-1/c} = y_0 \Rightarrow A = y_0^{-c}$

$\Rightarrow y = (y_0^{-c} - ckt)^{-1/c}$

(b) Find doomsday.

solve $y_0^{-c} - ckt = 0$

$\Rightarrow \frac{1}{y_0^c} = ckt$

$\Rightarrow t = \frac{1}{cky_0^c}$

$\lim_{t \to \frac{1}{cky_0^c}^-} \frac{1}{\sqrt[c]{\frac{1}{y_0^c} - ckt}} = \infty$

denom $\mapsto 0$.

(c) Find doomsday if $y' = ky^{1.01}$

$y_0 = 2$.

$16 = (2^{-0.01} - 0.01k(3))^{-1/0.01}$

$\frac{\sqrt[-100]{16} - 2^{-0.01}}{-0.01(3)} = k = 0.6813$

so doomsday is

$t = \frac{1}{0.01(0.6813)2^{0.01}}$

$= 145.77$ mo.

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15 $$\frac{dP}{dt} = \underbrace{0.08}_{\text{initial growth rate}} P\left(1 - \frac{P}{\underset{\text{karrying kapacity}}{1000}}\right) - \underset{\text{constant rate @ which the pop. is harvested.}}{15}$$

(a) (-15) represents the constant rate @ which fish are harvested.

(b) The pop. is stable when $\frac{dP}{dt} = 0$.

$$0 = 0.08P - \frac{0.08P^2}{1000} - 15$$

$$\Rightarrow 0 = 80P - 0.08P^2 - 15000$$

$$\Rightarrow 0 = 0.08P^2 - 80P + 15000$$

$$\Rightarrow P = \frac{80 \pm \sqrt{6400 - 4(0.08)(15000)}}{2(0.08)}$$

$= 250$ or 750.