LESSON 10.1 – QUADRATIC EQUATIONS I

OVERVIEW

Here's what you'll learn in this lesson:

Solving by Factoring

a. The standard form of a quadratic equation

b. Putting a quadratic equation into standard form: $ax^2 + bx + c = 0$

c. Solving quadratic equations of the form $ax^2 + bx = 0$ by factoring

d. Solving quadratic equations of the form $ax^2 + bx + c = 0$ by factoring

Solving by Square Roots

a. Finding square roots

b. Solving quadratic equations of the form $ax^2 = b$

c. Solving quadratic equations of the form $(ax + b)^2 = c$

A baseball coach analyzes the path of a baseball to help his players with their batting. A building contractor is insulating the walls of an A-frame and needs to calculate the amount of insulation required to make the house more energy efficient. A police officer is measuring the length of skidmarks to determine how fast a driver was going.

Each of these people can find the information they are looking for by solving equations known as quadratic equations, or equations in one variable of degree two.

In this lesson you will learn how to solve quadratic equations by factoring and by using square roots.

EXPLAIN

SOLVING BY FACTORING

Summary

Quadratic Equations

A quadratic equation in one variable is an equation that can be written in the form $ax^2 + bx + c = 0$ where a, b, and c are any real numbers, and $a \neq 0$.

The variable in a quadratic equation can be any letter, not just x. For example: $3y^2 + 5y - 9 = 0$ is a quadratic equation.

Quadratic equations are also called second degree equations or equations of degree 2.

Every quadratic equation has two solutions. If the two solutions are equal, the quadratic equation is said to have a solution of multiplicity two.

A quadratic equation written in the form $ax^2 + bx + c = 0$ is said to be in standard form. This means the right side is 0, and the terms on the left side are written in descending order.

Solving Quadratic Equations by Factoring

You have learned how to solve linear equations like $2x - 7 = 0$. You have also learned how to factor polynomials. You can combine these skills to solve some quadratic equations by factoring. However, not all quadratic equations can be solved by factoring.

When you solve quadratic equations by factoring you use the zero product property. This property states that if the product of two numbers (or polynomials) is equal to 0, then one (or both) factors must be equal to 0.

That is, if $P \cdot Q = 0$, then $P = 0$ or $Q = 0$ (or both P and $Q = 0$).

For example: if $3 \cdot x = 0$, then $x = 0$

if $a \cdot b = 0$, then $a = 0$ or $b = 0$

if $y(y - 3) = 0$, then $y = 0$ or $y - 3 = 0$

To solve a quadratic equation by factoring:

1. Write the quadratic equation in standard form.
2. Factor the left side.
3. Use the zero product property to set each factor equal to 0.
4. Finish solving for x.

If the left side can't be factored when the equation is in standard form, then the equation can't be solved by factoring.

For example, to solve $4x^2 = 3x$ by factoring:

1. Write the equation in standard form. $4x^2 - 3x = 0$
2. Factor the left side. $x(4x - 3) = 0$
3. Use the zero product property to set each factor equal to 0. $x = 0$ or $4x - 3 = 0$
4. Finish solving for x. $x = 0$ or $4x = 3$, $x = \frac{3}{4}$

So the two solutions of the equation $4x^2 = 3x$ are $x = 0$ or $x = \frac{3}{4}$.

You can check these solutions by substituting them into the original equation.

Check $x = 0$:

Is $4(0)^2 = 3(0)$?

Is $0 = 0$? Yes.

Check $x = \frac{3}{4}$:

Is $4\left(\frac{3}{4}\right)^2 = 3\left(\frac{3}{4}\right)$?

Is $4\left(\frac{9}{16}\right) = \frac{9}{4}$?

Is $\frac{9}{4} = \frac{9}{4}$? Yes.

As another example, to solve $x^2 + 4x = -4$ by factoring:

1. Write the equation in standard form. $x^2 + 4x + 4 = 0$
2. Factor the left side. $(x + 2)(x + 2) = 0$
3. Use the zero product property to set each factor equal to 0. $x + 2 = 0$ or $x + 2 = 0$
4. Finish solving for x. $x = -2$ or $x = -2$

So the solution of the equation $x^2 + 4x = -4$ is $x = -2$. This is a solution of multiplicity two.

Check the solution by substituting it into the original equation.

Is $(-2)^2 + 4(-2) = -4$?

Is $4 - 8 = -4$?

Is $-4 = -4$? Yes.

Sample Problems

1. Solve $x^2 - 6x = 0$ for x:

   ☑ a. Write the equation in standard form. $x^2 - 6x = 0$

   ☐ b. Factor the left side. _____ $= 0$

   ☐ c. Use the zero product property to set each factor equal to 0. _____ $= 0$ or _____ $= 0$

   ☐ d. Finish solving for x. $x = 0$ or $x =$ _____

2. Solve $3x^2 + 10x = 8$ by factoring:

   ☐ a. Write the equation in standard form. _____ $= 0$

   ☐ b. Factor the left side. (_____)(_____) $= 0$

   ☐ c. Use the zero product property to set each factor equal to 0. (_____) $= 0$ or (_____) $= 0$

   ☐ d. Finish solving for x. $x =$ _____ or $x =$ _____

3. Solve $10x(x - 3) - x - 8 = 4 - 5x$ by factoring:

   ☐ a. Write the equation in standard form. _____ $= 0$

   ☐ b. Factor the left side. $2($_____$) = 0$
   $2($_____$)($_____$) = 0$

   ☐ c. Use the zero product property to set each factor equal to 0. (_____) $= 0$ or (_____) $= 0$

   ☐ d. Finish solving for x. $x =$ _____ or $x =$ _____

Answers to Sample Problems

b. $x(x - 6)$

c. x, $(x - 6)$ (in either order)

d. 6

a. $3x^2 + 10x - 8$

b. $3x - 2$, $x + 4$ (in either order)

c. $3x - 2$, $x + 4$ (in either order)

d. $\frac{2}{3}$, -4

a. $10x^2 - 26x - 12$

b. $5x^2 - 13x - 6$
$5x + 2$, $x - 3$ (in either order)

c. $5x + 2$, $x - 3$ (in either order)

d. $\frac{-2}{5}$, 3 (in either order)

SOLVING BY SQUARE ROOTS

Summary

Square Roots

Another way to solve some quadratic equations is to use the square root property.

Here are some facts about square roots that you will need to know before you can solve quadratic equations.

1. Every positive number b has two square roots, a positive number a and a negative number $-a$, where $a^2 = b$ and $(-a)^2 = b$.

   For example, the square roots of 81 are 9 and –9 because $9^2 = 81$ and $(-9)^2 = 81$.

2. The radical symbol, $\sqrt{\ }$, is used to represent the positive square root: $\sqrt{81} = 9$. A negative sign is included to represent the negative square root: $-\sqrt{81} = -9$.

   You can use the radical symbol to represent the square roots of every nonnegative number.

   For example, the square roots of 17 are $\sqrt{17}$ and $-\sqrt{17}$ because $\left(\sqrt{17}\right)^2 = 17$ and $\left(-\sqrt{17}\right)^2 = 17$.

3. The square root of a product is the product of the square roots. That is, $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$ where a and b are nonnegative real numbers.

   For example: $\sqrt{16 \cdot 9} = \sqrt{16} \cdot \sqrt{9} = 4 \cdot 3 = 12$.

4. The square root of a quotient is the quotient of the square roots. That is, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ where a and b are nonnegative real numbers and $b \neq 0$.

   For example: $\sqrt{\frac{225}{25}} = \frac{\sqrt{225}}{\sqrt{25}} = \frac{15}{5} = 3$

Note that these properties can be used to simplify radicals.

For example $\sqrt{45}$ can be simplified by writing it as:

$\sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

and $\sqrt{\frac{108}{4}}$ can be simplified by writing it as:

$\frac{\sqrt{108}}{\sqrt{4}} = \frac{\sqrt{108}}{\sqrt{4}} = \frac{\sqrt{36} \cdot \sqrt{3}}{\sqrt{4}} = \frac{6\sqrt{3}}{\sqrt{4}} = 3\sqrt{3}$

Solving Quadratic Equations by Square Roots

The square root property states that if $x^2 = a$ then $x = \sqrt{a}$ or $x = -\sqrt{a}$.

To solve a quadratic equation using the square root property:

1. Write the quadratic equation in the form $x^2 = a$.
2. Use the square root property: If $x^2 = a$ then $x = \sqrt{a}$ or $-\sqrt{a}$.
3. Finish solving for x.

For example, to solve $x^2 - 49 = 0$ for x:

1. Write the equation in the form $x^2 = a$. $x^2 = 49$
2. Use the square root property. $x = \sqrt{49}$ or $x = -\sqrt{49}$
3. Finish solving for x. $x = 7$ or $x = -7$

As another example, to solve $z^2 + 6z + 9 = 24$ for z:

1. Write the equation in the form $x^2 = a$. $(z + 3)^2 = 24$
2. Use the square root property. $z + 3 = \sqrt{24}$ or $z + 3 = -\sqrt{24}$
3. Finish solving for x. $z + 3 = 2\sqrt{6}$ or $z + 3 = -2\sqrt{6}$

   $z = -3 + 2\sqrt{6}$ or $z = -3 - 2\sqrt{6}$

Sample Problems

1. Solve for b: $b^2 = 32$

   ☑ a. Write the equation in the form $x^2 = a$. $b^2 = 32$

   ☐ b. Use the square root property. $b =$ _____ or $b =$ _____

   ☐ c. Finish solving for b. $b =$ _____ or $b =$ _____

2. Solve for x: $8x^2 = 128$

   ☐ a. Write the equation in the form $x^2 = a$. $x^2 =$ _____

   ☐ b. Use the square root property. $x =$ _____ or $x =$ _____

   ☐ c. Finish solving for x. $x =$ _____ or $x =$ _____

3. Solve for s: $5(3s - 8)^2 = 10$

   ☐ a. Write the equation in the form $x^2 = a$. $(3s - 8)^2 =$ _____

   ☐ b. Use the square root property. _____ $= \sqrt{2}$ or _____ $= -\sqrt{2}$

   ☐ c. Finish solving for s. $s =$ _____ or $s =$ _____

Answers to Sample Problems

b. $\sqrt{32}, -\sqrt{32}$ (in either order)

c. $4\sqrt{2}, -4\sqrt{2}$ (in either order)

a. 16 or $\frac{128}{8}$

b. $\sqrt{16}, -\sqrt{16}$

c. 4, –4

a. 2

b. $3s - 8, 3s - 8$

c. $\frac{8 + \sqrt{2}}{3}, \frac{8 - \sqrt{2}}{3}$

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain
Solving by Factoring

1. Write $x(2x-3)=5$ in standard form.
2. Solve $x^2-5x=0$ by factoring.
3. Solve $2y^2+8y=0$ by factoring.
4. Write $2x-3x(x-5)+1=7x-10$ in standard form.
5. Circle the equations below that are quadratic.

   $-11x^2=0$

   $2a(a+5)=4$

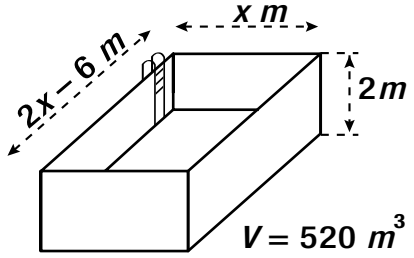
   $x(7x^2-2x+1)=0$

   $10x^2+3x-6=5x(2x+7)$

   $6x-9x^2=8$
6. Solve $x^2+x=12$ by factoring.
7. Solve $z^2-25=0$ by factoring.
8. Solve $4b^2-12b+9=0$ by factoring.
9. The average depth of the Huang's rectangular swimming pool is 2 meters. If the pool holds 520 m^3 of water, and the length of one side is 6 meters less than 2 times the length of the other side, what are the dimensions of the swimming pool?

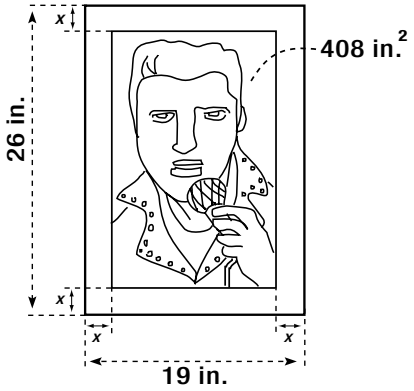
   Hint: Volume = depth · length · width

   $520 = 2 \cdot x \cdot (2x-6)$



10. Lucy just put a mat around her new Elvis poster. If the matted poster is 19-by-26 inches, and there is 408 $in.^2$ of the poster showing, how wide is the mat?

    Hint: Find x in this equation: $(19-2x)(26-2x)=408$



11. Solve $3x^2+4x=5+2x$ by factoring.
12. Solve $3r(4r-7)-2=4$ by factoring.

Solving by Square Roots

13. Solve $a^2=100$ using the square root property.
14. Simplify:

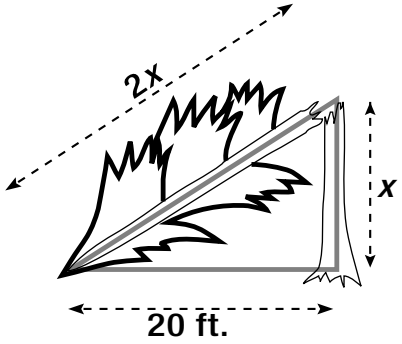
    a. $\sqrt{\frac{108}{16}}$

    b. $\sqrt{675}$

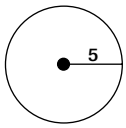
    c. $\sqrt{49+16}$
15. Solve $3x^2=108$ using the square root property.
16. Solve $c^2-112=0$ using the square root property.
17. Solve $2x^2-162=0$ using the square root property.
18. Solve $(x-6)^2=36$ using the square root property.
19. Solve $x^2-2x+1=75$ using the square root property.
20. Solve $9y^2=7$ using the square root property.

21. A tree is hit by lightning. The trunk of the tree breaks and the top of the tree touches the ground 20 ft. from the base of the tree. If the top part of the tree is twice as long as the bottom part, approximately how tall was the tree before it was hit by lightning?

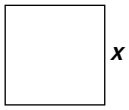
(Hint: In a right triangle, $a^2 + b^2 = c^2$, where a and b are the legs of the triangle, and c is the hypotenuse. So $20^2 + x^2 = (2x)^2$.)



22. Malia is making a cake and she is supposed to pour the batter into a circular pan whose diameter is 10 inches. She doesn't have a circular pan, but she knows that one of her square pans has the same base area as a 10-inch circular pan. What is the length of a side of the square pan?



Area is 25π square inches



Area is x^2 square inches

23. Solve $4x^2 - 36x + 81 = 5$ using the square root property.

24. Solve $4(z + 13)^2 = 17$ using the square root property.

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Solving by Factoring

1. Solve $x^2 + 7x = 0$ by factoring.
2. Solve $x^2 + 13x = 0$ by factoring.
3. Solve $6x^2 - 24x = 0$ by factoring.
4. Solve $5x^2 + 35x = 0$ by factoring.
5. Solve $8x^2 + 40x = 0$ by factoring.
6. Solve $10x^2 - 70x = 0$ by factoring.
7. Solve $7x^2 - 42x = 0$ by factoring.
8. Solve $x^2 + 8x + 7 = 0$ by factoring.
9. Solve $x^2 - 7x + 6 = 0$ by factoring
10. Solve $x^2 + 12x + 11 = 0$ by factoring.
11. Solve $x^2 + 12x + 35 = 0$ by factoring.
12. Solve $x^2 - 17x + 66 = 0$ by factoring.
13. Solve $x^2 + 9x + 18 = 0$ by factoring.
14. Solve $x^2 - 3x - 40 = 0$ by factoring.
15. Solve $x^2 - 7x - 18 = 0$ by factoring.
16. Solve $x^2 - 4x - 21 = 0$ by factoring.
17. Solve $x^2 - 5x = 150$ by factoring.
18. Solve $x^2 - 8x = 33$ by factoring.
19. Solve $x^2 - 3x = 10$ by factoring.
20. Solve $x^2 + 7x = 44$ by factoring.
21. Solve $x^2 + 2x = 120$ by factoring.
22. Solve $x^2 + 2x = 24$ by factoring.
23. Solve $x^2 - 3x = 54$ by factoring.
24. Solve $x^2 + 2x = 99$ by factoring.
25. Solve $x^2 - x = 30$ by factoring.
26. Solve $x^2 = 5x + 66$ by factoring.
27. Solve $x^2 = 3x + 180$ by factoring.
28. Solve $x^2 = 4x + 32$ by factoring.

Solving by Square Roots

29. Solve $x^2 = 100$ using the square root property.
30. Solve $x^2 = 81$ using the square root property.
31. Solve $x^2 = 256$ using the square root property.
32. Solve $x^2 = 144$ using the square root property.
33. Solve $x^2 = 48$ using the square root property.
34. Solve $x^2 = 50$ using the square root property.
35. Solve $x^2 = 32$ using the square root property.
36. Solve $5x^2 = 245$ using the square root property.
37. Solve $4x^2 = 324$ using the square root property.
38. Solve $3x^2 = 108$ using the square root property.
39. Solve $7x^2 = 126$ using the square root property.
40. Solve $2x^2 = 90$ using the square root property.
41. Solve $5x^2 = 60$ using the square root property.
42. Solve $5x^2 - 180 = 0$ using the square root property.
43. Solve $2x^2 - 162 = 0$ using the square root property.
44. Solve $3x^2 - 147 = 0$ using the square root property.
45. Solve $(x + 5)^2 = 49$ using the square root property.
46. Solve $(x - 4)^2 = 225$ using the square root property.
47. Solve $(x + 9)^2 = 81$ using the square root property.
48. Solve $(x + 8)^2 = 10$ using the square root property.
49. Solve $(x - 3)^2 = 13$ using the square root property.
50. Solve $(x - 2)^2 = 7$ using the square root property.
51. Solve $x^2 + 6x + 9 = 64$ using the square root property.
52. Solve $x^2 - 10x + 25 = 121$ using the square root property.
53. Solve $x^2 + 4x + 4 = 49$ using the square root property.
54. Solve $4x^2 + 28x + 49 = 32$ using the square root property.
55. Solve $25x^2 - 40x + 16 = 75$ using the square root property.
56. Solve $9x^2 - 30x + 25 = 18$ using the square root property.

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Write this quadratic equation in standard form and identify a, b, and c.

   $$1 + 2x(x - 8) = x + 3$$

2. Solve the equation $6x^2 - 24x = 0$ by factoring.

3. Circle the quadratic equations.

   $x = 22 + 1$

   $2 = (x - 3)^2$

   $x^2 = x^2 + \frac{3}{x^2} + 8x$

   $x(x + 9) = 4$

   $x^2 - 9 = 7x + 2$

4. Solve $2x^2 - x - 15 = 0$.

5. Circle the expressions below that are equal to 8.

   $-\sqrt{64}$ $\qquad$ $\frac{\sqrt{256}}{\sqrt{4}}$

   $\sqrt{(-8)^2}$ $\qquad$ $\frac{\sqrt{192}}{3}$

   $\sqrt{9 + 16}$ $\qquad$ $\sqrt{\frac{64}{5}} \cdot \sqrt{5}$

6. Solve $x^2 = 343$ using the square root property.

7. Solve this equation for x:

   $$x = \frac{\frac{\sqrt{20}}{\sqrt{3}}}{\frac{\sqrt{8}}{\sqrt{3}}}$$

8. Solve $(x - 5)^2 = 164$ using the square root property.