Section 9.2: Rational Exponents
Academic Systems

Exponent rules: Assuming that variables satisfy basic conditions outline in the text book, we have the following rules:

a.) $a^x a^y = a^{x+y}$

b.) $\frac{a^x}{a^y} = a^{x-y}$

c.) $\left(a^x\right)^y = a^{x \cdot y}$

d.) $a^{-x} = \frac{1}{a^x}$

e.) $a^0 = 1,\ a \neq 0$ and 0^0 is undefined.

f.) $x^{1/n} = \sqrt[n]{x}, n > 0$

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1.) Evaluate $\sqrt[5]{-32}$

$= \sqrt[5]{(-2)^5}$

$= -2$

2.) Evaluate $\sqrt[4]{\frac{625}{1296}}$

$= \sqrt[4]{\frac{5^4}{2^4\ 3^4}}$

$= \frac{5}{6}$

3.) Rewrite $\sqrt[5]{312^4}$ using rational exponents:

$= \sqrt[5]{(2^3 \cdot 3 \cdot 13)^4}$

$= \sqrt[5]{2^{12} \cdot 3^4 \cdot 13^4}$

$= 4\sqrt[5]{2^2 \cdot 3^4 \cdot 13^4}$

4.) Find $x^{\frac{1}{5}} \cdot x^{\frac{2}{3}}$

$= x^{\frac{1}{5} + \frac{2}{3}}$

$= x^{\frac{3}{15} + \frac{10}{15}}$

$= x^{13/15}$

5.) Simplify $\sqrt[3]{192a^3b^5c^9}$

$= \sqrt[3]{2^6 \cdot 3\, a^3 b^5 c^9}$

$= 4 \cdot 2^2 a b c^3 \sqrt[3]{3b^2}$

6.) Simplify $\sqrt[3]{128x} + 2\sqrt[3]{16x^4} - \sqrt[3]{54x}$

$= \sqrt[3]{2^7 x} + 2\sqrt[3]{2^4 x^4} - \sqrt[3]{3^3 \cdot 2x}$

$= 4\sqrt[3]{2x} + 4x\sqrt[3]{2x} - 3\sqrt[3]{2x}$

$= \sqrt[3]{2x}(4 + 4x - 3)$

$= (1 + 4x)\sqrt[3]{2x}$

7.) Simplify $3\sqrt{2y}\left(7\sqrt{10y} + 4\sqrt{3}\right)$

$= 21\sqrt{20y^2} + 12\sqrt{6y}$

$= 42y\sqrt{5} + 12\sqrt{6y}$

$=$

8.) Simplify $\left(5\sqrt{2y} - \sqrt{3x}\right)\left(5\sqrt{2y} + \sqrt{3x}\right)$

$= 25(2y) + 5\sqrt{6xy} - 5\sqrt{6xy} - 3x$

$= 50y - 3x$

9.) Simplify $\dfrac{x - \sqrt{3}}{x + \sqrt{3}}$

$= \dfrac{x - \sqrt{3}}{x + \sqrt{3}} \cdot \dfrac{x - \sqrt{3}}{x - \sqrt{3}}$

$= \dfrac{x^2 - 2\sqrt{3}x + 3}{x^2 - 3}$

10.) Evaluate the expression $\left(x^{-\frac{2}{3}} y^{\frac{3}{5}} z^{-\frac{4}{7}}\right)^3$ using only positive exponents.

$= \left(\dfrac{y^{3/5}}{x^{\frac{2}{3}} z^{4/7}}\right)^3$

$= \dfrac{y^{9/5}}{x^2 z^{12/7}}$