7.1.2

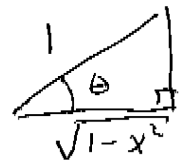
$\boxed{\text{Partial fractions}}$ OR $\boxed{\text{Integration by Parts}}$

$$\int \frac{du}{1-u^2} = \int \frac{\frac{1}{2}}{1+u} + \frac{\frac{1}{2}}{1-u}\, du$$
$$= \frac{1}{2}\ln|1+u| - \frac{1}{2}\ln|1-u|$$

Example 1: Use Trig Substitution (from 7.3) to find $\int \frac{\sqrt{1-x^2}}{x}\,dx$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$



$$\int \frac{\sqrt{1-x^2}}{x}\,dx = \int \frac{\sqrt{1-\sin^2\theta}}{\sin\theta}\cdot\cos\theta\, d\theta$$
$$= \int \frac{\cos^2\theta}{\sin\theta}\, d\theta$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$$= \int \frac{\cos^2\theta \cdot \sin\theta}{1-\cos^2\theta}\, d\theta$$
$$= \int \frac{u^2}{1-u^2}\, du$$
$$= \int \frac{u^2-1+1}{1-u^2}\, du$$
$$= \int \frac{-(1-u^2)}{1-u^2} + \frac{1}{1-u^2}\, du$$
$$= -u + \frac{1}{2}\ln\left|\frac{1+u}{1-u}\right| + C$$
$$= -\cos\theta + \frac{1}{2}\ln\left|\frac{1+\cos\theta}{1-\cos\theta}\right| + C$$
$$= -\sqrt{1-x^2} + \frac{1}{2}\ln\left|\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\right| + C$$

Example 2: Use Trig Substitution (from 7.3) to find $\int \sqrt{t^2-6t+13}\,dt$

	t	-3
t	t^2	$-3t$
-3	$-3t$	9

$t^2 - 6t + 13$

$= t^2 - 6t + 9 + 4$

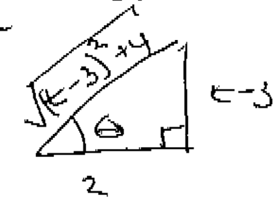
$= (t-3)^2 + 4$

$$\int \sqrt{t^2-6t+13}\,dt$$
$$= \int \sqrt{(t-3)^2+4}\,dt$$

$t - 3 = 2\tan\theta$

$dt = 2\sec^2\theta\, d\theta$

AND $\frac{t-3}{2} = \tan\theta$



$$= \int 2\sqrt{\tan^2\theta+1}\cdot 2\sec^2\theta\, d\theta$$
$$= 4\int \sec^3\theta\, d\theta$$
$$= 4\left(\frac{1}{2}(\sec\theta\tan\theta + \ln|\sec\theta+\tan\theta|)\right)$$
$$= 2\left(\frac{\sqrt{(t-3)^2+4}}{2}\cdot\frac{t-3}{2} + \ln\left|\frac{\sqrt{(t-3)^2+4}}{2} + \frac{t-3}{2}\right|\right) + C$$