9.1: Intro to O.D.E.

A differential equation is an eqt. that contains an unknown fct & some of its derivatives. These are very applied w/ apps including:

(1) Population growth.
Radioactive decay
Free fall
Newton's Law of Cooling
RL circuits (Resistor & Inductor)
(2) Spring motion

Each eqt. mixes fct. values & ROC of the fct values.

(1) Population Growth:

(A) Under certain conditions, a population P grows @ a rate proportional to the size of the population.

$$\frac{dP}{dt} = kP$$

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(B) most populations don't grow w/o bound. so, how do we model a population growth is approximately proportional to population size when P is small. But, the population size decreases above a certain pt:

$$(\text{small } P) \qquad \frac{dP}{dt} \approx kP$$

$$(P > k) \qquad \frac{dP}{dt} < 0$$

Logistic eqt: $\frac{dP}{dt} = kP\left(1 - \frac{P}{k}\right)$

Q: what does k look like in real life.

(2) Spring motion. (Hooke's Law, undamped)

$$F = -kx$$

$$\Rightarrow ma = -kx$$

$$\Rightarrow m\frac{d^2x}{dt^2} = -kx \qquad \text{(2nd order O.D.E.)}$$

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Specific v. General Solutions.

Ex1: Solve $\frac{dy}{dt} = \sin t$

$$y = -\cos t + C$$ (a general solution).

Ex2: Construct the ODE.

(skip?) a) The RoC of the area of a disk over time is proportional to the product of r & the roc of r.

$$\frac{dA}{dt} = k \cdot r \frac{dr}{dt}$$

b) The acceleration of a particle is inversely proportional to its velocity.

$$\frac{d^2x}{dt^2} = \frac{k}{\frac{dx}{dt}} \quad \text{or} \quad x'' = \frac{k}{x'}$$

Ex 3: Show $y = \tan(t)$ is a solution to $y' = 1 + y^2$.

Q: Is this the only solution? Try...

$$y = \alpha \tan(t + \beta) + \gamma$$

Ex 4: A fct y satisfies $\frac{dy}{dt} = y^4 - 6y^3 + 5y^2$.

(a) What are the const. solutions.

(b) For what y is y increasing?
decreasing?

Ex 5: Show $y = \frac{1}{\frac{t^3}{3} + c}$ is a solution to $\frac{dy}{dt} = -t^2 y^2$. (What does c do?)

Q: What does an initial value do? (IVP).