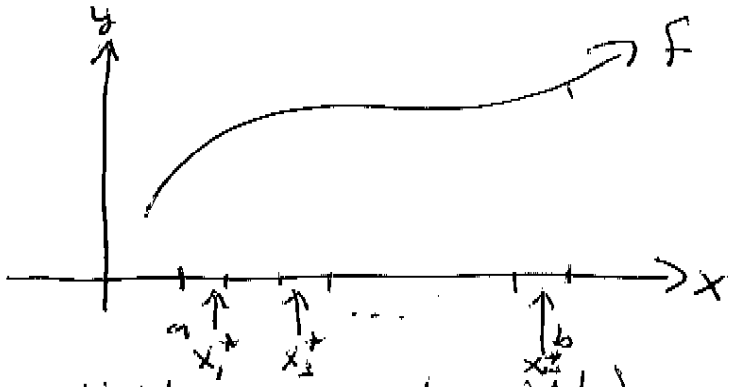
6.5 : Ave. value of a fct

To average n values $y_1, \dots, y_n$

$$y_{ave} = \frac{y_1 + y_2 + \dots + y_n}{n} = \frac{y_1}{n} + \frac{y_2}{n} + \dots + \frac{y_n}{n}$$

If we want to find the average of a fct $f(x)$ on $[a, b]$



(1) subdivide (equal widths) $\Delta x = \frac{b-a}{n} \Rightarrow n = \frac{b-a}{\Delta x}$

(2) sample points

(3) differential fct elements

$$f_{ave} \approx \frac{f(x_1^*)}{n} + \dots + \frac{f(x_n^*)}{n} = \frac{f(x_1^*) + \dots + f(x_n^*)}{n}$$

$$= \frac{f(x_1^*) + \dots + f(x_n^*)}{\frac{b-a}{\Delta x}}$$

$$= \frac{1}{b-a}\left(f(x_1^*)\Delta x + \dots + f(x_n^*)\Delta x \right)$$

$$= \frac{1}{b-a} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

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(4) limit of the Riemann sums

$$f_{ave} = \frac{1}{b-a} \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

(5) write as a definite integral.

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx$$

Ex1: Find the average value of $f(x) = (x-3)^2$ on $[2, 5]$

$$f_{ave} = \frac{1}{5-2}\int_2^5 (x-3)^2\,dx$$

$$= \frac{1}{5-2}\cdot\left[\frac{1}{3}(x-3)^3\right]_2^5$$

$$= \frac{1}{9}\left(8 - (-1)\right)$$

$$= 1.$$

MVT for Integrals: If f is cont. on $[a,b]$, then $\exists\ c \in [a,b]$ s.t.

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

or

$$\int_a^b f(x)dx = f(c)(b-a).$$

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Ex1 rev: Find c s.t. $f(c) = f_{ave}$

solve $f(c) = f_{ave}$

$\Rightarrow (c-3)^2 = 1$

$\Rightarrow c - 3 = \pm 1$

$\Rightarrow c = 3 \pm 1$

$\Rightarrow c = 2$ or $c = 4$

Draw the picture.