LESSON EII.E – GRAPHING LINES

OVERVIEW

Here's what you'll learn in this lesson:

Graphing Lines

a. The Cartesian coordinate system

b. The distance formula

c. Graphing linear equations

d. Finding the x- and y-intercepts

e. Finding the slope of a line

Finding Equations

a. The point-slope form of a line

b. The standard form of a line

c. The slope-intercept form of a line

d. Horizontal lines

e. Vertical lines

f. Parallel and perpendicular lines

In this lesson you will review how to graph linear equations and how to find the equation of a line. In the process, you will reacquaint yourself with the Cartesian coordinate system.

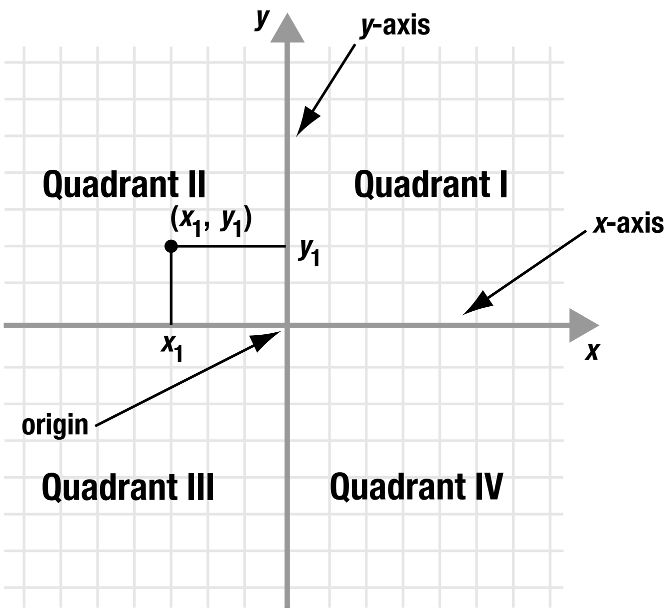
EXPLAIN

GRAPHING LINES

Summary

The Cartesian Coordinate System

You have learned how to graph real numbers on a number line. Now you'll review how to plot ordered pairs of numbers. In the 17th century, the French philosopher and mathematician René Descartes invented the Cartesian coordinate system as a means to represent ordered pairs of numbers. The Cartesian coordinate system, or xy-plane, consists of two number lines placed at right angles to each other that break the plane into four quadrants. The horizontal number line is called the x-axis and the vertical number line is called the y-axis. These axes intersect at the point (0, 0), called the origin.



Given an ordered pair of real numbers (x_1, y_1) where x_1 is the x-coordinate (abscissa) and y_1 is the y-coordinate (ordinate), you can plot this ordered pair as a point in the xy-plane.

To plot a point:

1. Draw a vertical line through the x-coordinate of the point.
2. Draw a horizontal line through the y-coordinate of the point.
3. Plot the point where these two lines intersect.

As an example, points P (1, 3) and Q (–2, –2) are plotted in Figure EII.E.1.

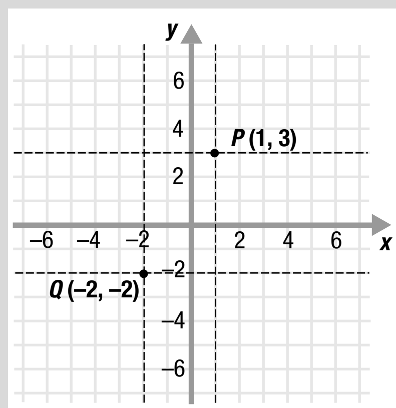


Figure EII.E.1

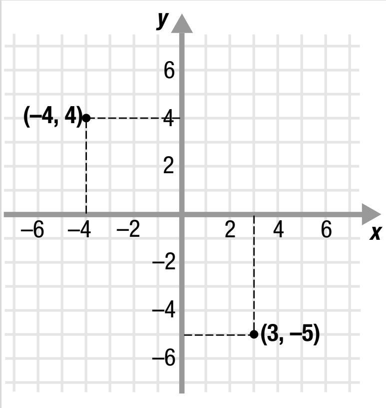


Figure EII.E.2

Drawing horizontal and vertical lines is a very formal method of plotting points or finding coordinates. With experience, you will be able to do both without actually drawing the lines.

If a line is horizontal, the y-coordinate will be the same for every point on the line. That is, $y_1 = y_2$. Substituting this into the distance formula gives the horizontal distance formula.

Similarly, if a line is vertical, the x-coordinate will be the same for every point on the line. That is, $x_1 = x_2$. Once again, if you substitute this into the distance formula and simplify, you will get the vertical distance formula.

It doesn't matter which point you call (x_1, y_1) or which point you call (x_2, y_2). Either way, you'll get the same answer.

Similarly, given a point in the *xy*-plane, you can find its coordinates. To find the coordinates of a point:

1. Draw a vertical line from the point to the *x*-axis. The line intersects the *x*-axis at the *x*-coordinate.
2. Draw a horizontal line from the point to the *y*-axis. The line intersects the *y*-axis at the *y*-coordinate.

For an example, look at Figure EII.E.2, where the points (–4, 4) and (3, –5) are given.

The Distance Formula

Once you can plot points, you can find the distance between points. In general, you can always find the distance between any two points (x_1, y_1) and (x_2, y_2) in the plane. This distance, *d*, is given by the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

If the points lie on a vertical or horizontal line, this formula reduces to simpler expressions to calculate the distance.

On a horizontal line, the distance, *d*, between two points (x_1, y_1) and (x_2, y_2) is given by:

$$d = |x_2 - x_1|$$

On a vertical line, the distance, *d*, between two points (x_1, y_1) and (x_2, y_2) is given by:

$$d = |y_2 - y_1|$$

To find the distance, *d*, between two points:

1. Label one of the points (x_1, y_1) and the other (x_2, y_2).
2. Determine if the points lie on a horizontal or vertical line.
   - If the points lie on a horizontal line, their *y*-values are equal.
   - If the points lie on a vertical line, their *x*-values are equal.
3. Apply the appropriate distance formula.

For example, to find the distance, *d*, between the points (–1, 7) and (5, 7):

1. Label one of the points (x_1, y_1) and the other (x_2, y_2).	Let $(x_1, y_1) = (-1, 7)$ Let $(x_2, y_2) = (5, 7)$
2. Determine if the points lie on a horizontal or vertical line. See the grid in Figure EII.E.3.	Are the *x*-values equal? No. Are the *y*-values equal? Yes. The points lie on a horizontal line.

3. Apply the horizontal distance formula.

$$\begin{aligned} d &= |x_2 - x_1| \\ &= |5 - (-1)| \\ &= |5 + 1| \\ &= |6| \\ &= 6 \end{aligned}$$

So the distance between the points (–1, 7) and (5, 7) is 6.

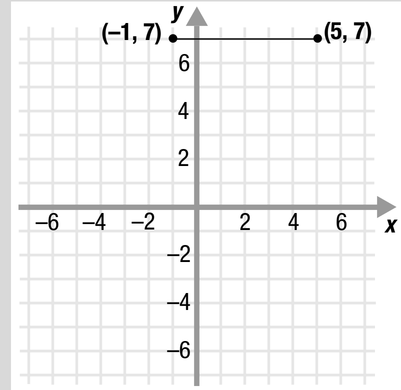


Figure EII.E.3

As another example, to find the distance, *d*, between the points (–1, 7) and (5, 1):

1. Label one of the points (x_1, y_1) and the other (x_2, y_2). — Let $(x_1, y_1) = (-1, 7)$. Let $(x_2, y_2) = (5, 1)$
2. Determine if the points lie on a horizontal or vertical line. See the grid in Figure EII.E.4. — Are the *x*-values equal? No. Are the *y*-values equal? No. The points do not lie on a vertical or a horizontal line.
3. Apply the distance formula.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[(5 - (-1)]^2 + (1 - 7)^2} \\ &= \sqrt{(6)^2 + (-6)^2} \\ &= \sqrt{36 + 36} \\ &= \sqrt{72} \\ &= 6\sqrt{2} \end{aligned}$$

So the distance between the points (–1, 7) and (5, 1) is $6\sqrt{2}$.

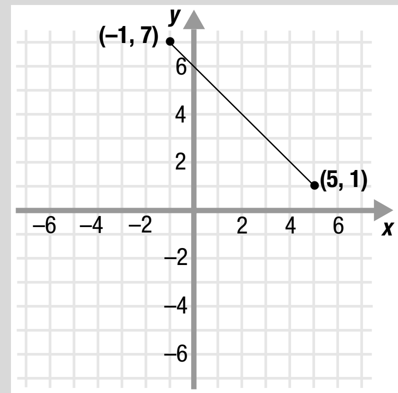


Figure EII.E.4

Graphing Linear Equations

The graph of a linear equation is a line. To graph a linear equation:

1. Make a table of ordered pairs that satisfy the equation.
2. Plot these ordered pairs.
3. Draw a line through the plotted points.

Since a line is determined by two points, you don't need more than two points to graph a line. However, plotting extra points will help you avoid errors.

For example, to graph the linear equation $x + 3y = 6$:

1. Make a table of ordered pairs that satisfy the equation.
2. Plot these ordered pairs.
3. Draw a line through the plotted points. See the grid in Figure EII.E.5

x	y
6	0
3	1
0	2
–3	3

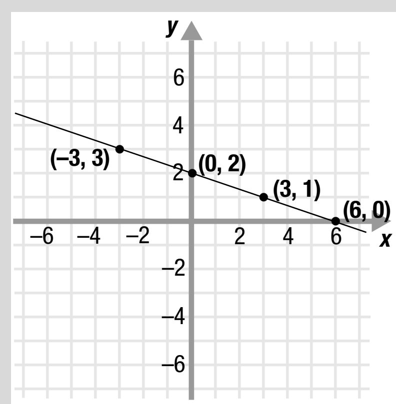


Figure EII.E.5

Here's another example. To graph the linear equation $4x - 2y = 6$:

The points in this example are two special points called the x-intercept and the y-intercept. You'll learn more about these in the next section.

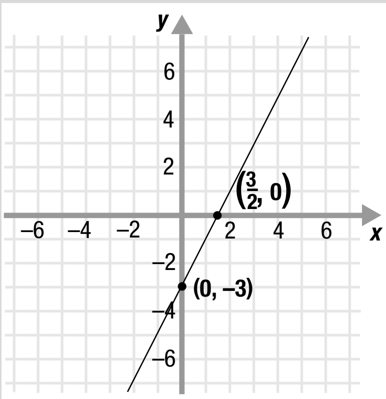


Figure EII.E.6

1. Make a table of ordered pairs that satisfy the equation.

x	y
$\frac{3}{2}$	0
0	–3

Choose $y = 0$:

$4x - 2(0) = 6$

$4x = 6$

$x = \frac{6}{4}$

$= \frac{3}{2}$

Choose $x = 0$:

$4(0) - 2y = 6$

$-2y = 6$

$y = \frac{6}{-2}$

$= -3$

2. Plot the ordered pairs.

3. Draw a line through the plotted points. See the grid in Figure EII.E.6.

Finding the x- and y-Intercepts of a Line

When graphing a line, two points on the line that are often easy to find are the x-intercept and the y-intercept. The x-intercept is the point where the line crosses the x-axis. Similarly, the y-intercept is the point where the line crosses the y-axis.

To find the intercepts of a line:

1. Set $y = 0$ and solve for x. This gives the x-intercept, $(x, 0)$.
2. Set $x = 0$ and solve for y. This gives the y-intercept, $(0, y)$.

For example, to find the intercepts of the line $5x + 2y = -1$:

1. Set $y = 0$ and solve for x.

$5x + 2(0) = -1$

$5x = -1$

$x = -\frac{1}{5}$

The x-intercept is $\left(-\frac{1}{5}, 0\right)$.

2. Set $x = 0$ and solve for y.

$5(0) + 2y = -1$

$2y = -1$

$y = -\frac{1}{2}$

The y-intercept is $\left(0, -\frac{1}{2}\right)$.

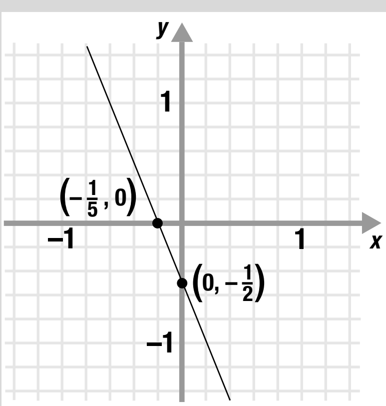


Figure EII.E.7

The intercepts of the line $5x + 2y = -1$ are plotted in Figure EII.E.7.

Here's another example. To find the intercepts of the line $y = 3x + 6$:

1. Set $y = 0$ and solve for x.

$$0 = 3x + 6$$
$$-3x = 6$$
$$x = \frac{6}{-3}$$
$$= -2$$

The x-intercept is (–2, 0).

2. Set $x = 0$ and solve for y.

$$y = 3(0) + 6$$
$$y = 6$$

The y-intercept is (0, 6).

The intercepts of the line $y = 3x + 6$ are plotted in Figure EII.E.8.

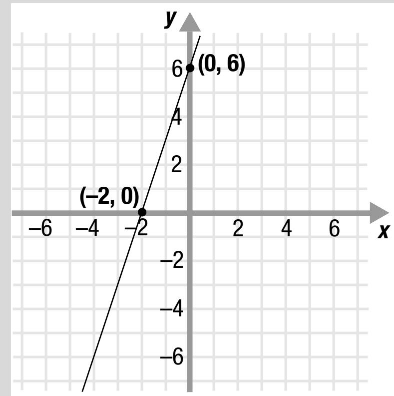


Figure EII.E.8

Finding the Slope of a Line

The slope of a line is a number that describes the steepness of the line. It is the ratio of the rise to the run in moving from one point on the line to another point on the line.

The slope, m, of the line containing the points (x_1, y_1) and (x_2, y_2) is given by:

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

To find the slope of a line through two points (x_1, y_1) and (x_2, y_2):

1. Label one of the points (x_1, y_1) and the other (x_2, y_2).
2. Substitute the coordinates of the points into the definition of slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$.
3. Simplify.

Again, it doesn't matter which point you call (x_1, y_1) or which point you call (x_2, y_2). Either way, you get the same slope.

For example, to find the slope of the line joining the points (8, 2) and (6, 5):

1. Label one of the points (x_1, y_1) and the other (x_2, y_2).

Let $(x_1, y_1) = (8, 2)$
Let $(x_2, y_2) = (6, 5)$

2. Substitute the coordinates of the points into the definition of slope.

$$m = \frac{5 - 2}{6 - 8}$$

3. Simplify.

$$m = \frac{3}{-2}$$
$$m = -\frac{3}{2}$$

So the slope of the line through the points (8, 2) and (6, 5) is $-\frac{3}{2}$.

Here's another example. To find the slope of the line passing through the points (3, –1) and (3, 14):

1. Label one of the points (x_1, y_1) and the other (x_2, y_2). Let $(x_1, y_1) = (3, -1)$ Let $(x_2, y_2) = (3, 14)$
2. Substitute the coordinates of the points into the definition of slope. $m = \frac{14 - (-1)}{3 - 3}$
3. Simplify. $m = \frac{14 + 1}{3 - 3}$

   $m = \frac{15}{0}$

The slope of a vertical line is undefined.
The slope of a horizontal line is zero.

Division by 0 is undefined, so the slope of the line through the points (3, –1) and (3, 14) is undefined.

Answers to Sample Problems

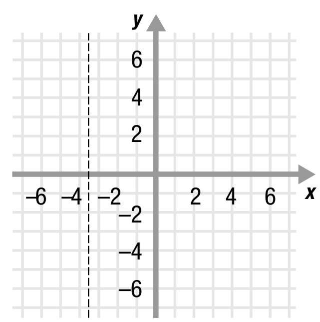
Sample Problems

1. On the grid below, plot the point Q (–3.5, 4).

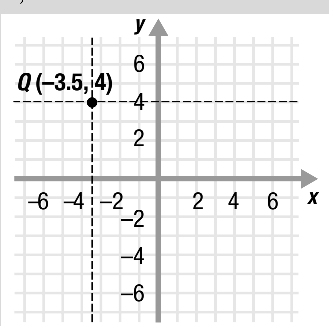
   ✔ a. Draw a vertical line through –3.5, the x-coordinate of Q.

   ☐ b. Draw a horizontal line through 4, the y-coordinate of Q.

   ☐ c. Plot a point where the lines intersect. Label the point Q.



b., c.



2. Find the distance, d, between the point (–1, –2) and the point (7, 10).

   ✔ a. Label one of the points (x_1, y_1) and the other (x_2, y_2). Let $(x_1, y_1) = (-1, -2)$ Let $(x_2, y_2) = (7, 10)$

   ☐ b. Determine if the points lie on a horizontal or vertical line. Are the x-values equal? ______ Are the y-values equal? ______ The points do not lie on a horizontal or vertical line.

   b. No.
   No.

   ✔ c. Substitute the coordinates of the points into the distance formula. $d = \sqrt{[7 - (-1)]^2 + [10 - (-2)]^2}$

   ☐ d. Simplify. $d =$ ______________

   d. $\sqrt{208}$ or $4\sqrt{13}$

3. Graph the linear equation $5x + 4y = 20$.

✔ a. Make a table of ordered pairs that satisfy the equation.

Choose $y = 0$.

$x =$ ______

Choose $x = 0$.

$y =$ ______

x	y
__	0
0	__

☐ b. Plot the ordered pairs.

☐ c. Draw a line through the plotted points.

4. Find the x- and y-intercepts of the line $y - 7 = -9(x + 2)$.

✔ a Set $y = 0$ and solve for x.

$$0 - 7 = -9(x + 2)$$
$$-7 = -9x - 18$$
$$11 = -9x$$
$$x = -\frac{11}{9}$$

☐ b. Write the x-intercept. The x-intercept is (_____, _____).

☐ c. Set $x = 0$ and solve for y.

$$y - 7 = -9(0 + 2)$$
$y =$ ____

☐ d. Write the y-intercept. The y-intercept is (_____, _____).

5. Find the slope of the line containing the points (4, 8) and (12, 6).

☐ a. Label one of the points (x_1, y_1) and the other (x_2, y_2).

Let $(x_1, y_1) = (4, 8)$

Let $(x_2, y_2) =$ (_____, _____)

☐ b. Substitute the coordinates of the points into the definition of slope.

$$m = \frac{6 - 8}{\_\_\_\_ - \_\_\_\_}$$

☐ c. Simplify.

$m =$ _____

Answers to Sample Problems

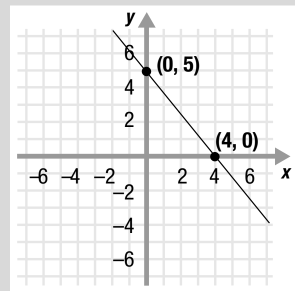
a. 4

5

4

5

b., c.



b. $-\frac{11}{9}$*, 0*

c. –11

d. 0, –11

a. 12, 6

b. 12, 4

c $-\frac{1}{4}$

FINDING EQUATIONS

Summary

The Point-Slope Form of the Equation of a Line

You can find the equation of a line if you know the slope, m, of the line and one point, (x_1, y_1), on the line.

To find the point-slope form for the equation of the line through (x_1, y_1) with slope m:

1. Pick any point on the line. Call it (x, y).
2. Substitute the slope, m, and the points (x_1, y_1) and (x, y) into the definition of slope: $m = \frac{y - y_1}{x - x_1}$
3. Multiply both sides of the equation by the denominator, $(x - x_1)$.
4. Rewrite the equation as $(y - y_1) = m(x - x_1)$.

The equation $(y - y_1) = m(x - x_1)$ is the point-slope form of the equation of a line.

For example, to find the equation of the line through the point (1, 4) with slope –7.

1. Pick any point on the line.	(x, y)
2. Substitute the slope, $m = -7$, the point (1, 4), and the point (x, y) into the definition of slope.	$\frac{y-4}{x-1} = -7$
3. Multiply both sides by $x - 1$.	$(x-1) \cdot \frac{y-4}{x-1} = (x-1) \cdot (-7)$
4. Rewrite the equation.	$y - 4 = -7(x - 1)$

So the equation of the line through the point (1, 4) with slope –7 is $y - 4 = -7(x - 1)$.

It is not necessary to develop the equation from scratch every time. Having derived the point-slope form, you can just substitute the given slope, m, and the point (x_1, y_1) into the final form of the equation $(y - y_1) = m(x - x_1)$.

Here's an example. To find the equation of the line through the point (–3, 8) with slope 14:

1. Substitute the slope $m = 14$ and the point (–3, 8) into the point-slope form.	$y - 8 = 14[x - (-3)]$
2. Simplify.	$y - 8 = 14(x + 3)$

So the equation of the line through the point (–3, 8) with slope 14 is $y - 8 = 14(x + 3)$.

If you are given two points, you can still use the point-slope form to find the equation of the line.

To find the point-slope form of the equation of a line through two points:

1. Find the slope of the line by substituting the coordinates of the two points into the definition of slope.

2. Use the slope and either one of the points to write the equation in point-slope form.

For example, to find the equation of the line through the points (–2, –3) and (0, 5):

1. Find the slope of the line.

$$m = \frac{5-(-3)}{0-(-2)} = \frac{5+3}{2} = \frac{8}{2} = 4$$

2. Use the slope, $m = 4$, and one of the points, say (–2, –3), to write the equation in point-slope form.

$$y-(-3) = 4[x-(-2)]$$
$$y+3 = 4(x+2)$$

So the equation of the line through the points (–2, –3) and (0, 5) is $y + 3 = 4(x + 2)$.

The Standard Form of the Equation of a Line

The equation of a line in point-slope form can be rewritten in several different forms. One of these is the standard form. The equation of a line in standard form is $Ax + By = C$. Here, A and B are not both equal to zero.

To change a linear equation in another form to standard form:

1. Distribute (if necessary) to remove parentheses.

2. Move all the x- and y-terms to the left side of the equation.

3. Move all of the constant terms to the right side of the equation.

4. If necessary, rearrange the terms on the left so the equation is in the form $Ax + By = C$.

For example, to rewrite the equation of the line $y - 8 = 11(x - 2)$ in standard form:

1. Distribute the 11. $\quad y - 8 = 11x - 22$

2. Subtract $11x$ from both sides. $\quad y - 8 - 11x = -22$

3. Add 8 to both sides. $\quad y - 11x = -14$

4. Switch the x- and y-terms. $\quad -11x + y = -14$

So the equation of the line $y - 8 = 11(x - 2)$ in standard form is $-11x + y = -14$.

Here's a different type of example. To write the equation of the line through the point (7, 6) with slope $m = -5$ in standard form:

1. Substitute the slope, $m = -5$, and the point (7, 6) into the point-slope form. $\quad y - 6 = -5(x - 7)$
2. Distribute the -5. $\quad y - 6 = -5x + 35$
3. Add $5x$ to both sides. $\quad y - 6 + 5x = 35$
4. Add 6 to both sides. $\quad y + 5x = 41$
5. Switch the x- and y-terms. $\quad 5x + y = 41$

So the standard form of the equation of the line through (7, 6) with slope -5 is $5x + y = 41$.

The Slope-Intercept Form of the Equation of a Line

The slope-intercept form of the equation of a line is $y = mx + b$. Here, m is the slope of the line and $(0, b)$ is the y-intercept.

To change a linear equation in another form to the slope-intercept form:

1. Solve the equation for y.
2. If necessary, rearrange the terms on the right so the equation is in the form $y = mx + b$.

For example, to find the slope-intercept form of the equation $3x + 12y = 6$:

1. Solve the equation for y.

$$3x + 12y = 6$$
$$3x + 12y - 3x = 6 - 3x$$
$$12y = 6 - 3x$$
$$\frac{12y}{12} = \frac{6}{12} - \frac{3}{12}x$$
$$y = \frac{1}{2} - \frac{1}{4}x$$

2. Rearrange the terms on the right. $\quad y = -\frac{1}{4}x + \frac{1}{2}$

So the slope-intercept form of the equation $3x + 12y = 6$ is $y = -\frac{1}{4}x + \frac{1}{2}$.

Here's another example. To write the equation of the line with slope 2 through the point $\left(\frac{1}{2}, 7\right)$ in slope-intercept form:

1. Write the point-slope form of the equation of the line. $\quad y - 7 = 2\left(x - \frac{1}{2}\right)$
2. Solve the equation for y.

$$y = 2\left(x - \frac{1}{2}\right) + 7$$
$$y = 2x - 1 + 7$$
$$y = 2x + 6$$

So the slope-intercept form of the line through the point $\left(\frac{1}{2}, 7\right)$ with slope 2 is $y = 2x + 6$.

To find the slope-intercept form of a linear equation when you are given the slope, m, and the y-intercept, $(0, b)$:

Note that when $x = 0$, $y = m(0) + b$ or $y = b$.

1. Start with the equation $y = mx + b$.
2. Substitute the slope for m.
3. Substitute the y-coordinate of the y-intercept for b.

Here's an example. To find the slope-intercept form of the line with slope $m = \frac{6}{11}$ and y-intercept $(0, 2)$:

1. Start with the equation $y = mx + b$. $\quad y = mx + b$
2. Substitute the slope for m. $\quad y = \frac{6}{11}x + b$
3. Substitute the y-coordinate of the y-intercept for b. $\quad y = \frac{6}{11}x + 2$

So the slope-intercept form of the line with slope $m = \frac{6}{11}$ and y-intercept $(0, 2)$ is $y = \frac{6}{11}x + 2$.

Horizontal Lines

The equation of a horizontal line is $y = c$. Here, c is a constant.

To find the equation of a horizontal line:

1. Find the y-coordinate of any point on the line.
2. Substitute the value of the y-coordinate for c in the equation $y = c$.

For example, to find the equation of the horizontal line through the point $(–18, 4)$:

1. Find the y-coordinate of any point on the line. $\quad 4$
2. Substitute 4 for c. $\quad y = 4$

So the equation of the horizontal line through $(–18, 4)$ is $y = 4$.

Vertical Lines

The equation of a vertical line is $x = c$. Here, c is a constant.

To find the equation of a vertical line:

1. Find the x-coordinate of any point on the line.
2. Substitute the value of the x-coordinate for c in the equation $x = c$.

For example, to find the equation of the vertical line through the point $(7, –7)$:

1. Find the x-coordinate of any point on the line. $\quad 7$
2. Substitute 7 for c. $\quad x = 7$

So the equation of the vertical line through $(7, –7)$ is $x = 7$.

Parallel and Perpendicular Lines

To find the negative reciprocal of a fraction, just switch the numerator and the denominator and change the sign. For example, the negative reciprocal of $\frac{3}{7}$ is $-\frac{7}{3}$. The negative reciprocal of 5 is $-\frac{1}{5}$.

You know how to find the equation of a line given a point on the line and the slope of the line. Sometimes you may not be given the slope of a line when you need it. However, in some cases, these facts may help you find the slope:

- Parallel lines have the same slope.
- Perpendicular lines have slopes that are negative reciprocals of each other.

For example, to find the equation of the line that is parallel to $2x + y = -9$ and passes through the point (0, –6):

1. Find the slope of the line $2x + y = -9$.

   Convert to slope-intercept form:
   $$2x + y = -9$$
   $$y = -2x - 9$$
   So the slope, m, is –2.

2. Find the slope of the parallel line.

   Parallel lines have the same slope, so $m = -2$.

3. Find the equation of the parallel line. Use the slope-intercept form, since you have the slope and the y-intercept.

   $$y = mx + b$$
   $$y = -2x - 6$$

So the equation of the line that is parallel to $2x + y = -9$ and passes through the point (0, –6) is $y = -2x - 6$.

Here's another example. To find the equation of the line through $\left(\frac{3}{5}, \frac{4}{5}\right)$ that is perpendicular to $y = -\frac{1}{5}x + 8$:

1. Find the slope of the line $y = -\frac{1}{5}x + 8$.

   This equation is in slope-intercept form, so the slope, m, is $-\frac{1}{5}$.

2. Find the slope of the perpendicular line.

   Perpendicular lines have slopes that are negative reciprocals of each other, so:
   $$m = -\frac{1}{-\frac{1}{5}}$$
   $$= -1 \cdot -\frac{5}{1}$$
   $$= 5$$

3. Find the equation of the perpendicular line. Use the point-slope form, since you have a point and the slope.

   $$y - y_1 = m(x - x_1)$$
   $$y - \frac{4}{5} = 5\left(x - \frac{3}{5}\right)$$

So the equation of the line perpendicular to $y = -\frac{1}{5}x + 8$ that passes through $\left(\frac{3}{5}, \frac{4}{5}\right)$ is $y - \frac{4}{5} = 5\left(x - \frac{3}{5}\right)$.

Sample Problems

Answers to Sample Problems

1. Find the standard form of the equation of the line with slope $-\frac{1}{4}$ that passes through the point (3, 1).

   ☐ a. Write the point-slope form of the linear equation. $y - \_\_\_\_ = -\frac{1}{4}(x - \_\_\_\_)$ — *a. 1, 3*

   ☑ b. Distribute. $y - 1 = -\frac{1}{4}x + \frac{3}{4}$

   ☑ c. Move the x-term to the left side. $y - 1 + \frac{1}{4}x = \frac{3}{4}$

   ☐ d. Move the constant terms to the right side. $y + \frac{1}{4}x = \_\_\_\_$ — *d.* $\frac{7}{4}$

   ☐ e. Rearrange the left side of the equation. $\_\_\_\_\_\_ = \frac{7}{4}$ — *e.* $\frac{1}{4}x + y$

2. Rewrite $12x + 7y = 49$ in slope-intercept form.

   ☐ a. Solve the equation for y. If necessary, rearrange the terms on the right. $y = \_\_\_\_\_\_$ — *a.* $-\frac{12}{7}x + 7$

3. Find the equation of the line parallel to the y-axis that passes through the point (12, 20).

   ☑ a. Determine if the line is vertical or horizontal. The line parallel to the y-axis is a vertical line.

   ☐ b. Write the equation of a vertical line. ______ — *b.* $x = c$

   ☐ c. Find the x-coordinate of any point on the line. ______ — *c. 12*

   ☐ d. Substitute the value of the x-coordinate for c. $x = \_\_\_\_$ — *d. 12*

4. Find the equation of the line through the point (4, –9) that is parallel to the line through the points (1, 3) and (5, 7).

   ☐ a. Find the slope of the line through the points (1, 3) and (5, 7). $m = \_\_\_\_$ — *a.* $\frac{7-3}{5-1}$ *or 1*

   ☐ b. Find the slope of the parallel line. $m = \_\_\_\_$ — *b. 1*

   ☐ c. Find the equation of the parallel line. Use point-slope form. $y - \_\_\_\_ = \_\_\_\_(x - \_\_\_\_)$; $y + \_\_\_ = x - \_\_\_$ — *c. (–9), 1, 4; 9, 4*

5. Find the equation of the line through (8, 7) that is perpendicular to the line $2x + 3y = 1$.

   ☐ a. Find the slope of the line $2x + 3y = 1$.
   $3y = -2x + 1$
   $y = -\frac{2}{3}x + \frac{1}{3}$
   $m = \_\_\_\_$ — *a.* $-\frac{2}{3}$

   ☐ b. Find the slope of the perpendicular line. $m = \_\_\_\_$ — *b.* $\frac{3}{2}$

   ☐ c. Find the equation of the perpendicular line. Use point-slope form. ______ — *c.* $y - 7 = \frac{3}{2}(x - 8)$

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

Graphing Lines

1. Find the coordinates of the points P, Q, and R as shown in Figure EII.E.9.

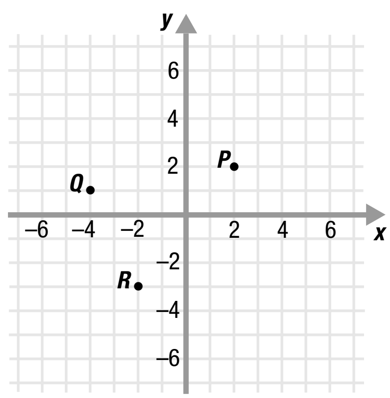


Figure EII.E.9

2. Find the distance between the points (0, 0) and (0, 7).

3. Plot the points in the table below, and then draw a line through the points.

x	y
–2	–2
0	–1
2	0
4	1

4. Find the x-intercept of the line $6x + 9y = 54$.

5. Find the distance between the points (–2, –3) and (4, –3).

6. Graph the line $y - 2 = -3(x + 1)$.

7. Find the slope, m, of the line through the points (12, 7) and (5, 4).

8. Find the distance between the points (–2, 9) and (–5, 5).

9. Graph the line $6x - 2y = -3$.

10. Find the x- and y-intercepts of the line $y + 3 = 16(x - 2)$.

11. Of the points A (3, 3), B (–2, 2), and C (1, 1), which two are closest together?

12. Graph the line with slope $m = -\frac{5}{2}$ that passes through the point (3, –2).

Finding Equations

13. Find the point-slope form of the equation of the line through the point (1, 2) with slope 5.

14. Write the linear equation $y = -16x + 9$ in standard form.

15. Find the equation of the horizontal line through the points (1, 12) and (6, 12).

16. Find the point-slope form of the equation of the line through the points (–7, 3) and (–4, 2).

17. Write the linear equation $y + 8 = 2(x + 6)$ in standard form.

18. Find the equation of the vertical line that crosses the x-axis 3 units to the left of the y-axis.

19. Find the slope-intercept form of the equation of the line through (0, 9) that is parallel to the line $y - 7 = 18(x + 14)$.

20. Find the standard form of the equation of the line passing through the points (2, 11) and (3, 12).

21. Find the equation of the line through $\left(\frac{3}{7}, \frac{1}{8}\right)$ that is perpendicular to the y-axis.

22. Find the point-slope form of the equation of the line through (4, –6) that is parallel to the line $36x + 18y = -42$.

23. Find the standard form of the equation of the line through (–1, 1) that is perpendicular to the line $6x - y = 11$.

24. Find the equation of the line perpendicular to the line $-5x - 40y = 10$ that has the same y-intercept as this line.

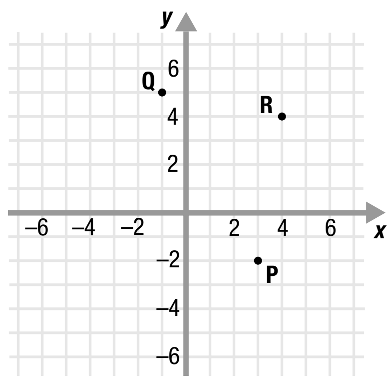
APPLY

Practice Problems

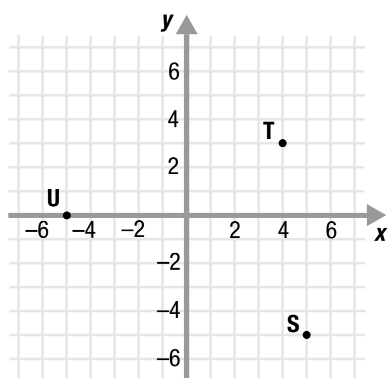
Here are some additional practice problems for you to try.

Graphing Lines

1. Find the coordinates of the points *P*, *Q*, and *R* below.



2. Find the coordinates of the points *S*, *T*, and *U* below.



3. Circle the point below that is in Quadrant II.

   (4, 6) (–3, 4)

   (0, –2) (5, –7)

4. Circle the point below that is in Quadrant III.

   (–3, –2) (4, –4)

   (7, 0) (–4, 6)

5. Find the distance between the points (8, 4) and (3, 4).
6. Find the distance between the points (–13, 12) and (5, 12).
7. Find the distance between the points (–3, 7) and (–3, –2).
8. Find the distance between the points (9, 2) and (3, 10).
9. Find the distance between the points (8, –5) and (–4, 0).
10. Find the distance between the points (–4, 1) and (–1, –3).
11. Graph the line $y + 3 = -2x$.
12. Graph the line $y - 5 = 3x$.
13. Graph the line $y + 4 = -\frac{1}{2}x$.
14. Find the slope m of the line that passes through the points (4, 7) and (8, 13).
15. Find the slope m of the line that passes through the points (3, 5) and (–5, 7).
16. Find the slope m of the line passing through the points (5, –3) and (–3, –3).
17. Find the x- and y-intercepts of the line $2x - y = 10$.
18. Find the x- and y-intercepts of the line $3x - 5y = -15$.
19. Find the x- and y-intercepts of the line $4x + 3y = 21$.
20. Graph the line $y + 3 = -2(x - 4)$.
21. Graph the line $y - 2 = \frac{1}{2}(x + 6)$.
22. Graph the line $y + 4 = -\frac{2}{3}(x - 3)$.
23. Circle each equation below that represents a line that has slope 2.

    $y + 2 = -2(x - 7)$ $y = \frac{1}{2}x + 10$

    $y - 2x = 15$ $2x + 2y = 2$

24. Circle each equation below that represents a line that has slope –3.

    $y = -\frac{1}{3}x + 5$ $y - 6 = 3(x + 3)$

    $x + y = -3$ $3x + y = 9$

25. Circle each equation below that represents a line that has slope –1.

$y = x - 1$ $\quad$ $2x + 2y = 14$

$y - 5 = x + 3$ $\quad$ $y = -x + 2$

26. Graph the line that passes through the point (0, 3) with slope $m = 2$.

27. Graph the line with slope $m = -\frac{1}{2}$ that passes through the point (0, 1).

28. Graph the line that passes through the point (3, –1) with slope $m = \frac{4}{3}$.

Finding Equations

29. Find the point-slope form of the equation of the line that passes through the point (–3, 5) with slope 2.

30. Find the point-slope form of the equation of the line that passes through the point (2, –4) with slope $\frac{1}{4}$.

31. Find the equation of the horizontal line and the equation of the vertical line that pass through the point (–2, 6).

32. Find the equation of the horizontal line and the equation of the vertical line that pass through the point (0, 8).

33. Find the equation of the horizontal line and the equation of the vertical line that pass through the point (–3, 0).

34. Find the equation of the vertical line that crosses the x-axis 5 units to the right of the y-axis.

35. Find the equation of the horizontal line that crosses the y-axis 4 units below the x-axis.

36. Find the slope-intercept form of the equation of the line that passes through the point (3, 1) with slope –5.

37. Find the slope-intercept form of the equation of the line that passes through the point (–2, 2) with slope 4.

38. Find the slope-intercept form of the equation of the line that passes through the point (–2, 7) with slope $\frac{1}{2}$.

39. Find the slope-intercept form of the equation of the line that passes through the points (3, –2) and (1, 4).

40. Find the slope-intercept form of the equation of the line that passes through the points (3, –3) and (5, 1).

41. Find the slope-intercept form of the equation of the line that passes through the points (–6, 3) and (–3, 2).

42. Find the x-intercept of the line that passes through the point (2, 4) with slope –2.

43. Find the y-intercept of the line that passes through the point (3, 6) with slope $-\frac{1}{3}$.

44. Find the y-intercept of the line that passes through the point (–5, 2) with slope $-\frac{1}{5}$.

45. Find the standard form of the equation of the line that passes through the points (0, –2) and (4, –6).

46. Find the standard form of the equation of the line that passes through the points (0, 1) and (–5, –9).

47. Find the standard form of the equation of the line that passes through the points (3, 5) and (–6, 8).

48. Find the equation in slope-intercept form of the line that passes through (4, 2) and is perpendicular to $y = 2x$.

49. Find the equation in slope-intercept form of the line that passes through (5, –2) and is perpendicular to $y = \frac{1}{3}x + 7$.

50. Find the equation in slope-intercept form of the line that passes through (–4, –9) and is perpendicular to $y = -\frac{2}{5}x - 17$.

51. Find the equation in slope-intercept form of the line that passes through (–2, 2) and is parallel to $y = -3x$.

52. Find the equation in slope-intercept form of the line that passes through (6, 7) and is parallel to $y = \frac{2}{3}x + 10$.

53. Find the equation in slope-intercept form of the line that passes through (8, –3) and is parallel to $y = -\frac{1}{4}x - 15$.

54. Find the equation in slope-intercept form of the line that has x-intercept (4, 0) and y-intercept (0, –8).

55. Find the equation in slope-intercept form of the line that has x-intercept (–2, 0) and y-intercept (0, 6).

56. Find the equation in slope-intercept form of the line that has x-intercept (–6, 0) and y-intercept (0, –4).

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Look at Figure EII.E.10. Determine in which quadrant the points *P*, *Q*, and *R* lie.

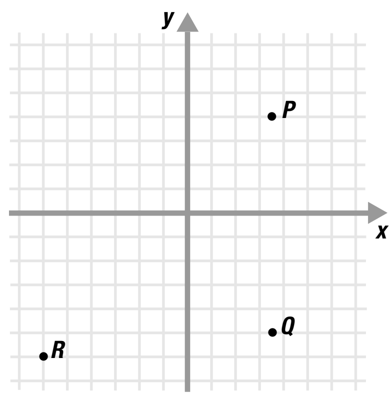


Figure EII.E.10

2. Find the distance, *d*, between the point (6, 11) and the point (–3, –1).

3. Find both the rise (change in *y*) and the run (change in *x*) in moving from the point (2, 3) to the point (5, 1).

4. A line passes through the points (0, 6) and (–3, –3). Find its slope.

5. Circle the equation of the line through the point (20, –7) that has slope 9.

$$y - 7 = 9(x - 20)$$

$$y - 20 = 9(x + 7)$$

$$x + 7 = 9(y - 20)$$

$$y + 7 = 9(x - 20)$$

6. Find the *y*-intercept of the line that has slope $m = \frac{1}{2}$ and passes through the point $\left(-3, \frac{1}{2}\right)$.

7. Find the equation of the vertical line and the equation of the horizontal line that pass through the point (–11, 17).

8. Find the standard form of the equation of the line that passes through the point (0, 5) and is perpendicular to $y = -\frac{1}{8}x + 30$.