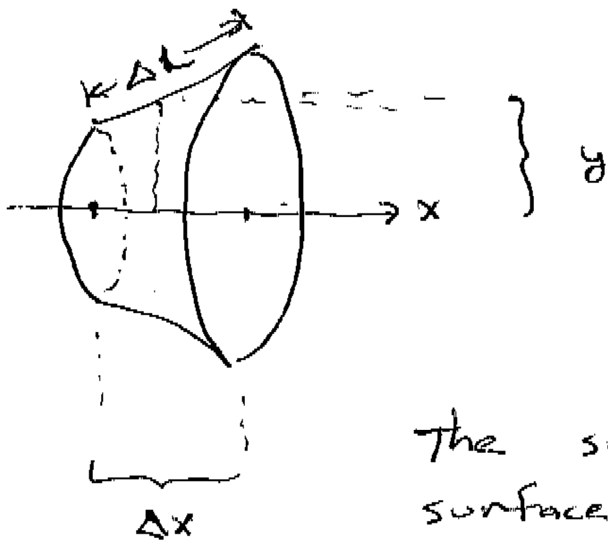
8.2: Area of a surface of revolution.

recall $\Delta L = \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\,\Delta x$ and

$$dL = \sqrt{1 + (f'(x))^2}\,dx$$

if we rotate about the x-axis



$$\Delta S = 2\pi y\,\Delta L$$

$$dS = 2\pi y\sqrt{1 + (y')^2}\,dx$$

The surface area of the surface obtained by rotating $y = f(x)$ on $[a, b]$ about the x-axis...

$$S = \int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\,dx$$

& if $x = g(y)$ on $[c, d]$ is rotated about the y-axis

$$S = \int_c^d 2\pi g(y)\sqrt{1 + (g'(y))^2}\,dy.$$

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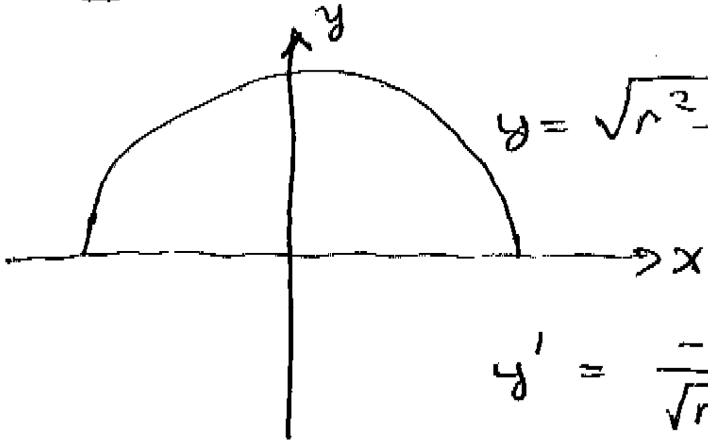
ex1: Verify the SA of a sphere.



$$y = \sqrt{r^2 - x^2}$$

$$y' = \frac{-x}{\sqrt{r^2-x^2}}$$

$$SA = \int_{-r}^{r} 2\pi\sqrt{r^2-x^2}\left(1 + \left(\frac{x}{\sqrt{r^2-x^2}}\right)^2\right)^{1/2} dx$$

$$= 4\pi\int_0^r \sqrt{r^2-x^2}\sqrt{\frac{r^2-x^2+x^2}{r^2-x^2}}\, dx$$

$$= 4\pi\int_0^r \sqrt{r^2-x^2}\cdot\frac{r}{\sqrt{r^2-x^2}}\, dx$$

$$= 4\pi r^2.$$

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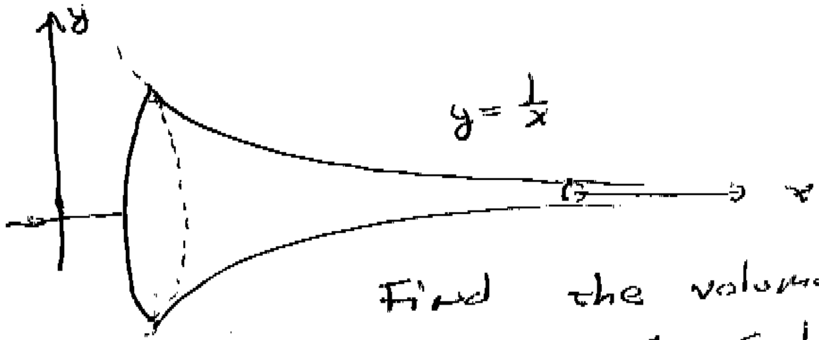
ex2: Gabriel's Trumpet



Find the volume & surface area of Gabriel's Trumpet.

$$V = \int_1^\infty \pi \left(\frac{1}{x}\right)^2 dx$$

$$= \int_1^\infty \pi x^{-2}\, dx$$

$$= \lim_{t\to\infty} \left(\left[-\pi \frac{1}{x} \right]_1^t \right)$$

$$= \lim_{t\to\infty} \left(\pi - \frac{\pi}{t} \right)$$

$$= \pi$$

$$SA = \int_1^\infty 2\pi \left(\frac{1}{x}\right) \sqrt{1 + \left(\frac{-1}{x^2}\right)^2}\, dx$$

$$\frac{x^4+1}{x^4}$$

$x^2 = \tan\theta$

$2x\,dx = \sec^2\theta\, d\theta$

$$= \int_1^\infty 2\pi \cdot \frac{1}{x^3} \sqrt{x^4+1}\, dx$$

$$= \int \pi \cot^2\theta \cdot \sec\theta \cdot \sec^2\theta\, d\theta$$

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$$= \pi \int \frac{\cos^2\theta}{\sin^2\theta \cos^3\theta}\, d\theta$$

$$= \pi \int \frac{\cos\theta}{\sin^2\theta(1-\sin^2\theta)}\, d\theta \qquad v = \sin\theta, \quad dv = \cos\theta\, d\theta$$

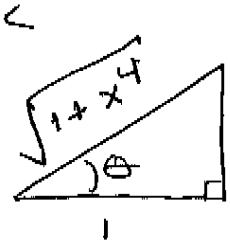
$$= \pi \int \frac{dv}{v^2(1-v^2)}$$

$$= \pi \int \left(\frac{1}{v^2} + \frac{1/2}{1-v} + \frac{1/2}{1+v} \right) dv$$

$$= \pi \left(-\frac{1}{v} + \frac{1}{2}\ln(1-v) + \frac{1}{2}\ln(1+v) \right) + C$$

$$= \pi \left(-\csc\theta + \frac{1}{2}\ln(\underbrace{1-\sin^2\theta}_{\cos^2\theta}) \right) + C$$

$$= \pi \left(-\frac{\sqrt{1+x^4}}{x^2} + \frac{1}{2}\ln\left(\frac{1}{1+x^4}\right) \right) + C$$



$u = x^2$