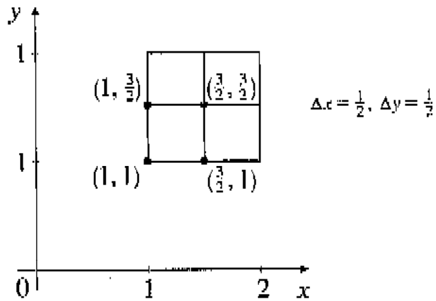
1. $f(x, y) = x^y$

(a)



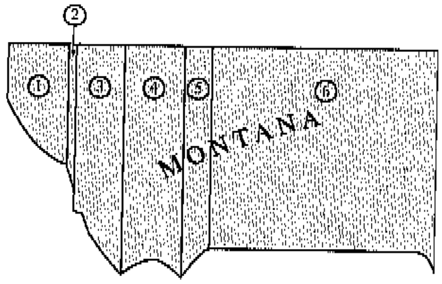
$$\int_1^2 \int_1^2 f(x, y)\, dy\, dx \approx f(1, 1) \cdot \tfrac{1}{2} \cdot \tfrac{1}{2} + f\left(\tfrac{3}{2}, 1\right) \cdot \tfrac{1}{2} \cdot \tfrac{1}{2} + f\left(1, \tfrac{3}{2}\right) \cdot \tfrac{1}{2} \cdot \tfrac{1}{2} + f\left(\tfrac{3}{2}, \tfrac{3}{2}\right) \cdot \tfrac{1}{2} \cdot \tfrac{1}{2}$$
$$= \tfrac{1}{4}\left[1 + \tfrac{3}{2} + 1 + \left(\tfrac{3}{2}\right)^{3/2}\right] \approx \tfrac{1}{4}(5.3375) \approx 1.344$$

(b) This estimate is an underestimate since the function is increasing in the x- and y-directions as x and y go from 1 to 2.

2. (a) $$\int_0^{\pi/2} \int_0^{\pi/2} \frac{1}{(1 + \sin^2 x)(1 + \sin^2 y)}\, dx\, dy = \left(\int_0^{\pi/2} \frac{dx}{1 + \sin^2 x}\right)\left(\int_0^{\pi/2} \frac{dy}{1 + \sin^2 y}\right)$$
$$= \left(\frac{\pi}{2\sqrt{3}}\right)^2 = \frac{\pi^2}{12}$$

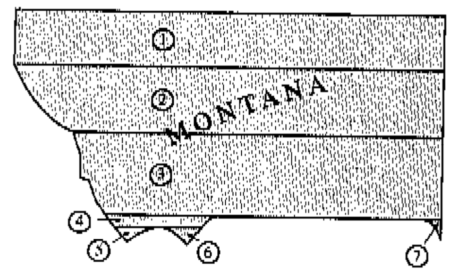
(b) $$\int_0^{\pi/2}\int_0^{\pi/2}\int_{1/(1+\sin^2 x)}^{1/(1+\sin^2 y)} dz\,dx\,dy = \int_0^{\pi/2}\int_0^{\pi/2}\left(\frac{1}{1+\sin^2 y}-\frac{1}{1+\sin^2 x}\right)dx\,dy$$
$$= \int_0^{\pi/2}\int_0^{\pi/2}\left(\frac{1}{1+\sin^2 y}\right)dx\,dy - \int_0^{\pi/2}\int_0^{\pi/2}\left(\frac{1}{1+\sin^2 x}\right)dx\,dy$$
$$= \frac{\pi^2}{4\sqrt{3}} - \frac{\pi^2}{4\sqrt{3}} = 0$$

3. (a)



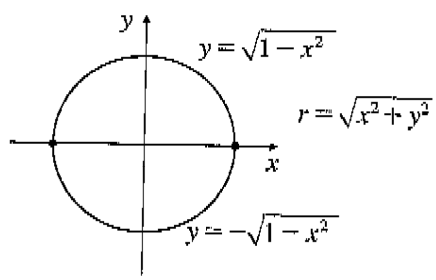
Type I

(b)



Type II

4. $\int_0^{2\pi}\int_0^1 r^2\,dr\,d\theta = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\sqrt{x^2+y^2}\,dy\,dx$



5. $\int_0^{2\pi}\int_1^2 \cos(r^2)\,r\,dr\,d\theta = \pi(\sin 4 - \sin 1)$

6. $x^2+2y^2=1$

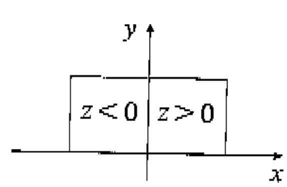
(a) $r^2(\cos^2\theta + 2\sin^2\theta) = r^2(1+\sin^2\theta) = 1, r \ge 0$

(b) $$\int_0^{2\pi}\int_0^{1/\sqrt{1+\sin^2\theta}} r\,dr\,d\theta$$

7. Since the parallelepiped has volume 60, we have $\iiint_R 10\,dV = 600$.

$\iiint_R x\,dV = 12\int_0^5 x\,dx = 12\left(\frac{25}{2}\right) = 150$

8. (a) $\int_0^1\int_{-1}^1\int_0^{xy} 1\,dz\,dx\,dy = \int_0^1\int_{-1}^1 xy\,dx\,dy = \int_0^1\left[\frac{1}{2}x^2y\right]_{-1}^{1}dy = 0$. The region between $z=0$ and $z=xy$ in the first quadrant is above the xy-plane, while a symmetric region is below the xy-plane in the second quadrant.



(b) $\int_0^1 \int_{-1}^1 \int_0^{|xy|} 1\, dz\, dx\, dy = 2\int_0^1 \int_0^1 \int_0^{xy} dz\, dx\, dy = 2\int_0^1 \left[\frac{1}{2}x^2 y\right]_0^1 dy = 2\int_0^1 \frac{1}{2}y\, dy = \frac{1}{2}y^2\Big]_0^1 = \frac{1}{2}$.
This is the total volume between $z = 0$ and $z = xy$. Because we take the absolute value, the volumes do not cancel.

9. Since the surface area is 4π, we need to find ϕ so that the area lit is π.
$\pi = \int_0^\phi \int_0^{2\pi} \sin\phi\, d\theta\, d\phi = 2\pi \int_0^\phi \sin\phi\, d\phi = 2\pi(-\cos\phi + \cos 0)$, so $\frac{1}{2} = 1 - \cos\phi \Rightarrow \cos\phi = \frac{1}{2}$
$\Rightarrow \phi = \frac{\pi}{3}$.

10. (a) $\int_0^1 \int_{-\sqrt{1-(x-1)^2}}^{x} (x+y)\, dy\, dx + \int_1^2 \int_{-\sqrt{1-(x-1)^2}}^{2-x} (x+y)\, dy\, dx$

(b) $\int_{-1}^0 \int_{1-\sqrt{1-y^2}}^{1+\sqrt{1-y^2}} (x+y)\, dx\, dy + \int_0^1 \int_y^{x} (x+y)\, dx\, dy$. [← 2-y]

Note that the circular part of the curve is $y = -\sqrt{1-(x-1)^2}$ or $x = 1 \pm \sqrt{1-y^2}$.

11. (a) $\int_0^{\pi/2} \int_0^{1/(\sin\theta+\cos\theta)} r^3 \sin\theta\cos\theta\, dr\, d\theta + \int_{\pi/2}^{\pi} \int_0^{1/(\sin\theta-\cos\theta)} r^3 \sin\theta\cos\theta\, dr\, d\theta$

(b) 0

12. $\displaystyle\iint_R \frac{1}{9-(x^2+y^2)^{3/2}}\, dA$

(a) $\frac{1}{2}(4\pi - \pi) = \frac{3\pi}{2}$

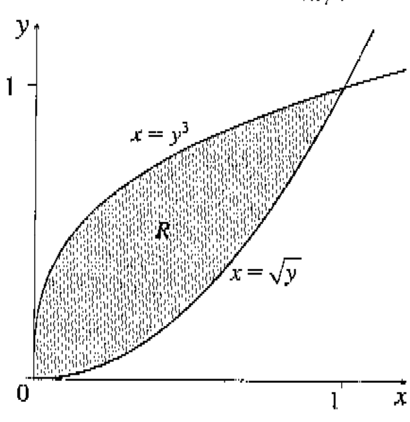
(b) Since the semicircles satisfy $x^2+y^2 = 1$ and $x^2+y^2 = 4$, we have on $x^2+y^2=1$,
$\dfrac{1}{9(x^2+y^2)^{3/2}} = \dfrac{1}{8}$ and on $x^2+y^2 = 4$, $\dfrac{1}{9(x^2+y^2)^{3/2}} = 1$.

(c) A lower bound is the minimum value times the area, that is, $\frac{1}{8}\cdot\frac{3\pi}{2} = \frac{3\pi}{16}$.
An upper bound is the maximum value times the area, that is, $1\cdot\frac{3\pi}{2} = \frac{3\pi}{2}$.

13. (a) $\left\{(r,\theta) \mid 0 \le r \le 1, \frac{\pi}{4} \le \theta \le \frac{7\pi}{4}\right\}$

(b) $\iint_{\text{Pac-Man}} x\, dA = \int_0^1 \int_{\pi/4}^{7\pi/4} r^2\cos\theta\, d\theta\, dr = \int_0^1 \left[r^2\sin\theta\right]_{\pi/4}^{7\pi/4} dr = -\sqrt{2}\int_0^1 r^2\, dr = -\frac{\sqrt{2}}{3}$
$\iint_{\text{Pac-Man}} y\, dA = \int_0^1 \int_{\pi/4}^{7\pi/4} r^2\sin\theta\, d\theta\, dr = \int_0^1 \left[-r^2\cos\theta\right]_{\pi/4}^{7\pi/4} dr = 0$

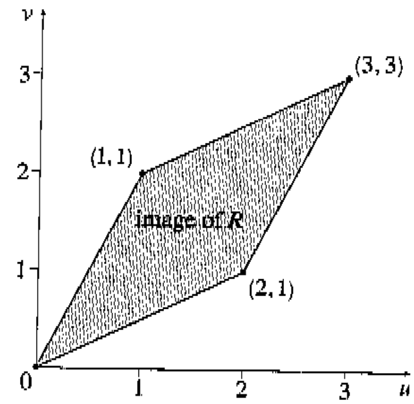
14. (a)



(b) $\int_0^1 \int_{y^3}^{\sqrt{y}} \int_0^{xy} dz\, dx\, dy = \int_0^1 \int_{x^2}^{\sqrt[3]{x}} \int_0^{xy} dz\, dy\, dx$

15. $x = 2u + v$, $y = u + 2v$

(a)

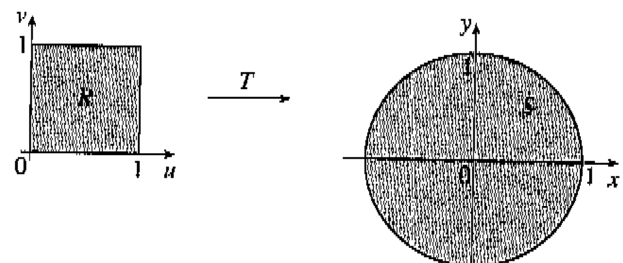


(b) The Jacobian is $\left\| \begin{matrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{matrix} \right\| = \left\| \begin{matrix} 2 & 1 \\ 1 & 2 \end{matrix} \right\| = 3$, so

$$\begin{aligned} \iint_S (3x+2y)\,dA &= \int_0^1 \int_0^1 [3(2u+v)+2(u+2v)]\,3\,du\,dv \\ &= 3\int_0^1 \left[3u^2+3uv+u^2+4uv\right]_0^1 dv \\ &= 3\int_0^1 (3+3v+1+4v)\,dv = 3\left[4v+\tfrac{7}{2}v^2\right]_0^1 = \tfrac{45}{2} \end{aligned}$$

16. $\int_1^9 \int_0^{\pi/2} \int_{\pi/6}^{\pi/4} \rho^2 \sin\phi\,d\phi\,d\theta\,d\rho = \frac{\pi}{2}\int_1^9 \left[-\rho^2\cos\phi\right]_{\pi/6}^{\pi/4} d\rho = \frac{\pi}{2}\int_1^9 \frac{\sqrt{3}-\sqrt{2}}{2}\rho^2\,d\rho$
$= \frac{\sqrt{3}-\sqrt{2}}{2}\left[\frac{1}{3}\rho^3\right]_1^9 = \frac{182}{3}\left(\sqrt{3}-\sqrt{2}\right)\pi$

17. (a)



T maps the unit square in the uv-plane to the unit circle in the xy-plane.

(b) The area of S is π.

18. $f_{\text{ave}} = \dfrac{\int_{-1}^1 \int_{-1}^1 (ax+by)\,dy\,dx}{\int_{-1}^1 \int_{-1}^1 1\,dy\,dx} = \dfrac{\int_{-1}^1 2ax\,dx}{4} = 0$