1. Conservative; piecewise smooth; path independent; conservative; simple; curl; divergence; flux; irrotational

2. (a) C is the circle of radius 2 centered at the origin in the xy-plane. It has positive orientation if it is parametrized in the counterclockwise direction as viewed from above.

   (b) If S_1 is the disk of radius 2 centered at the origin with upward normal, then $C = \partial S_1$ with the same orientation.

   (c) By Stokes' Theorem, $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = \oint_{C=\partial S} \mathbf{F} \cdot d\mathbf{r} = \oint_{C=\partial S_1} \mathbf{F} \cdot d\mathbf{r} = \iint_{S_1} \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$. Since $z = 0$ on S_1, $\mathbf{F} = (x - 3y)\,\mathbf{i} + (y + 2x)\,\mathbf{j} + 0\mathbf{k}$, $\mathbf{n} = \mathbf{k}$, and $\operatorname{curl} \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x - 3y & y + 2x & 0 \end{vmatrix} = |5\mathbf{k}| = 5$.

   So $\iint_{S_1} \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = 5\left(\text{area of disk } x^2 + y^2 \le 4\right) = 20\pi$.

3. $\mathbf{r}(\phi, \theta) = 3 \sin\phi \cos\theta\, \mathbf{i} + 3 \sin\phi \sin\theta\, \mathbf{j} + 4 \cos\phi\, \mathbf{k}$

4. (a) $x^2 + \dfrac{y^2}{4} = 1$ can be parametrized counterclockwise by $\mathbf{F}(t) = \langle \cos t, 2 \sin t \rangle$, $0 \le t \le 2\pi$.

   (b) Note that if $\mathbf{F} = 0\mathbf{i} + x^3 y\mathbf{j}$, then $\partial Q/\partial x - \partial P/\partial y = 3x^2 y$. So

$$\begin{aligned} \iint_{0 \le x^2+y^2/4 \le 1} 3x^2 y\, dA &= \int_{\text{boundary}} P\, dx - Q\, dy = \int_0^{2\pi} \cos^3 2 + 2 \sin t\,(2 \cos t\, dt) \\ &= 4\int_0^{2\pi} \cos^4 t \sin t\, dt \quad (\text{Let } u = \cos t,\ du = -\sin t\, dt) \\ &= -4\int_{-1}^{1} u^4\, du = 0 \end{aligned}$$

   This can also be found directly, as follows:

$$\int_0^1 \int_{-2\sqrt{1-x^2}}^{2\sqrt{1-x^2}} 3x^2 y\, dy\, dx = \int_0^1 \left[\tfrac{3}{2} x^2 y^2\right]_{-2\sqrt{1-x^2}}^{2\sqrt{1-x^2}} dx = 0$$

5. $\mathbf{F}(x, y, z) = -\dfrac{y}{x^2 + y^2}\,\mathbf{i} + \dfrac{x}{x^2 + y^2}\,\mathbf{j} + z\,\mathbf{k}$.

   (a) $\mathbf{r}(t) = \langle \cos t, \sin t, 0 \rangle$, $0 \le t \le 2\pi$.
   $\oint_C \mathbf{F} \cdot d\mathbf{r} = \int_0^{2\pi} \langle -\sin t, \cos t, 0 \rangle \cdot \langle -\sin t, \cos t, 0 \rangle\, dt = \int_0^{2\pi} 1\, dt = 2\pi$

   (b) $\operatorname{curl} \mathbf{F}(x, y, z) = \left\langle 0 - 0, 0 - 0, \dfrac{y^2 - x^2}{(x^2 + y^2)^2} - \dfrac{y^2 - x^2}{(x^2 + y^2)^2} \right\rangle = \langle 0, 0, 0 \rangle$ everywhere except the z-axis (where $\mathbf{F}$ is undefined).

(c) Since **F** is not defined along the z-axis, we cannot find a surface such that C is its boundary and **F** is defined everywhere on the surface.

Another reason: If $P = -\frac{y^2}{x^2+y^2}$, then $\frac{\partial P}{\partial y} = \frac{y^2-x^2}{(x^2+y^2)^2}$, which does not have a limit at $(0, 0)$ and is discontinuous there.

6. (a) If we assume an outward normal, then by the Divergence Theorem,
$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iiint_B \operatorname{div}\mathbf{F}\cdot d\mathbf{V} = \iiint_B d\mathbf{V}$ (since $\operatorname{div}\mathbf{F} = 1$), which is simply the volume of B.

(b) Parametrize the sphere by $r(\theta,\phi) = \langle\cos\theta\sin\phi, \sin\theta\sin\phi, \cos\phi\rangle$. Then
$r_\theta \times r_\phi = \langle\sin^2\phi\cos\theta, \sin^2\phi\sin\theta, -\sin\phi\cos\phi\rangle$, which points outward, and
$f(r(\theta,\phi)) = \langle\cos\theta\sin\phi, 0, 0\rangle$, so
$\iint_S \mathbf{F}\cdot d\mathbf{S} = \int_0^\pi\int_0^{2\pi}\cos^2\theta\sin^3\phi\,d\theta\,d\phi = \int_0^{2\pi}\cos^2\theta\,d\theta\int_0^\pi\sin^3\phi\,d\phi = \frac{4}{3}\pi$.

(c) $\operatorname{div}\mathbf{F} = 12$, so $\iint_S \mathbf{F}\cdot d\mathbf{S} = 12\cdot\text{Volume}(B) = 16\pi$.

7. Using Green's Theorem with $P = \sin x + xy^2$ and $Q = e^y + \frac{1}{2}x^2$, we get

$$\text{Work} = \int_C P\,dx + Q\,dy = 2\int_{\text{Square}}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dA = 2\int_0^1\int_0^1 (x - 2xy)\,dx\,dy$$
$$= 2\int_0^1\left[\tfrac{1}{2}x^2 - x^2y\right]_0^1 dy = 2\int_0^1\left(\tfrac{1}{2} - y\right)dy = 2\left[\tfrac{1}{2}y - \tfrac{1}{2}y^2\right]_0^1 = 0$$

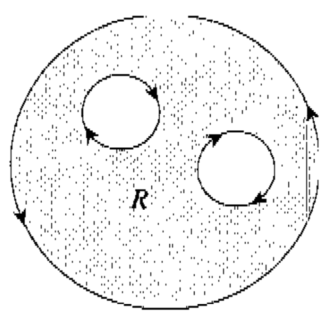
8. (a) $\operatorname{curl}\mathbf{F} = \mathbf{0}$

(b) Let $f(x,y,z) = x^2+y^2+z^2$. Then $\mathbf{F} = \nabla f$ and by the Fundamental Theorem for line integrals, $\int_C \mathbf{F}\cdot d\mathbf{r} = f(a) - f(0) = a_1^2 + a_2^2 + a_3^2 = \mathbf{a}\cdot\mathbf{a}$.

9. (a) If $\mathbf{G} = P\mathbf{i} + Q\mathbf{j}$, then computation gives
$$\operatorname{curl}\mathbf{G} = \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\mathbf{k} = \left[\frac{(y-3)^2-(x-2)^2}{(x-2)^2+(y-3)^2} - \frac{(y-3)^2-(x-2)^2}{(x-2)^2+(y-3)^2}\right]\mathbf{k} = \mathbf{0} \text{ for } (x,y)\neq(2,3).$$
Or: $\operatorname{curl}\mathbf{G} = \mathbf{0}$ since the vector field **G** is just **F** translated to the right 2 units and up 3 units.

(b) $\mathbf{F}+\mathbf{G}$ is defined at all points except $(0,0)$ and $(2,3)$, since **F** is not defined at $(0,0)$ and **G** is not defined at $(2,3)$. At all other points, $\operatorname{curl}(\mathbf{F}+\mathbf{G}) = \operatorname{curl}\mathbf{F} + \operatorname{curl}\mathbf{G} = \mathbf{0}$, and $\mathbf{F}+\mathbf{G}$ is irrotational.

10. (a)



(b) $\frac{1}{2}\left(\int_{\partial R} y\,dx - x\,dy\right) = \text{area}(R) = \pi\cdot 4^2 - 2\cdot\pi\cdot 1^2 = 14\pi$

(c) $\frac{1}{2}\left(\int_{\partial R_1} y\,dx - x\,dy\right) = 0$, since the two smaller circles have equal areas and opposite orientations.

11. If $x = 2\cos t \sin s$, $y = \sin t \sin s$, $z = \frac{1}{\sqrt{2}} \cos s$, then

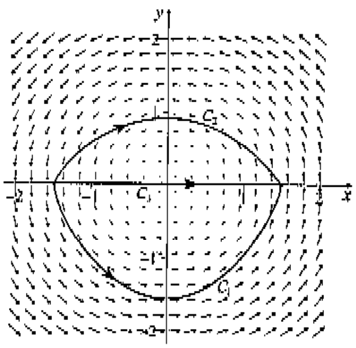
$\frac{1}{4}x^2 + y^2 = \frac{1}{4}\left(4\cos^2 t \sin^2 s\right) + \sin^2 t \sin^2 s = \sin^2 s \left(\cos^2 t + \sin^2 t\right) = \sin^2 s$, and so

$\frac{1}{4}x^2 + y^2 + 2z^2 = \sin^2 s + 2\left(\frac{1}{\sqrt{2}} \cos s\right)^2 = \sin^2 s + 2\left(\frac{1}{2}\cos^2 s\right) = \sin^2 s + \cos^2 s = 1.$

12. (a) Since **F** points in almost the same direction as vectors tangent to the path from A to B, $\mathbf{F}(t) \cdot \mathbf{r}'(t) > 0$ everywhere along the path, and hence the line integral $\int \mathbf{F} \cdot d\mathbf{r} > 0$.

(b) Since **F** is perpendicular to the path from C to D at every point, we have $\mathbf{F}(t) \cdot \mathbf{r}'(t) = 0$ everywhere along the path, and hence the line integral $\int \mathbf{F} \cdot d\mathbf{r} = 0$.

13.



14. (a) When $u = \frac{\pi}{2}$, $x = 0$, $y = 2 + \sin v$, and $z = \frac{\pi}{2} + \cos v$, so the center is $\left(0, 2, \frac{\pi}{2}\right)$ and the radius is 1.

(b) The normal vector at $P\left(0, 3, \frac{\pi}{2}\right)$ is $3\mathbf{j}$.

15. (a) $x = t$, $y = \sqrt{1 - t^2} \sin s$, and $z = \sqrt{1 - t^2} \cos s$ gives $y^2 + z^2 = 1 - t^2 = 1 - x^2$, or $x^2 + y^2 + z^2 = 1$, a sphere of radius 1.

(b) $x = t^2$, $y = s^2$, and $z = s^2 + t^2 = y + x$, $x \geq 0$, $y \geq 0$, part of a plane above the first quadrant.

16. If $z = \theta$, then $x = r\cos\theta$, $y = r\sin\theta$, $z = \theta$ and $\mathbf{R}(r, \theta) = r\cos\theta\,\mathbf{i} + r\sin\theta\,\mathbf{j} + \theta\mathbf{k}$, $r \geq 0$, $0 \leq \theta \leq 2\pi$ is a parametrization.

17. Both surfaces have the same boundary curve C: $x^2 + y^2 = 9$, $z = 0$. By Stokes' Theorem, $\iint_{S_1} \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{r} = \iint_{S_2} \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$.

18. $\mathbf{F}(u, v) = \langle u + v, u - v, 2u + 3v \rangle \Rightarrow \mathbf{F}_u = \langle 1, 1, 2 \rangle$, $\mathbf{F}_v = \langle 1, -1, 3 \rangle$, and $\mathbf{F}_u \times \mathbf{F}_v = \langle 5, -1, -2 \rangle$. Thus the surface area is $\int_0^1 \int_0^1 |\mathbf{F}_u \times \mathbf{F}_v| \, du \, dv = \int_0^1 \int_0^1 \sqrt{30} \, du \, dv = \sqrt{30}$.