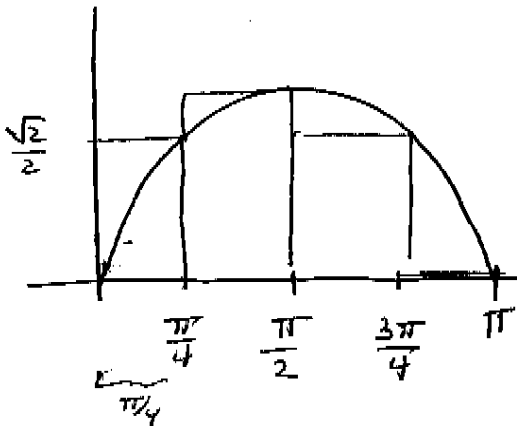


Practice Problems:

1.) Estimate $\int_0^\pi \sin(\theta) d\theta$ using R_4 . Give an exact answer (not a decimal approximation).



$$R_4 = \frac{\pi}{4} \left(2 \cdot \frac{\sqrt{2}}{2} + 1 \right)$$

$$= \frac{\pi}{4} (\sqrt{2} + 1)$$

2.) Use the definition of the definite integral to calculate $\int_{-1}^3 (2x^2 + 5x) dx$.

$$\Delta x = \frac{4}{n} ; x_i = \frac{4i}{n} - 1 ; x_i^2 = \frac{16i^2}{n^2} - \frac{8i}{n} + 1$$

$$\int_{-1}^3 (2x^2 + 5x) dx$$

$$= \lim_{n \rightarrow \infty} \left(2 \sum_{i=1}^n \left(\frac{4i}{n} - 1 \right)^2 \cdot \frac{4}{n} + 5 \sum_{i=1}^n \left(\frac{4i}{n} - 1 \right) \cdot \frac{4}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{128}{n^3} \sum_{i=1}^n i^2 + \frac{32}{n^2} \sum_{i=1}^n i - \frac{12}{n} \sum_{i=1}^n 1 \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{128}{n^3} \frac{n(n+1)(2n+1)}{6} + \frac{32}{n^2} \frac{n(n+1)}{2} - \frac{12}{n} \cdot n \right)$$

$$= \frac{128}{3} + \frac{32}{2} - 12 = \frac{116}{3}$$

3.) Evaluate the indefinite integral $\int \sin^5(3z) \cos(3z) dz$.

Let $u = \sin(3z)$

$$\frac{du}{3} = \cos(3z) dz$$

$$= \int \frac{1}{3} u^5 du$$

$$= \frac{1}{18} u^6 + C$$

$$= \frac{1}{18} \sin^6(3z) + C$$

4.) A car speeds up at a constant rate from 10 to 70 mph over a period of half an hour. Its fuel efficiency (in miles per gallon) at various speeds is shown in the table. Make left, right, and midpoint estimates of the quantity of fuel used during the half hour. Round to four decimal places.

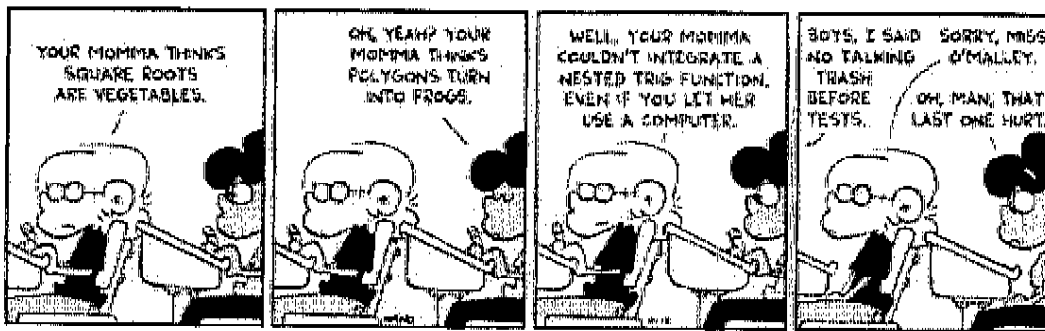
	$t=0$	5	10	15	20	25	$t=30$
Speed (mph)	10	20	30	40	50	60	70
Fuel efficiency (mpg)	15	18	21	23	24	25	26

Left endpoints.

$$\text{Gal} \approx 10 \left(\frac{1}{12}\right) \left(\frac{1}{15}\right) + 20 \left(\frac{1}{12}\right) \left(\frac{1}{18}\right) + \dots + 60 \left(\frac{1}{12}\right) \left(\frac{1}{25}\right)$$

$$T \cdot \frac{\text{mi}}{T} \cdot \frac{\text{gal}}{\text{mi}}$$

≈ 0.786 gal. (make sense... 30 min @ an average of 35 mph is 17.5 mi. The car gets around 20 mpg, so it would use less than



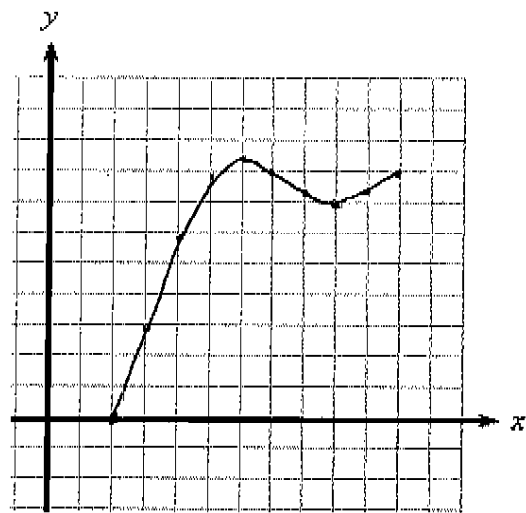
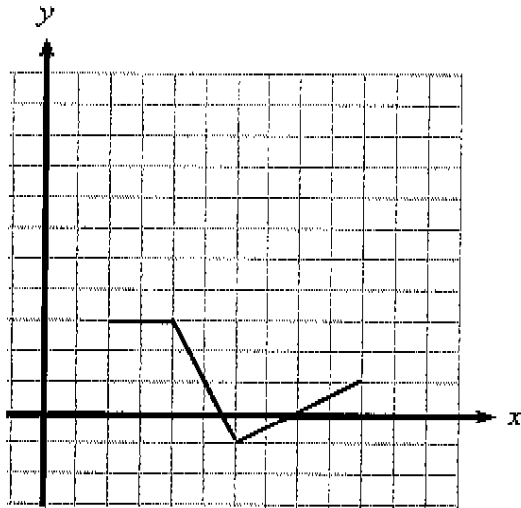
a gallon).

5.) Evaluate the definite integral $\int_0^8 t\sqrt{t+1} dt$, if it exists.

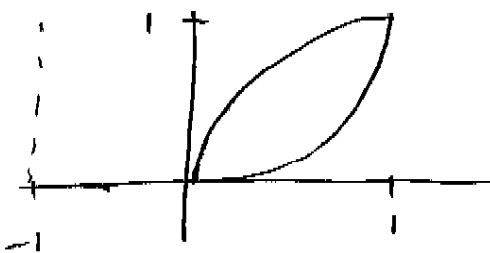
Let $u = t+1$
 $du = dt$

$$\begin{aligned} &= \int_1^9 (u-1)\sqrt{u} du \\ &= \int_1^9 (u^{3/2} - u^{1/2}) du \\ &= \left. \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} \right|_1^9 \\ &= \left(\frac{2}{5} (243) - \frac{2}{3} (27) \right) \\ &\quad - \left(\frac{2}{5} - \frac{2}{3} \right) \\ &= \frac{1192}{15} \end{aligned}$$

6.) Let $g(x) = \int_2^x f(t) dt$ on $2 \leq x \leq 10$ for the function $f(x)$ portrayed in the given graph. Given what you know, sketch a graph of $g(x)$ on the interval.

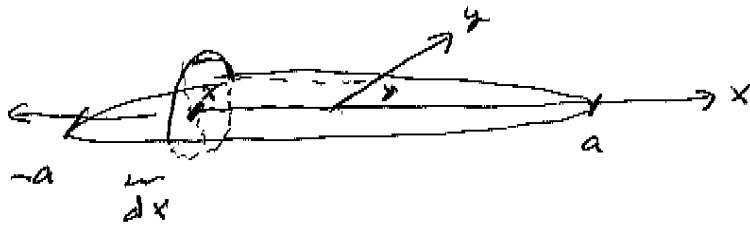


7.) Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = \sqrt{x}$ about the line $x = -1$



$$\begin{aligned}
 V &= 2\pi \int_0^1 (\cancel{1}x + 1)(\sqrt{x} - x^2) dx \\
 &= 2\pi \int_0^1 x^{3/2} - x^3 + x^{1/2} - x^2 dx
 \end{aligned}$$

8.) Find the volume of the solid formed in the following manner. The elliptical region with boundary curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is intersected with circular cross-sections perpendicular to the ellipse. The diameter of each circular cross-section is in the plane of the ellipse. (football.)



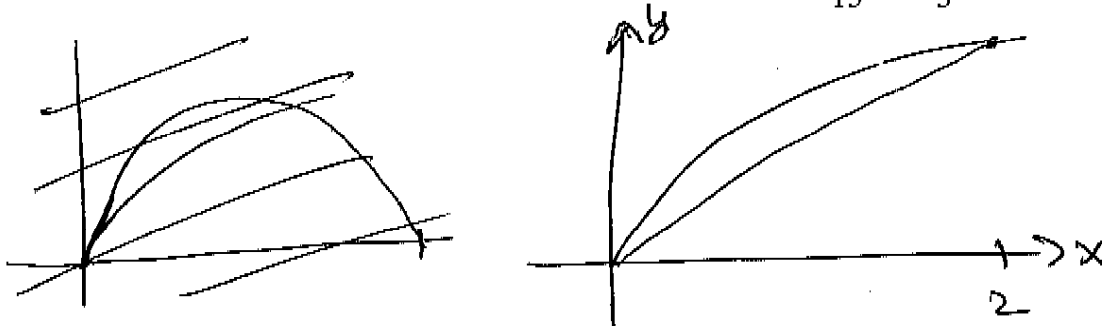
$$V = \int_{-a}^a \pi \left(\frac{b}{a} \sqrt{a^2 - x^2} \right)^2 dx$$

$$\frac{y}{b} = \pm \sqrt{1 - \frac{x^2}{a^2}}$$

$$y = \pm \frac{b}{a} \sqrt{a^2 - x^2}$$

9.) Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$ and $y = x - ax^2$ (when $a = \frac{1}{4}(2 - \sqrt{2})$) on $0 \leq x \leq 2$ about the x-axis.

Evaluate the integral in terms of a . Verify the volume is $\frac{14\pi}{15} - \frac{2\sqrt{2}\pi}{5}$.



$$V = \pi \int_0^2 \left[(\sqrt{x})^2 - (x - ax^2)^2 \right] dx$$