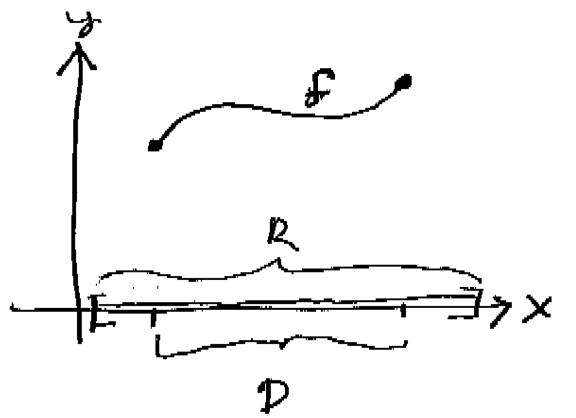
15.3 : Double Integrals over General Regions.

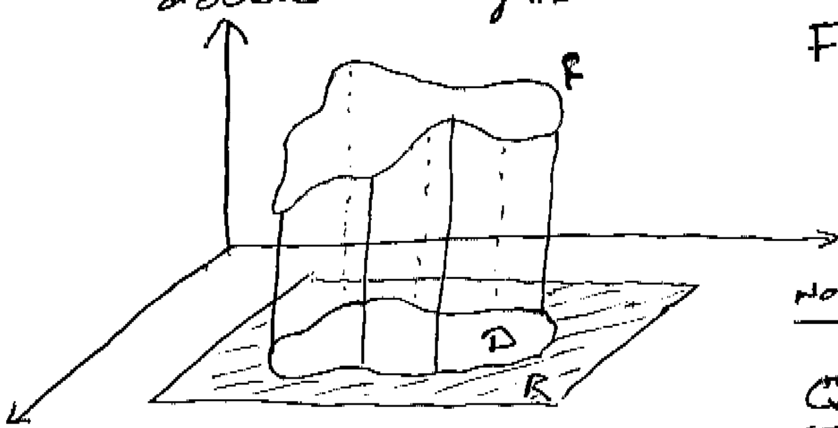
single integrals



$$F(x) = \begin{cases} f(x), & x \in D \\ 0, & x \in R \text{ and } x \notin D. \end{cases}$$

Note: D is bounded (otherwise we have an improper integral).

double integrals



$$F(x,y) = \begin{cases} f(x,y), & (x,y) \in D \\ 0, & (x,y) \in R \text{ and } (x,y) \notin D \end{cases}$$

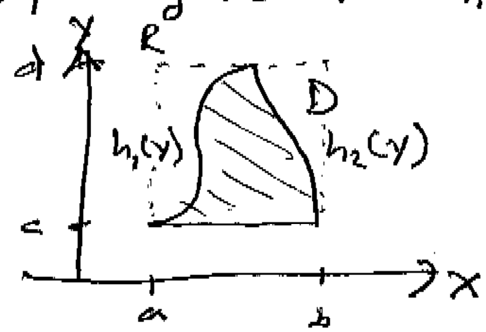
Note: D is bounded

Q: Is F cont?

☆ Fubini's Thm Applies.

If $D = \{(x,y) \mid c \le y \le d \text{ and } h_1(y) \le x \le h_2(y)\}$

That is:



$$\iint_D F(x,y)\,dA = \iint_R F(x,y)\,dA$$

$$= \int_c^d \int_a^b F(x,y)\,dx\,dy$$

$$= \int_c^d \int_{h_1(y)}^{h_2(y)} F(x,y)\,dx\,dy$$

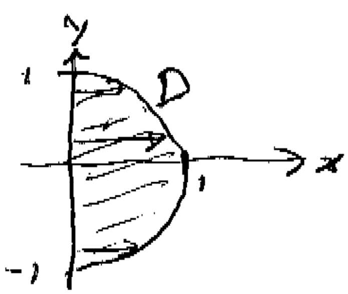
$$= \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy.$$

Note: the text refers to regions D of this sort as Type II regions.

Note: A double integral is a number associated w/ a fcn f & a region D, and this number exists & has a meaning independent of an particular method of computing it.

An iterated integral is a double integral plus a built in computational procedure.

Ex 1: $\iint_D xy^2\,dA$ where D is enclosed by $x=0$ & $x=\sqrt{1-y^2}$



$$D = \{(x,y) \mid -1 \le y \le 1 \text{ AND } 0 \le x \le \sqrt{1-y^2}$$

15.3
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$$\hookrightarrow \iint_D x y^2 \, dA = \int_{-1}^{1} \int_{0}^{\sqrt{1-y^2}} x y^2 \, dx \, dy.$$

$$= \int_{-1}^{1} \left[\frac{x^2}{2} y^2 \right]_0^{\sqrt{1-y^2}} dy$$

$$= \int_{-1}^{1} \frac{1-y^2}{2} \cdot y^2 \, dy$$

$$= \left[\frac{y^3}{6} - \frac{y^5}{10} \right]_{-1}^{1}$$

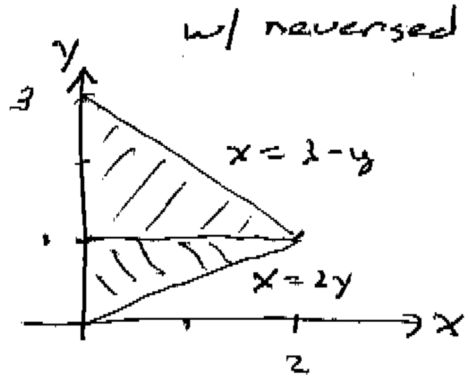
$$= \left(\frac{1}{6} - \frac{1}{10} \right) - \left(-\frac{1}{6} + \frac{1}{10} \right)$$

$$= \frac{1}{3} - \frac{2}{10} = \frac{4}{30} = \frac{2}{15}$$

Ex 2: In evaluating a double integral over D, the following was found.

$$\iint_D f(x,y)\, dA = \int_0^1 \int_0^{2y} f(x,y)\, dx\, dy + \int_1^3 \int_0^{3-y} f(x,y)\, dx\, dy.$$

sketch D & express the double integral w/ reversed order of integration.



$$\iint_D f(x,y)\, dA = \int_0^2 \int_{\frac{1}{2}x}^{3-x} f(x,y)\, dy\, dx$$

properties of double integrals.

$\boxed{11}$ If $m \le f(x,y) \le M$ in D, then

$$m\,A(D) \le \iint_D f(x,y)\,dA \le M\,A(D)$$

picture proof.