

Integration Strategies.

Some we know. - those on the table.

Some we don't

substitution.

parts.

trig. integrals.

trig. substitution.

partial fractions

Test 4

No calculators

study old tests.

NOTE ON HYPERBOLICS.

8.3 #14 and $-\operatorname{sech}^{-1}(x)$.

Strategy

- 1) Do you know it?
- 2) Can you simplify it?
- 3) Will substitution work?
- 4) What does it look like?

PARTS

LIATE

↓
Trig
Integral

↓
Trig
Substitution

↓
Partial
Fractions.

~~etc.~~

can it be
avoided?

becomes.

8.3: 2, 3, 4, 5, 6, 8, 9...

Table of Integration Formulas Constants of integration have been omitted.

- | | |
|---|--|
| 1. $\int x^n dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$ | 2. $\int \frac{1}{x} dx = \ln x $ |
| 3. $\int e^x dx = e^x$ | 4. $\int a^x dx = \frac{a^x}{\ln a}$ |
| 5. $\int \sin x dx = -\cos x$ | 6. $\int \cos x dx = \sin x$ |
| 7. $\int \sec^2 x dx = \tan x$ | 8. $\int \csc^2 x dx = -\cot x$ |
| 9. $\int \sec x \tan x dx = \sec x$ | 10. $\int \csc x \cot x dx = -\csc x$ |
| *11. $\int \sec x dx = \ln \sec x + \tan x $ | *12. $\int \csc x dx = \ln \csc x - \cot x $ |
| *13. $\int \tan x dx = \ln \sec x $ | *14. $\int \cot x dx = \ln \sin x $ |
| *15. $\int \sinh x dx = \cosh x$ | *16. $\int \cosh x dx = \sinh x$ |
| 17. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$ | 18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$ |
| *19. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left \frac{x-a}{x+a}\right $ | *20. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2} $ |

key

Group Work 1, Section 8.4
Partial Fractions (Version 3)

1. Compute the following integrals:

(a) $\int \frac{dx}{x+1} = \ln|x+1| + C$

(b) $\int \frac{dx}{x+2} = \ln|x+2| + C$

(c) $\int \frac{dx}{x^2+4} = \frac{1}{2} \arctan\left(\frac{x}{2}\right) + C$

2. Factor $x^4 + 3x^3 + 6x^2 + 12x + 8$. (Hint: see above)

$(x+1)(x+2)(x^2+4)$

3. Compute $\int \frac{20x^2 dx}{x^4 + 3x^3 + 6x^2 + 12x + 8}$

$20x^3 = A(x+2)(x^2+4) + B(x+1)(x^2+4) + (Cx+D)(x^2+4)$

$0 = 8A + 4B + 2D$

$0x = 2Cx + 4Ax + 4Bx + 3Dx \Rightarrow$

$0x^2 = 2Ax^2 + Bx^2 + 3Cx^2 + Dx^2$

$20x^3 = \int \frac{x^3 + Bx^2 + Cx + D}{x^4 + 3x^3 + 6x^2 + 12x + 8} dx$

x	$\begin{bmatrix} 8 & 4 & 0 & 2 & 0 \\ 4 & 4 & 2 & 3 & 0 \\ 2 & 1 & 3 & 1 & 20 \\ 1 & 1 & 1 & 0 & 20 \end{bmatrix}$	$A = $	4 4
x^2		$B = $	-10 -10
x^3		$C = $	6 6
		$D = $	4 4

4. Compute $\int \frac{x^3 + Bx^2 + Cx + D}{x^4 + 3x^3 + 6x^2 + 12x + 8} dx$. (Hint: what is the degree of the numerator or denominator?)

$= \int 1 dx + \int \frac{20x^2}{x^4 + 3x^3 + 6x^2 + 12x + 8} dx$

$= x + A //$