LESSON 4.3 – GRAPHING INEQUALITIES

OVERVIEW

Here's what you'll learn in this lesson:

Linear Inequalities

a. Ordered pairs as solutions of linear inequalities

b. Graphing linear inequalities

Suppose you want to know if you will get an A in your math course, despite a low score on your final. Or you want to know if your car will get you to work without running out of gas.

Both of these questions can be described by a linear inequality in two variables. For example, in the first question you could ask yourself, "What are the possible final scores that will ensure an A in this course, given my overall scores so far?" The second question is really asking, "How much gas should I have in the tank to guarantee that my car can travel the entire distance to work?" When you think of the questions like this you can see that there is going to be more than one possible answer. In fact, you can see that any value greater than or equal to some minimum score (or amount) will work!

Graphing linear inequalities can help you answer these questions.

EXPLAIN

LINEAR INEQUALITIES

Summary

Graphing Linear Inequalities

When you graph a linear equation, all of the points whose coordinates make the equation true lie on a line. For example, the linear equation $x + y = 3$ is plotted in Figure 4.3.1.

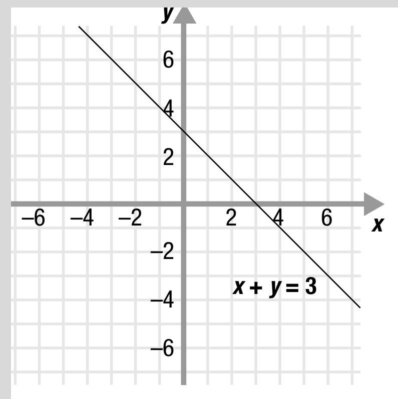


Figure 4.3.1

Notice that this line divides the *xy*-plane into three regions:

- points **above** the line
- points **on** the line
- points **below** the line

You know that the coordinates of any point **on** the line satisfy the equation $x + y = 3$. Using inequality symbols you can also describe which points lie **above** the line and which points lie **below** the line.

For example, try substituting the coordinates of some points not on the line $x + y = 3$ into the inequalities below to see which give you true statements:

	$x + y < 3$	$x + y > 3$
(0, 0)	Is $0 + 0 < 3$?	Is $0 + 0 > 3$?
	Is $0 < 3$? YES	Is $0 > 3$? NO
(6, 2)	Is $6 + 2 < 3$?	Is $6 + 2 > 3$?
	Is $8 < 3$? NO	Is $8 > 3$? YES
(–4, 1)	Is $-4 + 1 < 3$?	Is $-4 + 1 > 3$?
	Is $-3 < 3$? YES	Is $-3 > 3$? NO
(3, 5)	Is $3 + 5 < 3$?	Is $3 + 5 > 3$?
	Is $8 < 3$? NO	Is $8 > 3$? YES
(1, –3)	Is $1 + (-3) < 3$?	Is $1 + (-3) > 3$?
	Is $-2 < 3$? YES	Is $-2 > 3$? NO
(–2, 7)	Is $(-2) + 7 < 3$?	Is $(-2) + 7 > 3$?
	Is $5 < 3$? NO	Is $5 > 3$? YES
(–6, –4)	Is $(-6) + (-4) < 3$?	Is $(-6) + (-4) > 3$?
	Is $-10 < 3$? YES	Is $-10 > 3$? NO

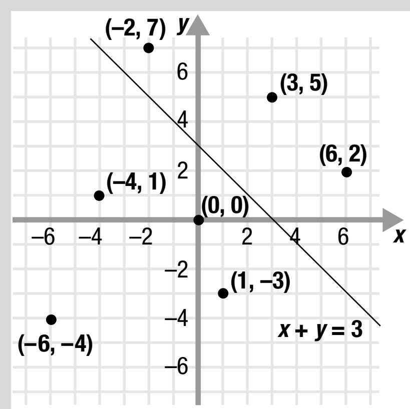


Figure 4.3.2

These points are plotted in Figure 4.3.2. Notice that the coordinates of any single point can make only one of the inequalities true. The points whose coordinates make the inequality $x + y < 3$ true lie below the line; the points whose coordinates make the inequality $x + y > 3$ true lie above the line.

The coordinates of any given point can make only one of these inequalities true since, in this example, a number cannot be both greater than 3 and less than 3.

If the coordinates of any point above the corresponding line make an inequality true, then the coordinates of **all** the points above the line make the inequality true. Similarly, if the coordinates of any point below the corresponding line make the inequality true, then the coordinates of **all** the points below the line make the inequality true. You can use this fact to figure out in which region the points that satisfy an inequality lie.

Why can't you use a test point that's on the line? Because, if you do, you still won't know which region to shade.

If (0, 0) doesn't lie on the line, it's a good test point to choose because when you substitute it into the inequality, the math is easy.

A dotted line is used if the inequality is "<" or ">" to show that points on the line do not satisfy the inequality.

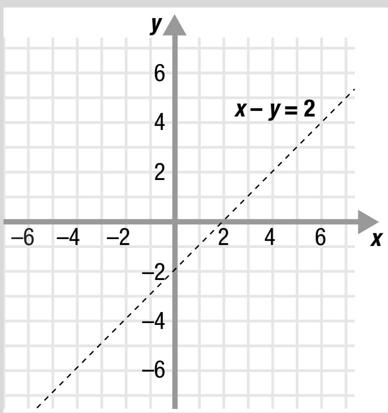


Figure 4.3.3

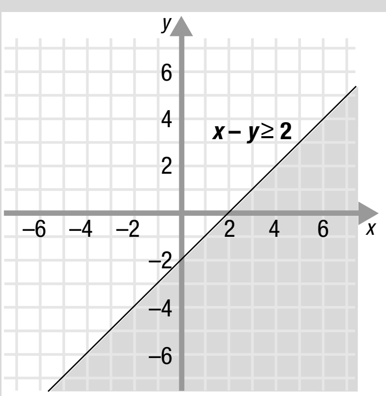


Figure 4.3.4

To find the region whose points satisfy an inequality, pick a point that does **not** lie on the corresponding line and substitute its coordinates into the inequality.

- If the coordinates of the point make the inequality true, then shade the region in which the point lies.
- If the coordinates of the point do **not** make the inequality true, then shade the region in which the point does **not** lie.

To graph a linear inequality:

1. Graph the linear equation that corresponds to the given inequality. Use a dotted line.
2. Use a test point that is not on the line to determine the region whose points satisfy the inequality.
3. Shade that region. If the inequality is $\leq$ or $\geq$, "shade" the line as well (that is, make it a solid line).

For example, to graph the linear inequality $x - y \geq 2$:

1. Graph the corresponding equation $x - y = 2$. Use a dotted line. See Figure 4.3.3.
2. Pick a point not on the line, say (0, 0), and substitute it into the inequality.

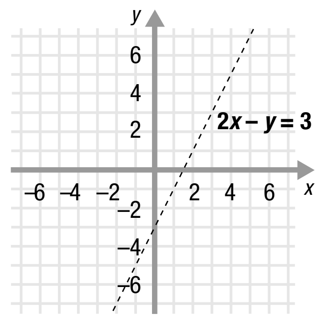
   Is $0 - 0 \geq 2$?
   Is $\quad 0 \geq 2$? No.

3. Since this statement isn't true, the point (0, 0) does not satisfy the inequality. So, shade the region in which (0, 0) does **not** lie. Shade the line because points on the line also satisfy the inequality $x - y \geq 2$. See Figure 4.3.4.

Sample Problems

1. Graph the inequality $2x - y < 3$.

   ✔ a. Graph the corresponding equation $2x - y = 3$.



   ☐ b. Substitute a test point into the inequality, say (0, 0).

   $2x \quad - \quad y \quad < 3$
   Is 2(___) – ___ < 3?
   Is ___ < 3? ____

   ☐ c. Shade the region whose points satisfy the inequality.

2. Graph the inequality $x - 3y \geq 0$.

   ☐ a. Graph the corresponding equation $x - 3y = 0$.

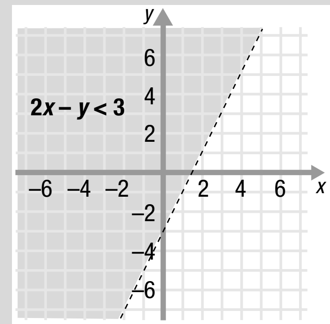
   ☐ b. Substitute a test point into the inequality, say (1, 1). (You can't substitute (0, 0) since it lies on the line $x - 3y = 0$.)

   $x \quad - \quad 3y \quad \geq 0$
   Is ___ – 3(___) ≥ 0?
   Is ____ ≥ 0? ____

   ☐ c. Shade the region whose points satisfy the inequality.

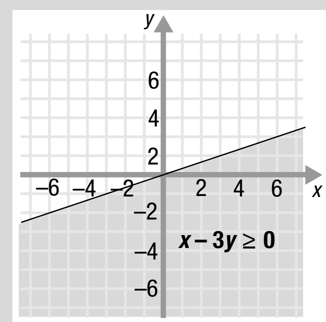
Answers to Sample Problems

c.



b. 0, 0
0, Yes

a., c.



b. 1, 1
–2, No

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

Linear Inequalities

1. In each row, check the boxes corresponding to the coordinates that make the statement true.

	(1, 7)	(4, 2)	(2, 5)	(0, 0)	(–6, 3)	(–4, –3)	(3, –6)
$y < 2x + 1$							
$y = 2x + 1$							
$y > 2x + 1$							

2. Graph the inequality $y \leq x + 2$.

3. Graph the inequality $x + y \geq -5$.

4. Find the coordinates of three points that satisfy the inequality $y \leq 3x - 2$.

5. Graph the inequality $y \geq \frac{2}{5}x - 2$.

6. Graph the inequality $3x - 2y > 12$.

7. In what ways do the graphs of the inequalities $x + y < 4$ and $x + y \leq 4$ differ?

8. Graph the inequality $2x > 7y$.

9. Janna has up to \$5.00 to spend on snacks at a new health food store. If guava chips are \$4.00 per pound and shredded coconut is \$2.50 per pound, graph the inequality that represents how much of each she can buy.

10. Shobana has up to \$12.00 to spend on junk food. If she can buy bulk candy for \$2.25 per pound and cookies for \$3.00 per pound, graph the inequality that represents how much of each she can buy.

11. Graph the inequality $5x - 4y \geq 0$.

12. In what ways do the graphs of the inequalities $2x - y > 3$ and $2x - y < 3$ differ?

APPLY

Practice Problems

Here are some additional practice problems for you to try.

1. In each row, check the boxes corresponding to the coordinates that make the statement true.

	(2, –1)	(4, –2)	(–5, 2)	(3, 8)	(–3, 1)	(4, 3)	(–1, 6)
$x - y < 1$							
$x - y = 1$							
$x - y > 1$							

2. In each row, check the boxes corresponding to the coordinates that make the statement true.

	(1, –3)	(3, –4)	(–6, 2)	(2, 7)	(–2, 1)	(6, 5)	(–2, 5)
$x + y < -1$							
$x + y = -1$							
$x + y > -1$							

3. In each row, check the boxes corresponding to the coordinates that make the statement true.

	(–3, 4)	(–5, 3)	(–1, 4)	(3, 5)	(3, –8)	(5, –1)	(–4, –5)
$2x - y < 1$							
$2x - y = 1$							
$2x - y > 1$							

4. In each row, check the boxes corresponding to the coordinates that make the statement true.

	(–2, 7)	(–4, 2)	(–1, 6)	(2, 5)	(2, –5)	(4, –2)	(–1, –2)
$3x + y < 1$							
$3x + y = 1$							
$3x + y > 1$							

5. Graph the inequality $x - y > 4$.
6. Graph the inequality $x - y < -3$.
7. Graph the inequality $x - y < 5$.
8. Graph the inequality $x + y > 1$.
9. Graph the inequality $x + y < -5$.
10. Graph the inequality $x + y < -2$.
11. Graph the inequality $x + y \leq -1$.
12. Graph the inequality $x + y \geq 5$.
13. Graph the inequality $x + y \geq 1$.
14. Graph the inequality $x - y \leq -4$.
15. Graph the inequality $x - y \geq 6$.
16. Graph the inequality $x - y \geq -3$.
17. Graph the inequality $\frac{2}{3}x + y < -3$.
18. Graph the inequality $\frac{1}{4}x + y > 1$.
19. Graph the inequality $\frac{1}{2}x + y > -2$.
20. Graph the inequality $3x - y \geq 2$.
21. Graph the inequality $-4x - y \leq -3$.
22. Graph the inequality $-2x - y \leq 1$.
23. Graph the inequality $4x - 3y < 12$.
24. Graph the inequality $2x + 5y > -5$.
25. Graph the inequality $3x + 2y > 8$.
26. Graph the inequality $-3x + \frac{1}{3}y \geq -2$.
27. Graph the inequality $2x - \frac{3}{2}y \leq 3$.
28. Graph the inequality $2x - \frac{1}{2}y \leq 1$.

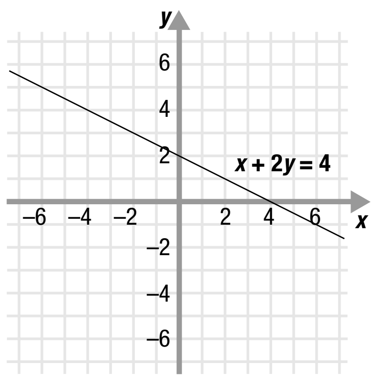
EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Graph the inequality $y > \frac{2}{3}x - 1$.

2. The graph of the line $x + 2y = 4$ is shown in Figure 4.3.5. Circle the point(s) below that satisfy the inequality $x + 2y \le 4$.



(0, 0)

(5, 2)

(–3, –1)

(8, –4)

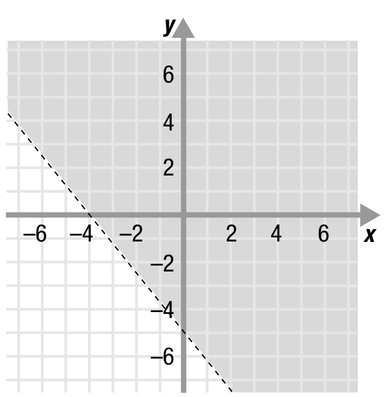
Figure 4.3.5

3. Graph the inequality $y \le 2x - 1$.

4. Circle the point(s) below that satisfy the linear inequality $y \le 4$.

(23, 56)	(0, 0)
(8, –14)	(–6, 7)

5. Circle the inequality below that has a solution represented on the graph shown in Figure 4.3.6.



$5x + 4y > -20$

$4x - 5y < 20$

$4x + 5y > -20$

$5x - 4y < 20$

Figure 4.3.6

6. Graph the inequality $y \le 2x + 3$.

7. The graph of the equation $y = -\frac{1}{2}x - 2$ is shown in Figure 4.3.7. Circle the point(s) below that satisfy the inequality $y > -\frac{1}{2}x - 2$.

(1, 2)

(–3, 3)

(–5, 6)

(2, –5)

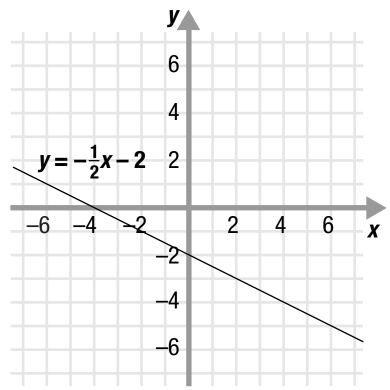
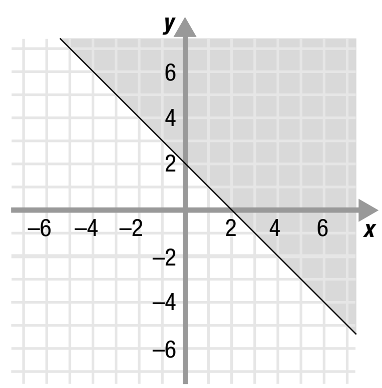


Figure 4.3.7

8. Circle the inequality below that has a solution represented on the graph shown in Figure 4.3.8.



$y > -x + 2$

$y < -x + 2$

$y \le -x + 2$

$y \ge -x + 2$

Figure 4.3.8

TOPIC 4 CUMULATIVE ACTIVITIES

CUMULATIVE REVIEW PROBLEMS

These problems combine all of the material you have covered so far in this course. You may want to test your understanding of this material before you move on to the next topic. Or you may wish to do these problems to review for a test.

1. Graph the equation $x - y = -2$.

2. Find the x- and y-intercepts of the line $2x + y = 3$.

3. Find the equation of the line through the point (9, 4) with slope $\frac{5}{4}$. Write the equation in standard form.

4. Graph the line through the point (3, 3) with slope 2.

5. Evaluate the expression $-5a^2b - 4a^3 + 7b^4$ when $a = 2$ and $b = -2$.

6. Find: $[3 + 2(5 - 7)^2] + 8$

7. Graph the inequality $y \geq 3x - 4$.

8a. Find the equation of the vertical line through the point (–4, 5).

b. What is the slope of this line?

c. Find the equation of the horizontal line through the point (–4, 5).

d. What is the slope of this line?

9. Write in lowest terms: $\frac{270}{405}$

10. Graph the line $y = 4$.

11. Find: $\left| -\frac{7}{9} + \frac{5}{6} \right|$

12. The sum of three consecutive numbers is –18. What are the numbers?

13. Graph the inequality $\frac{2}{5}x + y < 2$.

14. Find the x- and y-intercepts of the line $\frac{6}{7}x - \frac{4}{7}y = 2$.

15a. Find four points whose coordinates satisfy the inequality $x + 2y < 5$.

b. Find four points whose coordinates satisfy the inequality $x + 2y > 5$.

c. Find four points whose coordinates satisfy the equation $x + 2y = 5$.

16. Graph the equation $3x + 4y = 12$.

17. Find: $8\frac{2}{5} - 4\frac{1}{7}$

18. Solve for x: $2x - 4 = \frac{2}{3}(-3 + x)$

19. Find: $\frac{8}{11} \div \frac{16}{55}$

20. Graph the line through the point (–2, –5) with slope –1.

21. Find the equation of the line through the points (–7, 3) and (0, 4). Write your answer in standard form.

22. Write in lowest terms: $\frac{20}{35}$

23. Solve for y: $-5(y - 2) = \frac{2}{7}(6y - \frac{1}{2})$

24. Graph the inequality $\frac{2}{3}x - \frac{1}{2}y < -1$.

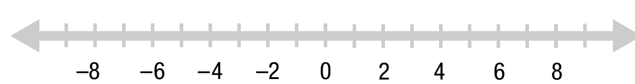
25. Paul is 21 years older than Rita. Ten years ago, Paul was twice as old as Rita was then. How old is each of them now?

26. The surface area, S, of a sphere is $S = 4\pi r^2$, where r is the radius of the sphere. Solve this formula for r.

27. Graph the line $x = -7$.

28. Evaluate the expression $2x^2 + 4xy - 3y + 1$ when $x = -3$ and $y = 5$.

29. Solve $-3 < 2x + 1 \leq 8$ for x, then graph its solution on the number line below.



30. Find the slope of the line through the points (3, 8) and (–2, 1).

31. Graph the inequality $7x - 5y > 0$.

32. Write in lowest terms: $\frac{42}{63}$

33. Solve for y: $y + 1 = -\frac{1}{5}(67 + 9y)$

34. Write the equation of the line through the point $(-8, -6)$ with slope $\frac{9}{4}$. Write your answer in slope-intercept form.

35. Find the slope of the line perpendicular to the line through the points $(7, -3)$ and $(4, 9)$.

36. Graph the equation $\frac{2}{5}x + \frac{1}{5}y = -1$.

37. Find the equation of the line through the point $(5, -1)$ with slope 4. Write your answer in point-slope form.

38. Find the slope and y-intercept of the line $3x + 2y = 8$.

39. Find: $3\frac{7}{9} + 1\frac{2}{3}$

40. Solve $\frac{3}{2} < 2 - x \leq 7$ for x, then graph its solution on the number line below.