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7.8: Improper Integrals

Ex1: Find the area under $y = \frac{1}{x^3}$ on $[1, t]$

we define $\int_1^{\infty} \frac{dx}{x^3} = \lim_{t\to\infty} \int_1^t \frac{dx}{t^3}$

Ex2: $\int_1^{\infty} \frac{dx}{x}$

Ex3: $\int_{-\infty}^{0} 2x e^{-x^2} dx$

Ex4: $\int_{-\infty}^{\infty} 2x e^{-x^2} dx$

The p-test: $\int_1^{\infty} \frac{1}{x^p} dx$

Ex5: $\int_1^9 \frac{dx}{\sqrt[3]{x-9}}$

end of day 1

Ex6: $\int_{-1}^{1} \frac{e^x}{e^x - 1} dx$

convergence vs. divergence

Ex7: $\int_0^1 \frac{dx}{\sqrt{x}}$ (p-test)

Ex8: $\int_0^1 \ln(x) dx$

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Comparison test for improper integrals.

Suppose f and g are cont. w/ $f(x) \geq g(x) \geq 0$ for $x \geq a$, then.

a) If $\int_a^\infty f(x)\,dx$ is convergent...

b) If $\int_a^\infty g(x)\,dx$ is divergent...

Ex 9: $\int_1^\infty \frac{x}{\sqrt{1+x^2}}\,dx$

Ex 10: $\int_0^1 \frac{e^{-x}}{\sqrt{x}}\,dx$

Ex 11: $\int_1^\infty \frac{2+e^{-x}}{x}\,dx$

Ex 12: $\int_0^{\pi/4} \frac{dx}{x \sin x}$.