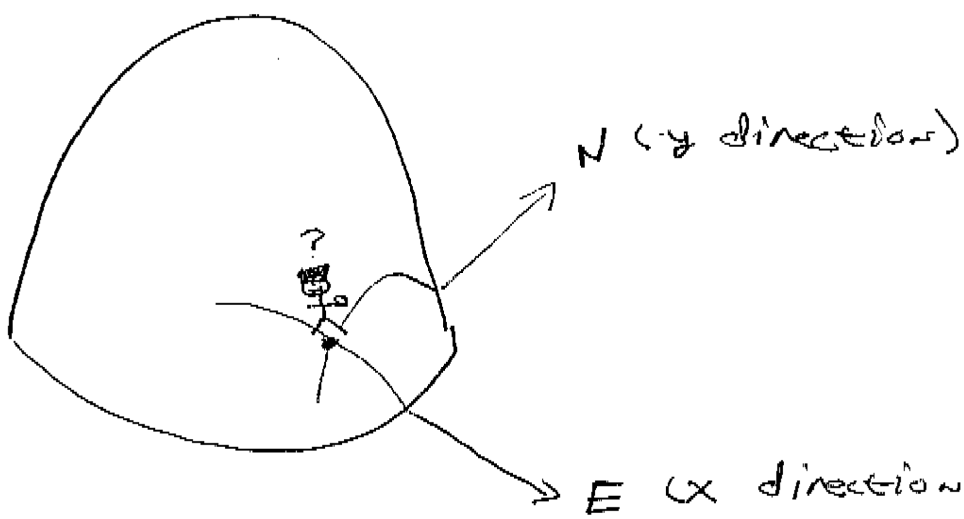
14.3: Partial Derivatives

Reading assignment covered the basics.

Picturing the partial derivative: Imagine you are on the side of a hill w/ a compass.



- my path N has an uphill ($f_y > 0$) slope
- my path E has an downhill ($f_x < 0$) slope.
- regardless, when looking one direction, I ignore (hold constant) other directions.
- NS & EW aren't the only directions for travel, but they're all we consider @ this point.

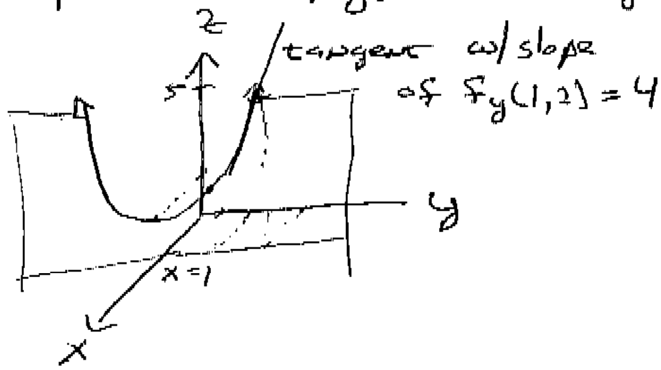
partials

Ex 1: Consider $f(x,y) = xy^2 + x^3$

a) find $f_x(x,y) = y^2 + 3x^2$

b) find & interpret $f_y(1,2) = 4$

to interpret, notice we are on the path $f(1,y)$ when $y=2$. $(f(1,y) = y^2+1)$



The surface & plane $x=1$ intersec on the parabola $z=y^2+1$; $x=1$. The slope of the tangent @ $(1,2,5)$ is 4.

Ex 2: If $u = xe^{x/y}$

a) $u_x(x,y) = u_x = e^{xy} + xe^{x/y} \cdot \frac{1}{y}$

b) $\frac{\partial u}{\partial y} = xe^{x/y} \cdot \frac{-x}{y^2}$

Ex 3: Find $\frac{\partial z}{\partial x}$ of $\cos(x \cdot y \cdot z) = 3x + 2y + z$.

implicit differentiation.

$\Rightarrow -\sin(xyz) \cdot \left(yz + xy\frac{\partial z}{\partial x}\right) = 3 + \frac{\partial z}{\partial x}$

$\Rightarrow -\sin(xyz)\,yz - xy\sin(xyz)\frac{\partial z}{\partial x} = 3 + \frac{\partial z}{\partial x}$

$\Rightarrow -3 - yz\sin(xyz) = \frac{\partial z}{\partial x}\left(1 + xy\sin(xyz)\right)$

$\Rightarrow \frac{\partial z}{\partial x} = -\frac{3 + yz\sin(xyz)}{1 + xy\sin(xyz)}$

14.8
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As w/ derivatives of fcts of 1 variable, we may be able to take higher order derivatives. However, we now must choose which variable to differentiate wrt @ each step.

Ex 4: consider $f(x) = x^3 e^{5y} + y\sin(2x)$

a) $f_x(x,y) = 3x^2 e^{5y} + 2y\cos(2x)$

$f_y = 5x^3 e^{5y} + \sin(2x)$

b) $f_{xx} = 6x e^{5y} - 4y\sin(2x)$

1st 2nd

$f_{xy} = 15x^2 e^{5y} + 2\cos(2x)$

$$\frac{\partial^2 f}{\partial y^2} = 25x^3 e^{5y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y} = 15x^2 e^{5y} + 2\cos(2x)$$

Notes

- pure v. mixed partials.
- mixed partials are sometimes equal.

Clairaut's Thm: Suppose f is defined on a disk D that contains the point (a,b). If the fcts f_{xy} & f_{yx} are both cont ~~at~~ on D, then $f_{xy}(a,b) = f_{yx}(a,b)$ (proof in Appendix F).

Clairaut's Thm gives a condition where the mixed partials are indeed equal.

While I enjoy PDE's such as the wave eqt & the Cobb-Douglas fct is a cool econ example, we will skip over them for now & come back to these topics ~~should~~ if & when the need arises.

Ex 5: Clarifying Clairaut's Thm (p 807)

Clarifying Clairaut's Theorem

Consider $f(x, y, z) = x^2 \cos(y^3 + z^2)$.

1. Why do we know that $f_{zyyxxx} = 0$ without doing any computation?

2. Do we also know, without doing any computation, that $f_{xyzzz} = 0$? Why or why not?

3. Suppose that $f_x = 3x + ay^2$, $f_y = bxy + 2y$, $f_y(1, 1) = 3$, and f has continuous mixed second partial derivatives f_{xy} and f_{yx}.

(a) Find values for a and b and thus equations for f_x and f_y. *Hint:* What does Clairaut's Theorem say about the mixed partial derivatives of a function? When does the theorem apply?

(b) Can you find a function $F(x, y)$ such that $\dfrac{\partial F}{\partial x} = f_x$ in part (a)?

(c) Can you find a function $G(x, y) = F(x, y) + k(y)$ such that $\dfrac{\partial G}{\partial y} = f_y$ in part (a)? What is $k(y)$?

(d) What is $\dfrac{\partial G}{\partial x}$? Can you now find $f(x, y)$?