

Quadratic Equations II

Practice Test #10: The sum of the solutions of a quadratic equation is $3/2$. The product of its solutions is 3. What is the equation?

Solution 1 (long): Let α & β be solutions to an arbitrary quadratic equation,

$$\alpha + \beta = \frac{3}{2} \quad \text{and} \quad \alpha\beta = 3$$

$$\Rightarrow \alpha \left(\frac{3}{2} - \alpha \right) = 3$$

$$\Rightarrow \frac{3}{2}\alpha - \alpha^2 = 3$$

$$\Rightarrow 3\alpha - 2\alpha^2 = 6$$

$$\Rightarrow 2\alpha^2 - 3\alpha + 6 = 0$$

$$\Rightarrow \alpha = \frac{3 \pm \sqrt{9 - 4(2)(6)}}{2(2)}$$

$$\Rightarrow \alpha = \frac{3 \pm \sqrt{-39}}{4}$$

$$\Rightarrow \alpha = \frac{3 \pm i\sqrt{39}}{4}$$

$$\text{wlog, } \alpha = \frac{3 + i\sqrt{39}}{4}$$

$$\text{AND } \beta = \frac{3 - i\sqrt{39}}{4}$$

so, a quadratic w/these solutions could be factored as

$$0 = a \left(x - \frac{3 + i\sqrt{39}}{4} \right) \left(x - \frac{3 - i\sqrt{39}}{4} \right)$$
 for $a \neq 0$

$$\Rightarrow 0 = a \left(x^2 - \frac{3 - i\sqrt{39}}{4} x - \frac{3 + i\sqrt{39}}{4} x + \frac{3 + i\sqrt{39}}{4} \cdot \frac{3 - i\sqrt{39}}{4} \right)$$

$$\Rightarrow 0 = a \left(x^2 - \frac{3}{2} x - 3 \right)$$

Solution 2 (short): Let α & β
be solutions to the arbitrary quadratic

$$\Rightarrow a(x - \alpha)(x - \beta) = 0$$

$$\Rightarrow a(x^2 - \alpha x - \beta x + \alpha\beta) = 0$$

$$\Rightarrow a(x^2 - \underbrace{(\alpha + \beta)}_{\frac{3}{2}}x + \underbrace{\alpha\beta}_3) = 0$$

sum product

$$\Rightarrow a(x^2 - \frac{3}{2}x + 3) = 0$$