LESSON 1.1 – THE REAL NUMBERS

OVERVIEW

Here is what you'll learn in this lesson:

Number Line and Notation

a. Subsets of real numbers: natural numbers, integers, rationals, irrationals

b. Graphing real numbers on a number line

c. Ordering symbols: $=, \neq, <, >, \leq, \geq$

d. The absolute value of a real number

e. Grouping symbols

f. Exponents

Numbers have been important to people since ancient times. Early cultures in Australia, South America, and South Africa all had basic counting systems. Later civilizations, including those of the Egyptians, Babylonians, Chinese, Greeks, and Mayans had sophisticated number systems, some of which form the basis of mathematics today.

You will begin your study of algebra by learning about different types of numbers. You will also learn to use a number line to compare numbers and to find distances. Finally, you will learn some mathematical notation.

EXPLAIN

NUMBER LINE AND NOTATION

Summary

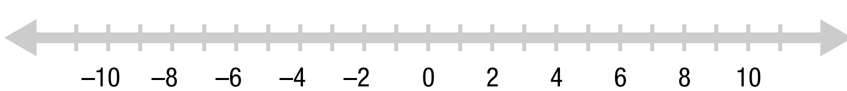
The Number Line

People in ancient civilizations used only the counting numbers: 1, 2, 3, 4, and so on. As their lives became more complex, they found they needed additional numbers to solve new problems and explain new situations. The types of numbers which have arisen over time—natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers—have given people the tools they need to solve problems in an increasingly complex life.

The table below lists some of the different types of numbers.

Types of Numbers	**Examples**
Natural Number	1, 2, 3, 4, …
Whole Numbers	0, 1, 2, 3, …
Integers	… –3, –2, –1, 0, 1, 2, 3, …
Rational Numbers	When written as decimals, rational numbers either end or repeat in a pattern. Some examples are: $\frac{9}{4} = 2.25$, $\sqrt{16} = 4$, and $-\frac{1}{3} = -.333\ldots$.
Irrational Numbers	When written as decimals, irrational numbers neither end nor repeat in a pattern. Some examples are: $\sqrt{2} = 1.41421\ldots$, $-\sqrt{5} = -2.23606\ldots$, and $\pi = 3.14159\ldots$.
Real Numbers	The rational and irrational numbers.

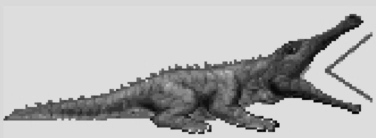
These different types of numbers can be pictured on a line called the number line.



Notation

Mathematicians use symbols as a shorthand way of expressing mathematical ideas.

Do you get confused about “<” and “>”? Think of a hungry alligator opening its jaws wide to swallow the bigger number.



Comparison Symbols

Certain mathematical symbols can help you compare the relative size of numbers. The table below summarizes some of these symbols.

Symbol	Meaning	Example
$=$	equal to	$3 = 3$
$\neq$	not equal to	$5 \neq 7$
$<$	less than	$2 < 9$
$>$	greater than	$7 > 1$
$\leq$	less than or equal to	$4 \leq 5$ $4 \leq 4$
$\geq$	greater than or equal to	$5 \geq 4$ $4 \geq 4$

On a number line:

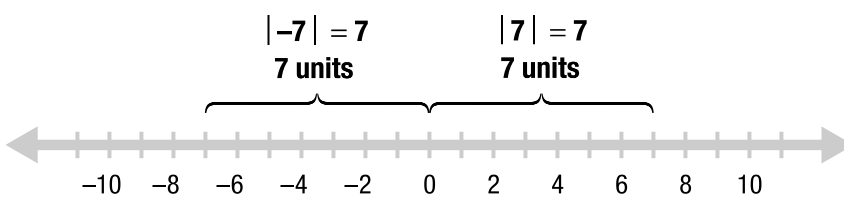
- Numbers to the left are less than numbers to the right. For example, $3 < 5$ means that 3 lies to the left of 5.
- Numbers to the right are greater than numbers to the left. For example, $5 > 3$ means that 5 lies to the right of 3.

Absolute Value

The absolute value of a number is the distance of that number from zero on the number line.

Vertical bars enclosing a number are used to denote absolute value.

For example, $|7| = 7$ and $|-7| = 7$.



Multiplication

Because the letter x is often used as a variable in algebra, it can be confusing to use it to indicate multiplication. For this reason, it is common to use a dot, parentheses, or brackets to denote multiplication.

For example, some ways to write 4 times 5 are:

$4 \cdot 5$ $(4)(5)$ $4(5)$ $(4)5$ $(4) \cdot (5)$

Exponents

Exponents are used to indicate repeated multiplication of the same number.

For example: $2^4 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2}_{\text{4 factors}}$

In this case, the number 2 is called the base and the number 4 is called the exponent. The exponent, 4, indicates that there are 4 factors of 2.

Power is another name for exponent.

2^2 is read as "two squared." 2^3 is read as "two cubed." Two to any exponent greater than 3 is read as "two to the fourth," "two to the fifth," "two to the sixth," etc. You could also say "two to the fourth power," "two to the fifth power," "two to the sixth power," etc.

Sample Problems

1. Find the value of 4^5.

☐ a. Write without exponents. $4^5 =$ ______________

☐ b. Multiply. $=$ _____

2. What is $|-\sqrt{2}|$?

☐ a. Find the absolute value. $|-\sqrt{2}| =$ _____

3. Use the numbers 4 and 5 and the symbols $=, \neq, <, >, \leq$, and $\geq$ to make true statements.

☐ a. Write statements using only the number 4.

$4 = 4$
$4 \leq 4$
4 __ 4

☐ b. Write statements using only the number 5.

5 __ 5
5 __ 5
5 __ 5

☐ c. Write statements using 4 and 5.

$4 \neq 5$
$5 > 4$
5 __ 4
4 __ 5

Answers to Sample Problems

1. a. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

b. 1024

2. a. $\sqrt{2}$

3. a. $\geq$

b. in any order: $\geq$, $=$, $\leq$

c. $\geq$ or $\neq$
$\leq$ or $<$

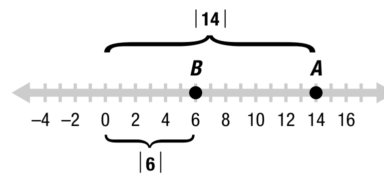
EXPLORE

Sample Problems

Answers to Sample Problems

On the computer you plotted points on the number line and observed the density of points and the correspondence between points and numbers. Below are some additional exploration problems.

1. On the number line, $A = 14$ and $B = 6$. Write an expression that represents the distance between A and B, then simplify your expression to find the distance.



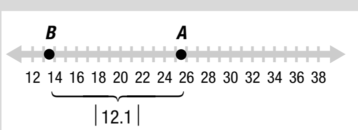
   ☑ a. Write an expression that represents the distance. $|14 - 6|$

b. 8

   ☐ b. Find the distance. = ___

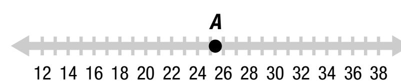
2. A and B are two points on the number line, and $A > B$. If $A = 25.5$ and the distance between the points is 12.1, what is the coordinate of B?

a.



b. 13.4

   ☐ a. Plot a point B 12.1 units to the left of A.



   ☐ b. Find the coordinate of B. B = _____

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain
Number Line and Notation

1. Circle the true statements.

   $3 < 12$ $\quad$ $4 \leq 4$

   $5 = \frac{20}{4}$ $\quad$ $6 \neq 7$

   $-2 < -6$

2. Find the absolute values:

   a. $|7|$

   b. $|-3|$

   c. $|0.4|$

   d. $|-1.6|$

   e. $|-0.72|$

3. Find the value of 5^4.

4. Circle the true statements.

   $2 \neq 3$ $\quad$ $-8 < -4$

   $9 > 6$ $\quad$ $2 \geq \frac{5}{8}$

   $8 < 4$

5. Find the absolute values:

   a. $|9|$

   b. $|-17|$

   c. $|2.3|$

   d. $|-4.8|$

   e. $|-0.485|$

6. Rewrite using exponents: $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$

7. Find: $|-2^3|$

8. Find: $3^2 \cdot 4^3$

9. Restaurants buy eggs in bulk by the box. Each box of eggs contains 12 cartons. Each carton has 12 rows and each row contains 12 eggs. Which of the following expresses the number of eggs in a box?

   $12 \cdot 3$

   12^3

   3^{12}

   $12\,(12 + 12 + 12)$

   $12 + 12 + 12$

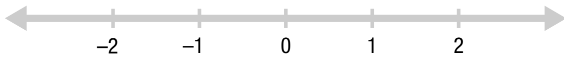
10. In a small town, 7 sisters each had 7 baskets.
    In each basket, there were 7 cats.
    Each cat had 7 kittens.
    In total, how many kittens were there?

11. Rewrite using exponents: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 7 \cdot 7 \cdot 7$

12. Find: $3^3 \cdot 5^2$

Explore

13. Plot the points 1, $-\sqrt{3}$, and $\sqrt{2}$ on a number line, then list them in order from smallest to largest.



14. On the number line, $A = 47$ and $B = 59$. Which expression represents the distance between A and B?

    $|47| - |59|$

    $|47 - 59|$

    $|59| + |47|$

    $\frac{|47 + 59|}{2}$

15. A and B are two points on the number line and $A > B$. If $A = 16.7$ and the distance between the points is 7.9, what is the coordinate of B?

16. Plot the points 5, $\sqrt{29}$, π, and $\sqrt{16}$ on a number line, then list them in order from smallest to largest.

17. On the number line, $A = 124$ and $B = -29$. Which expression represents the distance between A and B?

    $|124| - |-29|$

    $|124| + |-29|$

    $\frac{|124 - 29|}{2}$

    $|124 - 29|$

18. A and B are two points on the number line. If $A = 9.4$ and the distance between the points is 5.7, what are the two possibilities for the coordinate of B?

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Number Line and Notation

1. Circle the true statements.

   $9 = 9$

   $5 > 5$

   $7 \leq 11$

   $15 \leq 15$

   $2 < 0$

2. Circle the true statements.

   $5 \neq 5$

   $6 \leq 6$

   $7 < 7$

   $12 \geq 12$

   $1 > 0$

3. Circle the true statements.

   $7 \neq 7$

   $4 < 4$

   $6 \leq 12$

   $9 \geq 9$

   $10 > 15$

4. Find the absolute values.

   a. $|7|$

   b. $|-9|$

   c. $|0.25|$

   d. $|2.3|$

   e. $|-7.45|$

5. Find the absolute values.

   a. $|0|$

   b. $|100|$

   c. $|-0.001|$

   d. $|4.33|$

   e. $|-2.497|$

6. Find the absolute values.

   a. $|-6|$

   b. $|3|$

   c. $|0.5|$

   d. $|1.9|$

   e. $|-5.18|$

7. Find: 8^2

8. Find: 5^3

9. Find: 7^3

10. Find: 2^7

11. Find: 3^5

12. Find: 2^5

13. Rewrite using exponents: $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

14. Rewrite using exponents: $10 \cdot 10 \cdot 10 \cdot 10$

15. Rewrite using exponents: $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$

16. Given the sets P and Q below, determine whether the following statements are true or false.

    P = {3, 5, 7, 9, 11}

    Q = {1, 3, 6, 9, 12, 15}

    a. $P \subset Q$

    b. $Q \not\subset P$

    c. $3 \notin P$

    d. $3 \in Q$

17. Given the sets S and T below, determine whether the following statements are true or false.

    S = {2, 4, 6, 8, 10, 12, 14}

    T = {4, 8, 12}

    a. $T \subset S$

    b. $S \subset T$

    c. $4 \in S$

    d. $4 \in T$

18. Given the sets R and S below, determine whether the following statements are true or false.

    R = {1, 2, 5, 7, 8, 9}

    S = {1, 2, 5}

    a. $S \subset R$

    b. $R \not\subset S$

    c. $2 \in R$

    d. $2 \in S$

19. Find: $|-4^3|$

20. Find: $|-5^2| - |3^3|$

21. Find: $|3^2| - |2^3|$

22. Find: $3^4 \cdot 2^3$

23. Find: $5^3 \cdot 4^2$

24. Find: $2^4 \cdot 9^2$

25. On the number line, $A = 36$ and $B = -16$. Write an expression that represents the distance between A and B.

26. On the number line, $C = -36$ and $D = -17$. Write an expression that represents the distance between C and D.

27. A and B are two points on the number line. If $A = 31.7$ and the distance between the points is 7.3, what are the two possibilities for B?

28. E and F are two points on the number line. If $E = -25.6$ and the distance between the points is 4.7, what are the two possibilities for F?

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Circle the true statements.

   $3 > -4$

   $-5 > -7$

   $2 < 2$

   $0 \geq 3$

   $-6 \leq -6$

   $-1 \geq -1$

2. Find the absolute values:

   a. $|8|$

   b. $|-12.18|$

   c. $|-0.23|$

   d. $|15|$

   e. $|3.7|$

3. Which of the symbols, >, <, ≥, ≤, =, and ≠, could replace the ? below to make a true statement?

   $-7 \ ? \ -9$

4. Which of the following is a rational number between 0 and 1?

   $(.91)^2$

   $\sqrt{.91}$

   $\frac{1}{\sqrt{2}}$

   $-\left|\frac{2}{3}\right|$

5. The population of a colony of insects raised in a laboratory doubles every week. If you start with 2 insects, you will have 4 insects after 1 week, 8 insects after 2 weeks, and so on. How many insects will you have after 4 weeks?

   $5 \cdot 2$

   4^2

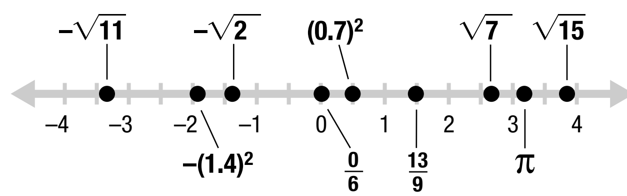
   $2 + 2 + 2 + 2$

   2^4

   2^5

6. *A* and *B* are two points on a number line, and $A < B$. If $A = -1$ and the distance between the two points is 2.5, what is the coordinate of *B*?

7. Find the points on the given number line which have an absolute value less than 2.



8. Which expression represents the distance on the number line between –47 and 36?

   $|-47 + 36|$

   $|36 - 47|$

   $|36 + 47|$

   $|-47| - |36|$