LESSON 9.1 – ROOTS AND RADICALS

OVERVIEW

Here's what you'll learn in this lesson:

Square Roots and Cube Roots

a. Definition of square root and cube root

b. Radicand, radical

c. Principal square root

d. Multiplication and division properties

e. Simplifying a square root or a cube root of a whole number

f. Simplifying square roots or cube roots of simple monomial expressions

Radical Expressions

a. Simplifying radical expressions

b. Like radical terms

c. Simplifying a sum or difference of radical expressions

d. Multiplying radical expressions

e. Conjugates

f. Rationalizing the denominator

g. Solving radical equations

In this lesson, you will learn about square roots and cube roots. You will find square roots of a number and also the cube root of a number, and you will use some of the properties of square roots and cube roots to simplify certain expressions. You will work with tables of square roots and cube roots to find the approximation of a number that is not a perfect square or a perfect cube.

Expressions that contain square roots or cube roots are called radical expressions. You will learn how to recognize whether a radical expression is in simplified form. You will learn how to add, subtract, multiply, and divide radical expressions. Also, you will learn how to simplify an expression that contains a radical in its denominator. Finally, you will learn how to solve certain equations that contain radical expressions.

EXPLAIN

SQUARE ROOTS AND CUBE ROOTS

Summary

Square Roots

A square root can be defined in terms of multiplication as well as with an exponent.
We say 4 is a square root of 16 because $4 \cdot 4 = 16$. Also, $4^2 = 16$.
Negative 4 is also a square root of 16 because $(-4) \cdot (-4) = 16$. Also, $(-4)^2 = 16$.

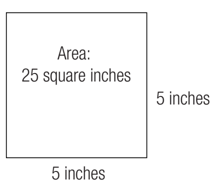
We say: 16 is 4 squared.
16 is a square number.
16 is a perfect square.

Every positive number has two square roots: a positive square root and a negative square root. The positive square root of a number is called the principal square root.

For example:

The positive square root of 16 is 4.
We also say the principal square root of 16 is 4.

Positive square roots can be interpreted nicely using geometry.



For example, the principal square root of 25 is 5. Geometrically, if a square has area 25 square inches, the length of the base (or “root”) of the square is 5 inches.

The radical symbol, $\sqrt{}$, is used to denote the positive square root of a number.
For example, $\sqrt{25} = 5$.

The negative of the radical symbol, $-\sqrt{}$, is used to denote the negative square root.
So, $-\sqrt{25} = -5$.

For example:

$\sqrt{49} = 7, \quad -\sqrt{49} = -7$

$\sqrt{144} = 12, \quad -\sqrt{144} = -12$

$\sqrt{1.44} = 1.2, \quad -\sqrt{1.44} = -1.2$

An entire expression such as $\sqrt{25}$ is called a radical. The number under the radical symbol is called the radicand. For example, the radicand of $\sqrt{25}$ is 25.

A negative number does not have a square root which is a real number, because no real number times itself equals a negative number.

The principal square root of 10 is not an integer because 10 is not a perfect square. However, 10 lies between two perfect squares, 9 and 16. We can use this information to estimate $\sqrt{10}$.

Number	Principal Square Root of Number
9	$\sqrt{9} = 3$
10	$\sqrt{10} \approx 3.2$
16	$\sqrt{16} = 4$

The symbol " ≈ " symbol means "is approximately equal to.

So the principal square root of 10 lies between 3 and 4.
The value of $\sqrt{10}$ is approximately 3.2.

Cube Roots

We say: 64 is 4 cubed
64 is a perfect cube

A cube root can be defined in terms of multiplication as well as with an exponent.
We say 4 is a cube root of 64 because $4 \cdot 4 \cdot 4 = 64$. Also, $4^3 = 64$.

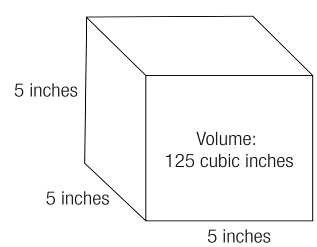
Write the cube root of 64 using the radical symbol:

$$4 = \sqrt[3]{64}$$

For a square root, the index 2 can also be written next to the radical symbol, but it is usually omitted. For example, the expressions $\sqrt{16}$ and $\sqrt[2]{16}$ each represent the principal square root of 16.

The number 3 to the left of the radical symbol in the expression $\sqrt[3]{64}$ is called the index of the radical. The radicand is 64.

The cube root of a positive number can also be interpreted geometrically.



For example, the cube root of 125 is 5. Geometrically, if a cube has volume 125 cubic inches, the length of the base (or "root") of the cube is 5 inches.

Every positive number has exactly one cube root which is a real number. This cube root is positive.
Every negative number has exactly one cube root which is a real number. This cube root is negative.

For example:

$5 = \sqrt[3]{125}$ since $5^3 = 125$

$-5 = \sqrt[3]{-125}$ since $(-5)^3 = -125$

The cube root of 100 is not an integer because 100 is not a perfect cube. However, 100 lies between two perfect cubes, 64 and 125. We can use this information to estimate $\sqrt[3]{100}$.

Number	Cube Root of Number
64	$\sqrt[3]{64} = 4$
100	$\sqrt[3]{100} \approx 4.6$
125	$\sqrt[3]{125} = 5$

The symbol "≈" means "is approximately equal to".

So the cube root of 100 lies between 4 and 5.
The value of $\sqrt[3]{100}$ is approximately 4.6.

Multiplication and Division Properties

When solving certain problems that involve square roots or cube roots, it is often helpful to use some of the basic properties of roots.

The multiplication property for roots states that the root of a product is the product of the roots:

$\sqrt{ab} = \sqrt{a}\sqrt{b}$; $a \geq 0$, $b \geq 0$

$\sqrt[3]{ab} = \sqrt[3]{a}\sqrt[3]{b}$; a, b real numbers

The multiplication property is useful "in both directions."
Here are two examples using the multiplication property in one direction:

$$\begin{aligned} &\sqrt{16 \cdot 5} \\ &= \sqrt{16} \cdot \sqrt{5} \\ &= 4\sqrt{5} \end{aligned}$$

$$\begin{aligned} &\sqrt[3]{8 \cdot 6} \\ &= \sqrt[3]{8} \cdot \sqrt[3]{6} \\ &= 2\sqrt[3]{6} \end{aligned}$$

The number 16 is a perfect square.
The number 8 is a perfect cube.

Here are two examples using the multiplication property in the other direction:

$$\begin{aligned} &\sqrt{3} \cdot \sqrt{27} \\ &= \sqrt{3 \cdot 27} \\ &= \sqrt{81} \\ &= 9 \end{aligned}$$

$$\begin{aligned} &\sqrt[3]{5} \cdot \sqrt[3]{25} \\ &= \sqrt[3]{5 \cdot 25} \\ &= \sqrt[3]{125} \\ &= 5 \end{aligned}$$

The number 81 is a perfect square.
The number 125 is a perfect cube.

The division property for roots states that the root of a quotient is the quotient of the roots:

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$; $a \geq 0$, $b > 0$

$\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$; a, b real numbers

The number 16 is a perfect square.

The number 27 is a perfect cube.
Square root;simplifying

Here are two examples:

$$\sqrt{\frac{5}{16}} \qquad \sqrt[3]{\frac{10}{27}}$$

$$= \frac{\sqrt{5}}{\sqrt{16}} \qquad = \frac{\sqrt[3]{10}}{\sqrt[3]{27}}$$

$$= \frac{\sqrt{5}}{4} \qquad = \frac{\sqrt[3]{10}}{3}$$

Simplifying a Square Root or a Cube Root of a Whole Number

The key idea in simplifying a radical is factoring.

To simplify the square root of a whole number:

1. Factor the number, trying to find perfect square factors.
2. Rewrite each factor under its own radical symbol.
3. Simplify the square root of each perfect square.

For example, to simplify: $\sqrt{600}$:

One way to find perfect square factors is to first find the prime factorization of the number.
*600 = **2** · **2** · 2 · 3 ·**5** · **5***
***2** · **2** and **5** · **5** are perfect squares.*

1. Factor the radicand, trying to find perfect square factors. $= \sqrt{\mathbf{4} \cdot 6 \cdot \mathbf{25}}$
2. Rewrite each factor under its own radical symbol. $= \sqrt{4} \cdot \sqrt{6} \cdot \sqrt{25}$
3. Simplify the square root of each perfect square. $= 2 \cdot \sqrt{6} \cdot 5$

$= 10\sqrt{6}$

Similarly, to simplify the cube root of a whole number:

1. Factor the number, trying to find perfect cube factors.
2. Rewrite each factor under its own radical symbol.
3. Simplify the cube root of each perfect cube.

For example, to simplify: $\sqrt[3]{250}$:

The number 125 is a perfect cube.

1. Factor the radicand, trying to find perfect cube factors. $= \sqrt[3]{\mathbf{125} \cdot 2}$
2. Rewrite each factor under its own radical symbol. $= \sqrt[3]{125} \cdot \sqrt[3]{2}$
3. Simplify the cube root of each perfect cube. $= 5\sqrt[3]{2}$

To simplify the square root of a quotient:

1. Rewrite the numerator and the denominator under their own radical symbols.
2. Simplify the numerator.
3. Simplify the denominator.

For example, to simplify: $\sqrt{\frac{32}{25}}$

1. Rewrite the numerator and the denominator under their own radical symbols. $= \frac{\sqrt{32}}{\sqrt{25}}$

   $= \frac{\sqrt{16 \cdot 2}}{\sqrt{25}}$

2. Simplify the numerator. $= \frac{4\sqrt{2}}{\sqrt{25}}$

3. Simplify the denominator. $= \frac{4\sqrt{2}}{5}$

Here's another example. Simplify: $\sqrt[3]{\frac{40}{27}}$

1. Rewrite the numerator and the denominator under their own radical symbols. $= \frac{\sqrt[3]{40}}{\sqrt[3]{27}}$

   $= \frac{\sqrt[3]{8 \cdot 5}}{\sqrt[3]{27}}$

2. Simplify the numerator. $= \frac{2\sqrt[3]{5}}{\sqrt[3]{27}}$

3. Simplify the denominator. $= \frac{2\sqrt[3]{5}}{3}$

Simplifying a Square Root or a Cube Root of a Monomial Expression

We can also simplify a radical expression in much the same way as we simplify the square root or the cube root of a number.

If x is a nonnegative number, then $\sqrt{x^2} = x$, because $x \cdot x = x^2$.

We can simplify the square root of any even power of a nonnegative number in a similar way.

For example:

$\sqrt{x^4} = x^2$ because $x^2 \cdot x^2 = x^4$

$\sqrt{x^{10}} = x^5$ because $x^5 \cdot x^5 = x^{10}$;

To simplify the square root of an expression:

1. Factor the expression, trying to find perfect square factors.
2. Rewrite each factor under its own radical symbol.
3. Simplify the square root of each perfect square.
4. Combine the remaining square roots.

For example, to simplify: $\sqrt{49x^3}$; x is a nonnegative number.

$$\sqrt{49x^3}$$

1. Factor the expression, trying to find perfect square factors. $= \sqrt{\mathbf{49} \cdot \boldsymbol{x^2} \cdot x}$
2. Rewrite each factor under its own radical symbol. $= \sqrt{49}\,\sqrt{x^2}\,\sqrt{x}$
3. Simplify the square root of each perfect square. $= 7x\sqrt{x}$

Here's another example. Simplify: $\sqrt{50x^7}$; x is a nonnegative number.

$$\sqrt{50x^7}$$

1. Factor the expression, trying to find perfect square factors. $= \sqrt{\mathbf{25} \cdot 2 \cdot \boldsymbol{x^6} \cdot x}$
2. Rewrite each factor under its own radical symbol. $= \sqrt{25}\,\sqrt{2}\,\sqrt{x^6}\,\sqrt{x}$
3. Simplify the square root of each perfect square. $= 5x^3\sqrt{2}\,\sqrt{x}$
4. Combine the remaining square roots. $= 5x^3\sqrt{2x}$

We can simplify the cube root of an expression in much the same way.

If x is a real number, $\sqrt[3]{x^3} = x$, because $x \cdot x \cdot x = x^3$.

Similarly, we can simplify the cube root of x raised to any power that is a multiple of three. For example:

$\sqrt[3]{x^6} = x^2$, because $x^2 \cdot x^2 \cdot x^2 = x^6$

$\sqrt[3]{x^{21}} = x^7$, because $x^7 \cdot x^7 \cdot x^7 = x^{21}$

To simplify the cube root of an expression:

1. Factor the expression, trying to find perfect cube factors.
2. Rewrite each factor under its own radical symbol.
3. Simplify the cube root of each perfect cube.
4. Combine the remaining cube roots.

For example, to simplify: $\sqrt[3]{8x^2y^6}$; x and y are real numbers.

$$\sqrt[3]{8x^2y^6}$$

1. Factor the expression, trying to find perfect cube factors. $= \sqrt[3]{\mathbf{2^3} \cdot x^2 \cdot \boldsymbol{y^3} \cdot \boldsymbol{y^3}}$
2. Rewrite each factor under its own radical symbol. $= \sqrt[3]{2^3}\,\sqrt[3]{x^2}\,\sqrt[3]{y^3}\,\sqrt[3]{y^3}$

3. Simplify the cube roots of each perfect cube. $= 2y\,y\sqrt[3]{x^2}$

4. Combine the remaining cube roots. $= 2y^2\sqrt[3]{x^2}$

Sample Problems

1. Identify the radical, the radicand, and the index of the following expression. Then evaluate the radical:

$\sqrt[3]{216}$

- ☐ a. The radical is: ____________
- ☐ b. The radicand is: ____________
- ☑ c. The index is: 3
- ☐ d. Evaluate: $\sqrt[3]{216}$ = _______ because ____ · ____ · ____ = 216

2. Find two consecutive integers between which $\sqrt{200}$ lies.

- ☐ a. Find the two perfect squares closest to 200. (________)^2 = 196
  $(\sqrt{200})^2$ = ____________
  15^2 = ____________
- ☐ b. $\sqrt{200}$ lies between: _______ and _______

3. Simplify: $\sqrt{\frac{75}{121}}$

- ☑ a. Rewrite the numerator and the denominator under their own radical symbols. $= \frac{\sqrt{75}}{\sqrt{121}}$
- ☐ b. Simplify the numerator, and write the entire expression. $= \frac{\sqrt{25 \cdot 3}}{\sqrt{121}}$
  $= \frac{\sqrt{25}\sqrt{3}}{\sqrt{121}}$
  = _____________
- ☐ c. Simplify the denominator, and write the entire expression. = _____________

4. Simplify: $\sqrt[3]{54x^3y^4}$
Here, x and y are real numbers.

- ☑ a. Factor the expression.
  - Try to find perfect cube factors. $= \sqrt[3]{27 \cdot 2 \cdot x^3 \cdot y^3 \cdot y}$
- ☐ b. Rewrite each factor under its own radical symbol. = ______________________
- ☐ c. Simplify the cube root of each perfect cube. = ______________________

Answers to Sample Problems

1. a. $\sqrt[3]{216}$

b. 216

d. 6, 6, 6, 6

2. a. 14
200
225

b. 14, 15

3. b. $\frac{5\sqrt{3}}{\sqrt{121}}$

c. $\frac{5\sqrt{3}}{11}$

4. b. $\sqrt[3]{27}\sqrt[3]{2}\sqrt[3]{x^3}\sqrt[3]{y^3}\sqrt[3]{y}$

c. $3xy\sqrt[3]{2}\sqrt[3]{y}$
or $3xy\sqrt[3]{2y}$

RADICAL EXPRESSIONS

Summary

Simplifying Radical Expressions

In order to add or subtract radical expressions, you will need to recognize whether an expression is in simplified form. When working with square roots or cube roots, a radical expression is in simplified form when there are:

- No perfect square factors under a square root
- No perfect cube factors under a cube root
- No fractions under a radical symbol
- No radicals in the denominator of a fraction

In each of the following examples, the radical expression is **not** in simplified form:

$\sqrt{48} = \sqrt{16 \cdot 3}$ is **not** in simplified form because 16 is a perfect square.

$\sqrt[3]{54} = \sqrt[3]{27 \cdot 2}$ is **not** in simplified form because 27 is a perfect cube.

$\sqrt{\frac{3}{8}}$ is **not** in simplified form because there is a fraction under the radical symbol.

$\frac{5}{2 + \sqrt[3]{2}}$ is **not** in simplified form because there is a radical in the denominator.

Identifying Like Radical Terms

When you add or subtract radical expressions, you will also need to recognize like radical terms. Like radical terms have the same index and the same radicand.

Here are some examples:

$6\sqrt[3]{4}$ and $-8\sqrt[3]{4}$ are like radical terms because in each radical, the index is 3 and the radicand is 4.

Remember that the index of a square root is 2, although the index is usually omitted.

$5\sqrt{6}$ and $9\sqrt{6}$ are like radical terms because in each radical, the index is 2 and the radicand is 6.

Adding and Subtracting Radical Expressions

To add or subtract terms in a radical expression:

1. Simplify each radical, if needed.
2. Identify like radical terms.
3. Combine like radical terms.

For example, to find: $\sqrt{24} + 6\sqrt{5} + 9\sqrt{6} + 4$:

1. Simplify the first term. $= \sqrt{4 \cdot 6} + 6\sqrt{5} + 9\sqrt{6} + 4$

   $= 2\sqrt{6} + 6\sqrt{5} + 9\sqrt{6} + 4$

2. Identify like radical terms. $= \mathbf{2\sqrt{6}} + 6\sqrt{5} + \mathbf{9\sqrt{6}} + 4$

3. Combine like radical terms. $= 11\sqrt{6} + 6\sqrt{5} + 4$

To combine like radical terms, add or subtract the numbers in front of the radical symbol, then write the radical symbol.

Here's another example. Find: $6\sqrt[3]{4} - 5 - 8\sqrt[3]{4} + 3\sqrt[3]{32}$:

1. Simplify the last term. $= 6\sqrt[3]{4} - 5 - 8\sqrt[3]{4} + 3\sqrt[3]{8 \cdot 4}$

   $= 6\sqrt[3]{4} - 5 - 8\sqrt[3]{4} + (3 \cdot 2)\sqrt[3]{4}$

   $= 6\sqrt[3]{4} - 5 - 8\sqrt[3]{4} + 6\sqrt[3]{4}$

2. Identify like radical terms. $= \mathbf{6\sqrt[3]{4}} - 5 - \mathbf{8\sqrt[3]{4}} + \mathbf{6\sqrt[3]{4}}$

3. Combine like radical terms. $= 4\sqrt[3]{4} - 5$

Multiplying Radical Expressions

To multiply two radical expressions, one of which has two terms:

1. Use the distributive property.

2. Simplify.

For example, to find: $\sqrt{5}(3\sqrt{6} - 4)$:

1. Use the distributive property. $= \sqrt{5}(3\sqrt{6}) - \sqrt{5}(4)$

   $= 3\sqrt{5}\sqrt{6} - 4\sqrt{5}$

2. Simplify. $= 3\sqrt{30} - 4\sqrt{5}$

This process is similar to finding the product of a monomial and a binomial.

Use the Multiplication Property of Radicals.

When you multiply polynomials, you can use the FOIL method for multiplying two binomials.
You will also use the FOIL method to multiply two radical expressions with two terms each.

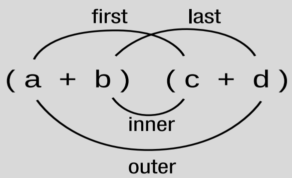
To multiply two radical expressions with two terms each:

1. Use the FOIL method.

2. Simplify.

This process is the same as finding the product of two binomials.

FOIL method: $(a + b)(c + d)$
$= ac + ad + bc + bd$



For example, to find: $(\sqrt{3} - 4)(3\sqrt{7} + 6)$:

1. Use the FOIL method. $= 3\sqrt{3}\sqrt{7} + 6\sqrt{3} - 4(3\sqrt{7}) - 4(6)$
2. Simplify. $= 3\sqrt{21} + 6\sqrt{3} - 12\sqrt{7} - 24$

Use the Multiplication Property of Radicals.

Here's another example. Find: $(5 + \sqrt{10})(2 - \sqrt{2})$:

1. Use the FOIL method. $= 5(2) - 5\sqrt{2} + 2\sqrt{10} - \sqrt{10}\sqrt{2}$
2. Simplify. $= 10 - 5\sqrt{2} + 2\sqrt{10} - \sqrt{20}$

$= 10 - 5\sqrt{2} + 2\sqrt{10} - \sqrt{4 \cdot 5}$

$= 10 - 5\sqrt{2} + 2\sqrt{10} - 2\sqrt{5}$

Conjugates

Sometimes when you multiply two irrational numbers you end up with a rational number.

Here's an example. Find: $(2 + \sqrt{11})(2 - \sqrt{11})$:

1. Use the FOIL method. $= 2(2) - 2\sqrt{11} + 2\sqrt{11} - \sqrt{11}\sqrt{11}$

$= 4 \quad + \quad 0 \quad - \sqrt{11}\sqrt{11}$

Note that $-2\sqrt{11} + 2\sqrt{11} = 0$
Note that $2\sqrt{6}\sqrt{10} - 2\sqrt{6}\sqrt{10} = 0$

2. Simplify. $= 4 - \sqrt{11}\sqrt{11}$

$= 4 - \sqrt{121}$

$= 4 - 11$

$= -7$

Radical expressions such as $2 + \sqrt{11}$ and $2 - \sqrt{11}$ are called conjugates of each other. The radical expression $2 + \sqrt{11}$ looks almost the same as $2 - \sqrt{11}$. The only difference is the + sign and the – sign.

When you multiply a radical expression by its conjugate, the result is a rational number.

Here are two more examples of radical expressions that are conjugates of one another:

$5 + 3\sqrt{7}$ and $5 - 3\sqrt{7}$

$8\sqrt{2} + 4\sqrt{3}$ and $8\sqrt{2} - 4\sqrt{3}$

Here's another example. Find: $(2\sqrt{6} - \sqrt{10})(2\sqrt{6} + \sqrt{10})$:

Note that $2\sqrt{6}\sqrt{10} - 2\sqrt{6}\sqrt{10} = 0$

1. Use the FOIL method. $= (2\sqrt{6})(2\sqrt{6}) + 2\sqrt{6}\sqrt{10} - 2\sqrt{6}\sqrt{10} - \sqrt{10}\sqrt{10}$
2. Simplify. $= 4\sqrt{6}\sqrt{6} - \sqrt{10}\sqrt{10}$

$= 4\sqrt{36} - \sqrt{100}$

$= 4(6) - 10$

$= 14$

Rationalizing the Denominator

When you divide one radical expression by another, you may be left with a radical in the denominator. For computational purposes, it is often useful to rewrite the result so that there are no radicals in the denominator.

To divide one radical expression by another, write the quotient as a fraction.

Then simplify the result, so there are no radicals in the denominator of the fraction.

The process of eliminating radicals from the denominator is called rationalizing the denominator. Radical expressions;rationalizing the denominator

For example, to rationalize the denominator in the fraction: $\frac{3}{\sqrt{8}}$:

1. Multiply the fraction by 1. $= \frac{3}{\sqrt{8}} \cdot \frac{\sqrt{8}}{\sqrt{8}}$

2. Do the multiplication. $= \frac{3\sqrt{8}}{(\sqrt{8})^2}$

$$= \frac{3\sqrt{8}}{8}$$

Write 1 in such a way that when you multiply, there will be no radical in the denominator of the fraction. Here, $1 = \frac{\sqrt{8}}{\sqrt{8}}$.

Remember that "squaring undoes the square root."

Here's another example. Rationalize the denominator in the fraction: $\frac{5}{\sqrt[3]{4}}$:

1. Multiply the fraction by 1. $= \frac{5}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{4} \cdot \sqrt[3]{4}}{\sqrt[3]{4} \cdot \sqrt[3]{4}}$

2. Do the multiplication. $= \frac{5 \cdot \sqrt[3]{4} \cdot \sqrt[3]{4}}{\sqrt[3]{4} \cdot \sqrt[3]{4} \cdot \sqrt[3]{4}}$

$$= \frac{5(\sqrt[3]{4})^2}{(\sqrt[3]{4})^3}$$

$$= \frac{5(\sqrt[3]{4})^2}{4}$$

Here, write $1 = \frac{\sqrt[3]{4} \cdot \sqrt[3]{4}}{\sqrt[3]{4} \cdot \sqrt[3]{4}}$

Remember that when you multiply a rational expression, such as $2 + \sqrt{7}$, by its conjugate, the result is a rational number. That is, the result has no radicals.

You can eliminate such a radical in the denominator of a fraction if you multiply the numerator and the denominator of the fraction by the conjugate of the denominator.

For example, to rationalize the denominator in the fraction: $\frac{9}{2 + \sqrt{7}}$:

1. Multiply the fraction by 1, using the conjugate of the denominator. $= \frac{9}{2 + \sqrt{7}} \cdot \frac{2 - \sqrt{7}}{2 - \sqrt{7}}$

2. Do the multiplication. $= \frac{9(2 - \sqrt{7})}{(2 + \sqrt{7})(2 - \sqrt{7})}$

$$= \frac{18 - 9\sqrt{7}}{4 - 7}$$

$$= \frac{18 - 9\sqrt{7}}{-3}$$

You want to choose 1 in such a way that you will multiply the numerator and denominator by the conjugate of the denominator. So, write $1 = \frac{2 - \sqrt{7}}{2 - \sqrt{7}}$.

Use the FOIL method to multiply the denominator by its conjugate.

Other possible answers:

$-6 + 3\sqrt{7}$, $\frac{6 - 3\sqrt{7}}{-1}$

Solving a Radical Equation

An equation that contains a radical term with a variable is called a radical equation. Here, you will learn how to solve a radical equation that contains a square root term with a variable.

To solve a radical equation that contains a square root term with a variable:

1. Get the square root by itself on one side of the equation.
2. Square both sides of the equation to eliminate the radical symbol.
3. Finish solving for the variable, if needed.

For example, to solve this radical equation for x: $\sqrt{x} - 3 = 7$

1. Get the square root by itself on the left side of the equation.
$$\sqrt{x} - 3 + 3 = 7 + 3$$
$$\sqrt{x} = 10$$

Remember that "squaring undoes the square root."

2. Square both sides of the equation to eliminate the radical symbol.
$$(\sqrt{x})^2 = 10^2$$
$$x = 100$$

3. The equation has been solved for x.

To check the answer, replace x with 100 in the original equation:

$$\sqrt{x} - 3 = 7$$

Is $\sqrt{100} - 3 = 7$?

Is $10 - 3 = 7$?

Is $7 = 7$? Yes.

Here is another example. Solve for x: $\sqrt{2x - 3} + 6 = 11$

1. Get the square root by itself on the left side of the equation.
$$\sqrt{2x - 3} + 6 - 6 = 11 - 6$$
$$\sqrt{2x - 3} = 5$$

Remember that "squaring undoes the square root."

2. Square both sides of the equation to eliminate the radical symbol.
$$(\sqrt{2x - 3})^2 = 5^2$$
$$2x - 3 = 25$$

3. Finish solving for x.
$$2x - 3 + 3 = 25 + 3$$
$$2x = 28$$
$$\frac{2x}{2} = \frac{28}{2}$$
$$x = 14$$

To check the answer, replace x with 14 in the original equation:

$$\sqrt{2x-3}+6=11$$

Is $\sqrt{2(14)-3}+6=11$?

Is $\sqrt{25}+6=11$?

Is $5+6=11$?

Is $11=11$? Yes.

Sample Problems

1. Find: $5\sqrt[3]{10}+9-\sqrt[3]{80}$

☐ a. Simplify each radical, as needed.

$= 5\sqrt[3]{10}+9-\sqrt[3]{8\cdot 10}$

$= 5\sqrt[3]{10}+9-\sqrt[3]{8}\sqrt[3]{10}$

$= 5\sqrt[3]{10}+9-$ ______ $\sqrt[3]{10}$

✔ b. Identify like radical terms.

$= \mathbf{5\sqrt[3]{10}}+9-\mathbf{2\sqrt[3]{10}}$

☐ c. Combine like radical terms.

$=$ ______ $\sqrt[3]{10}+9$

2. Find: $(6\sqrt{2}+1)(\sqrt{5}-4)$

☐ a. Use the FOIL method to multiply the expressions.

$= 6\sqrt{2}\sqrt{5}-24\sqrt{2}+1$___ $-$ 1__

☐ b. Simplify.

$=$ ______ $-$ ______ $+\sqrt{5}-4$

3. Rationalize the denominator: $\frac{5}{5-\sqrt{13}}$

☐ a. Multiply the fraction by 1. Use the conjugate of the denominator to write 1.

$= \frac{5}{5-\sqrt{13}}\cdot$ ____________

☐ b. Do the multiplication.

$=$ ________________________

4. Solve for x: $\sqrt{5x+1}-3=8$

✔ a. Get the square root by itself on the left side of the equation.

$\sqrt{5x+1}-3+3=8+3$

$\sqrt{5x+1}=11$

☐ b. Square both sides of the equation to eliminate the radical symbol.

$(\sqrt{5x+1})^2=11^2$

_______________ $=121$

☐ c. Finish solving for x.

$5x+1-1=121-1$

$5x=$ _______

$x=$ _______

Answers to Sample Problems

1a. 2

1c. 3

2a. $\sqrt{5}$, 4

2b. $6\sqrt{10}$, $24\sqrt{2}$

3a. $\frac{5+\sqrt{13}}{}$

3b. $\frac{25+5\sqrt{13}}{25-13}$ or $\frac{25+5\sqrt{13}}{12}$

4b. 5x + 1

4c. 120, 24

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

Square Roots and Cube Roots

1. For the expression $\sqrt[3]{27}$:

   a. Identify the radical.

   b. Identify the radicand.

   c. Identify the index.

   d. Evaluate the radical.

2. Evaluate: $\sqrt{49}$

3. Evaluate: $\sqrt[3]{-1000}$

4. Simplify: $\sqrt{162}$

5. Simplify: $\sqrt{\frac{108}{25}}$

6. Find two consecutive integers between which $\sqrt{50}$ lies.

7. Simplify: $\sqrt[3]{750}$

8. Simplify: $\sqrt[3]{\frac{80}{125}}$

9. The area of a circle is given by the formula $A = \pi r^2$, where A is the area of the circle and r is the radius of the circle. If the radius of a circle is $\sqrt{38}$ cm, what is the area of the circle?

10. If the volume of a cube is 216 cubic inches, what is the length of each edge of the cube? (Hint: The volume, V, of a cube with edges, each of length e is $V = e^3$.)

11. Simplify: $\sqrt{147a^{20}b^3}$

12. Simplify: $\sqrt[3]{216x^{21}y^4}$

13. Find: $2\sqrt{3} + \sqrt{13} - \sqrt{3} + 5\sqrt{13} + 3$

14. Find: $4\sqrt[3]{6} - 2\sqrt[3]{6} - 5\sqrt[3]{12} + 4\sqrt[3]{12}$

15. Find: $\sqrt{5}(3\sqrt{11} + 8)$

16. Find: $6\sqrt{98} - 2\sqrt{2} + 4 - \sqrt{128}$

17. Find: $4\sqrt[3]{24} - 2\sqrt[3]{81} + 5\sqrt[3]{3}$

18. Find: $(4\sqrt{5} + 3)(2\sqrt{7} - 5)$

19. Rationalize the denominator: $\frac{9}{11\sqrt{10}}$

20. Rationalize the denominator: $\frac{3}{6 + \sqrt{5}}$

21. The radius, r, of a sphere with surface area A is given by the formula $r = \frac{1}{2}\sqrt{\frac{A}{\pi}}$. If a sphere has radius 5 feet, what is its surface area?

22. The radius, r, of a sphere with volume V is given by the formula $r = \sqrt[3]{\frac{3V}{4\pi}}$. If a sphere has radius 3 inches, what is its volume?

23. Solve for x: $13 - \sqrt{x} = 4$

24. Solve for y: $\sqrt{3y - 2} - 5 = 5$

APPLY

Practice Problems

Here are some additional practice problems for you to try.

1. Find the negative square root of 25.
2. 25 and –25 are square roots of what number?
3. A square rug has area 110 square feet. Of the following values, which best approximates the length of an edge of the rug?
   a. 8.8 feet
   b. 10.5 feet
   c. 11.0 feet
   d. 27.5 feet
4. The area of a square table top is 6 square feet. Of the following values, which best approximates the length of an edge of the table top?
   a. 1.5 feet
   b. 2.0 feet
   c. 2.4 feet
   d. 3.0 feet
5. In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. Suppose the length of one leg of a right triangle is 5 cm and the length of the hypotenuse is 13 cm. What is the length of the other leg?
6. In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. Suppose the length of one leg of a right triangle is 24 inches and the length of the hypotenuse is 25 inches. What is the length of the other leg?
7. Simplify: $\frac{\sqrt{125}}{\sqrt{5}}$
8. Simplify: $\frac{\sqrt[3]{81}}{\;}$
9. Simplify: $\sqrt{x^5y^2}$
10. Simplify: $\sqrt[3]{-x^5y^2}$
11. Find: $\sqrt{3} - \sqrt{12} + \sqrt{27}$
12. Find: $\sqrt[3]{5} + \sqrt[3]{20} - \sqrt[3]{45}$
13. Use the FOIL method to do this multiplication: $(3 + \sqrt{5})(\sqrt{5} - 2)$
14. Use the FOIL method to do this multiplication: $(\sqrt{6} + 1)(3 - \sqrt{6})$
15. Multiply $4 + \sqrt{6}$ by its conjugate. What is the result?
16. Multiply $-4 + 3\sqrt{5}$ by its conjugate. What is the result?
17. Which of the following expressions, when multiplied by $\frac{3}{2 + \sqrt{14}}$, rationalizes the denominator of $\frac{3}{2 + \sqrt{14}}$?
   a. $\frac{2 - \sqrt{14}}{2 - \sqrt{14}}$
   b. $\frac{-2 + \sqrt{14}}{\;}$
   c. $\frac{-2 - \sqrt{14}}{-2 - \sqrt{14}}$
18. Which of the following expressions, when multiplied by $\frac{8}{\sqrt[3]{10}}$, rationalizes the denominator of $\frac{8}{\sqrt[3]{10}}$?
   a. $\frac{\sqrt{10} \cdot \sqrt{10}}{\;}$
   b. $\frac{\sqrt[3]{10} \cdot \sqrt[3]{10}}{\sqrt[3]{10} \cdot \sqrt[3]{10}}$
   c. $\frac{\sqrt[3]{10}}{\;}$
19. Solve this radical equation for x: $\sqrt{2x - 3} + 16 = 19$
20. Solve this radical equation for x: $\frac{\sqrt{3x + 1}}{5} = 2$

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. A positive number, a, has principal square root 1.1.
   a. What is another square root of a?
   b. What is the value of a?
2. The number $\sqrt[3]{-30}$ lies between what two consecutive integers?
3. Simplify the quotient of these square roots: $\frac{\sqrt{147}}{\sqrt{3}}$
4. Use prime factorization to simplify this radical: $\sqrt{28u^3}$ Here, u is a nonnegative number.
5. Find: $\sqrt{99} + \sqrt{44} - \sqrt{50} - \sqrt{2}$
6. Multiply $3 - 5\sqrt{2}$ by its conjugate. Enter the result.
7. For each given radical expression, rationalize the denominator.
   a. $\frac{5}{\sqrt[3]{-6}}$
   b. $\frac{3}{1 + \sqrt{7}}$
8. Solve this equation for x: $3\sqrt{5 - x} = 12$