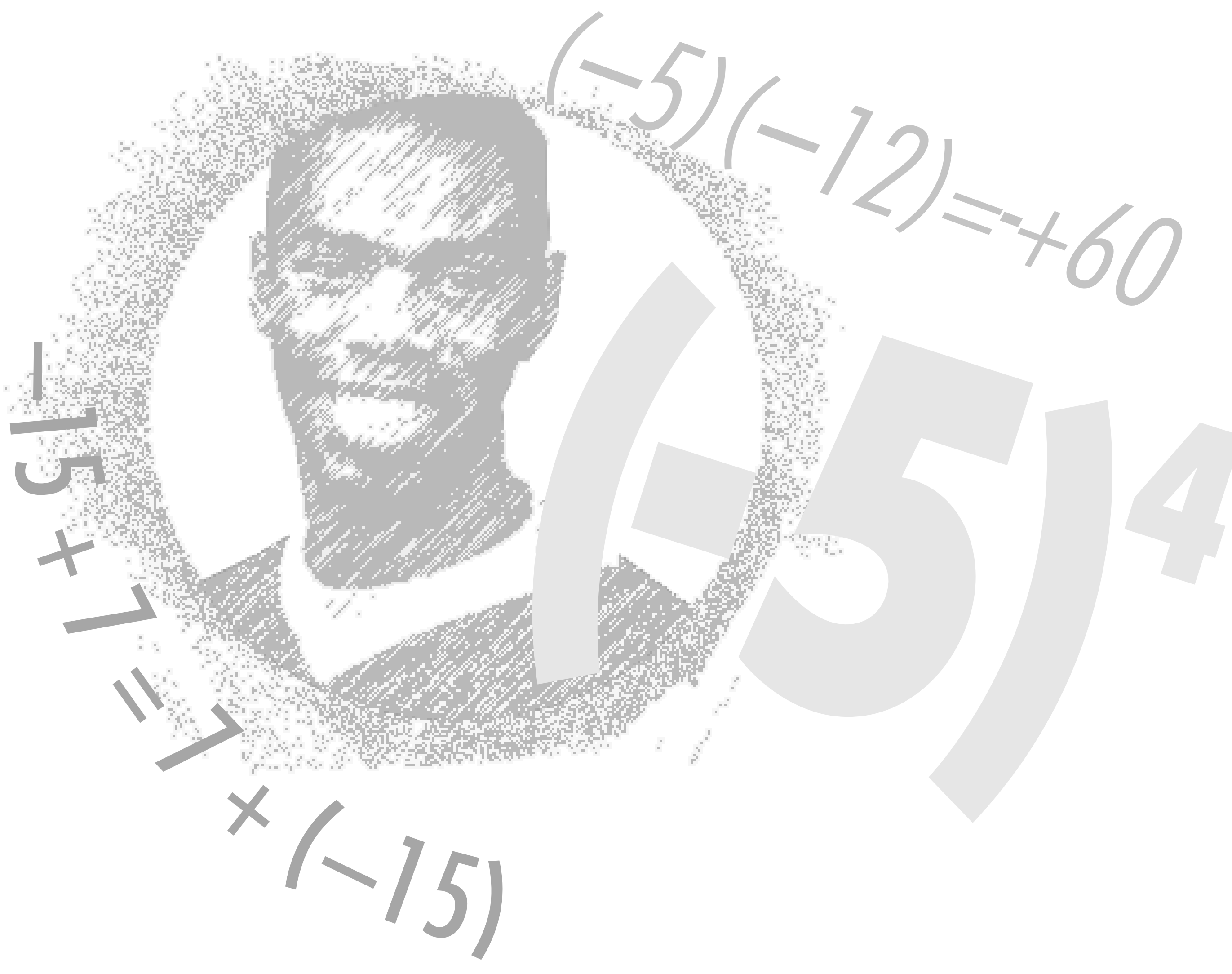
LESSON F4.2 – SIGNED NUMBERS II



Overview

You have already added and subtracted signed numbers. Now you will learn how to do more with signed numbers.

In this lesson you will learn how to multiply and divide, use exponents, and apply the properties of signed numbers. You will also learn how to combine signed numbers using the order of operations.

Before you begin, you may find it helpful to review the following mathematical ideas which will be used in this lesson:

To see these Review problems worked out, go to the Overview module of this lesson on the computer.

Review 1

Add signed numbers.

Do this addition: $-98 + 75$

Answer: -23

Review 2

Subtract signed numbers.

Do this subtraction: $-418 - (-365)$

Answer: -53

Review 3

Use the order of operations with whole numbers.

Use the order of operations to find the value of this expression. $12 \div (4 - 2) + 5 \times 2^3$

Answer: 46

Review 4

Apply properties of whole numbers.

Fill in the blank: $(23 + 7) + 18 = 23 + (\_\_\_ + 18)$

Answer: $x = 7$

Review 5

Combine like terms involving whole numbers.

Do this subtraction and addition: $13 + 25x - 2 - 5x$

Answer: $20x + 11$

Review 6

Solve an equation of the form $ax = b$.

Solve this equation for x: $7x = 56$

Answer: $x = 8$

Explain

In Concept 1: Multiplying and Dividing, you will find a section on each of the following:

- **Multiplying Two Numbers with Different Signs**
- **Multiplying Two Numbers with the Same Signs**
- **Multiplying more than Two Signed Numbers**
- **Dividing Two Numbers with Different Signs**
- **Dividing Two Numbers with the Same Sign**
- **Using Division to Solve Certain Equations that Contain Signed Numbers**

Remember, the dot in the expression $4 \cdot (-20)$ means multiply.

You may find these Examples useful while doing the homework for this section.

Concept 1: Multiplying and Dividing

Multiplying Two Numbers with Different Signs

You know how to add numbers with the same sign. For example,

$$(-20) + (-20) + (-20) + (-20) = -80$$

The repeated addition

$$(-20) + (-20) + (-20) + (-20)$$

can be written as multiplication:

$$4 \cdot (-20)$$

So, $4 \cdot (-20) = -80$.

$$\underset{\text{positive sign}}{\underset{\uparrow}{4}} \cdot \underset{\text{negative sign}}{\underset{\uparrow}{(-20)}} = \underset{\text{negative sign}}{\underset{\uparrow}{-80}}$$

When you multiply two numbers with different signs, you get a negative number.

To multiply two numbers with different signs:

- Ignore the signs and do the multiplication.
- Attach a negative sign.

Example 1

1. Find $(-16) \cdot 14$.

   To find the product:

   - *Ignore the signs and do the multiplication.* $16 \cdot 14 = 224$
   - *Attach a negative sign:* -224

   So, $(-16) \cdot 14 = -224$.

Example 2

2. Find $\frac{4}{5} \cdot -\frac{2}{3}$.

   To find the product:

   - *Ignore the signs and do the multiplication.* $\frac{4}{5} \cdot \frac{2}{3} = \frac{8}{15}$
   - *Attach a negative sign:* $-\frac{8}{15}$

   So, $\frac{4}{5} \cdot \left(-\frac{2}{3}\right) = -\frac{8}{15}$.

Multiplying Two Numbers with the Same Sign

You have seen how to multiply two numbers with different signs.

Now you will see what happens when you multiply two numbers with the same sign.

You already know how to multiply two positive numbers. For example,

$2 \cdot 5 = 10.$

When you multiply two positive numbers, the result is positive.

Now, look at the chart below:

$$\begin{aligned} 3 \cdot (-5) &= -15 \\ 2 \cdot (-5) &= -10 \\ 1 \cdot (-5) &= -5 \\ 0 \cdot (-5) &= 0 \end{aligned}$$

Notice that the numbers in the first column on the left: 3, 2, 1, 0 are decreasing by one.

Notice that the numbers in the second column on the left: –5 , –5, –5, –5 are all –5.

Notice that the numbers in the column on the right: –15, –10, –5, 0 are increasing by 5.

If the pattern is to continue, then the chart will become

$$\begin{aligned} -1 \cdot (-5) &= +5 \\ -2 \cdot (-5) &= +10 \\ -3 \cdot (-5) &= +15 \\ -4 \cdot (-5) &= +20 \end{aligned}$$

and so on.

Observe that when you multiply two negative numbers, the result is positive.

*But when you **add** two negative numbers, the result is **negative**.*

$(-5) + (-12) = -17$

To multiply two numbers with the same sign:

- Ignore the signs and do the multiplication.
- Attach a positive sign.

(You can also write the positive number without using +.)

You may find these Examples useful while doing the homework for this section.

Example 3

3. Find $(-24) \cdot (-10)$.

To find the product:

• *Ignore the signs and do the multiplication.*	$24 \cdot 10 = 240$
• *Attach a positive sign.*	$+240$

So, $(-24) \cdot (-10) = 240$.

+240 can also be written without using +.

Example 4

4. Find (–3.2) · (–4.1).

To find the product:

- *Ignore the signs and do the multiplication.* $3.2 \cdot 4.1 = 13.12$
- *Attach a positive sign.* $+13.12$

+13.12 can also be written without using +.

So, (–3.2) · (–4.1) = 13.12.

Multiplying More Than Two Signed Numbers

You have seen how to multiply two signed numbers.

Here's a way to multiply more than two signed numbers:

- Multiply from left to right, two numbers at a time.

You may find these Examples useful while doing the homework for this section.

Example 5

5. Find (–21) · (–3) · (–10).

To find the product:	$(-21) \cdot (-3) \cdot (-10)$
• *Multiply from left to right, two numbers at a time.*	
–21 and –3 have the same sign, so their product is positive.	$= \quad 63 \quad \cdot (-10)$
63 and –10 have different signs, so their product is negative.	$= \quad -630$

So, (–21) · (–3) · (–10) = –630.

*When you have an **odd** number of negative factors, the product of the factors is **negative**.*

Example 6

6. Find (–5) · (–20) · 9.

To find the product:	$(-5) \cdot (-20) \cdot 9$
• *Multiply from left to right, two numbers at a time.*	
–5 and–20 have the same sign, so their product is positive.	$= \quad 100 \quad \cdot 9$
100 and 9 have the same sign, so their product is positive.	$= \quad 900$

So, (–5) · (–20) · 9 = 900.

*When you have an **even** number of negative factors, the product of the factors is **positive**.*

The following example shows you how to use a calculator to multiply more than two signed numbers.

Example 7

7. Find 6 · (–4) · (–8).

Here's a way to find 6 · (–4) · (–8):	*On the calculator, you'll see...*
• *Enter 6*	*6*
• *Press "×"*	*6*
• *Enter 4*	*4*
• *Press "+/–"*	*–4*
• *Press "×"*	*–24*
• *Enter 8*	*8*
• *Press "+/–"*	*–8*
• *Press "="*	*192*

So, 6 · (–4) · (–8) = 192.

Notice that to enter –4, you use 4 followed by "+/–". Do not use "–" and 4.

Dividing Two Numbers with Different Signs

To multiply two signed numbers, you multiply the numbers without their signs, and attach the appropriate sign to the answer.

You can divide signed numbers in much the same way.

To divide two numbers with different signs:

- Ignore the signs and do the division.
- Attach a negative sign.

Example 8

You may find these Examples useful while doing the homework for this section.

8. Find 816 ÷ (–8).

To find the quotient:

- *Ignore the signs and do the division.* 816 ÷ 8 = 102
- *Attach a negative sign.* –102

So, 816 ÷ (–8) = –102.

Example 9

9. Find –4.8 ÷ 0.2.

To find the quotient:

- *Ignore the signs and do the division.* 4.8 ÷ 0.2 = 24
- *Attach a negative sign.* –24

So, –4.8 ÷ 0.2 = –24.

The following example shows how to use a calculator to divide two numbers with different signs.

Example 10

10. Use a calculator to find –36 ÷ 4.

To use a calculator to find –36 ÷ 4:	*On the calculator, you'll see...*
• *Enter 36*	*36*
• *Press "+/–"*	*–36*
• *Press "÷"*	*–36*
• *Enter 4*	*4*
• *Press "="*	*–9*

Dividing Two Numbers with the Same Sign

You have seen that to divide two numbers with different signs, you did much the same as when you multiply two numbers with different signs.

Likewise, when you divide two numbers with the same sign, it is much like when you multiply two numbers with the same sign.

To divide two numbers with the same sign:

- Ignore the signs and do the division.
- The result is positive.

Remember, division by zero is **not** allowed.

You may find these Examples useful while doing the homework for this section.

Example 11

11. Find (–360) ÷ (–12).

To find the quotient:

• *Ignore the signs and do the division.*	*360 ÷ 12 = 30*
• *The result is positive.*	*30*

So, (–360) ÷ (–12) = 30.

Example 12

12. Find $\left(-2\frac{3}{4}\right) \div \left(-1\frac{5}{6}\right)$.

To find the quotient:

• *Ignore the signs and do the division.*

$$\left(2\frac{3}{4}\right) \div \left(1\frac{5}{6}\right)$$

$$= \frac{11}{4} \cdot \frac{6}{11}$$

$$= \frac{3}{2}$$

• *The result is positive.* $= 1\frac{1}{2}$

So, $\left(-2\frac{3}{4}\right) \div \left(-1\frac{5}{6}\right) = 1\frac{1}{2}$.

Solving an Equation

In this section, you will solve some equations that contain signed numbers. The equations you will be solving will look like this:

$$ax = b$$

where a and b are signed numbers. When you solve such an equation, you will find the value of x that makes the equation true.

An example of such an equation is $-2x = 14$. A dot is not needed between the -2 and the x since $-2x$ means -2 times x.

To solve an equation for x:

- Get x by itself on one side of the equation and a number on the other side of the equation.
- Check by replacing x with this number in the original equation.

You may find these Examples useful while doing the homework for this section.

Example 13

13. Solve this equation for x: $9x = -90$

Here's one way to solve the equation:

- Get x by itself on one side of the equation.
- To do this, divide both sides of the equation by 9.

$$9x = -90$$
$$\frac{9x}{9} = -\frac{90}{9}$$
$$x = -10$$

When you divide both sides of an equation by the same nonzero number, you keep the solution the same.

- Check the answer.
- Replace x in the original equation with the value -10.

$9x = -90$
Is $9(-10) = -90$?
Is $-90 = -90$?
Yes.

So, $x = -10$.

Example 14

14. Solve this equation for x $-4x = -288$

Here's one way to solve the equation:

- Get x by itself on one side of the equation.
- To do this, divide both sides of the equation by -4. This keeps the left and right sides equal.

$$-4x = -288$$
$$\frac{-4x}{-4} = \frac{-288}{-4}$$
$$x = 72$$

You can use long division to divide 288 by 4.

- Check the answer.
- Replace x in the original equation with the value 72.

$-4x = -288$
Is $(-4)(72) = -288$?
Is $-288 = -288$?
Yes.

So, $x = 72$.

Example 15

15. Solve this equation for x: $\frac{x}{3} = -12$

Here's one way to solve the equation:

- Get x by itself on one side of the equation.
- To do this, multiply both sides of the equation by 3.

$$\frac{x}{3} = -12$$

$$3 \cdot \frac{x}{3} = 3 \cdot (-12)$$

$$\frac{3x}{3} = (3)(-12)$$

$$x = -36$$

- Check the answer.
- Replace x in the original equation with the value -36.

$$\frac{x}{3} = -12$$

Is $\frac{-36}{3} = -12$?

Is $-12 = -12$? Yes.

So, $x = -36$.

Explain

In Concept 2: Combining Operations, you will find a section on each of the following:

- **How to Evaluate an Exponential Expression with a Negative Base**
- **How to Use the Order of Operations**
- **The Commutative Property of Multiplication and the Commutative Property of Addition**
- **The Associative Property of Multiplication and the Associative Property of Addition**
- **How to Use the Distributive Property**
- **How to Simplify some Expressions that Include a Variable**

Here's one way to remember which number is the base and which number is the exponent.

You can picture the ***exponent****, 5, as the "upstairs" number.*

exponent upstairs

2^5

base in the basement

You can picture the ***base****, 2, in the* ***base****ment. The* ***base*** *is the "downstairs" number.*

Concept 2: Combining Operations

Exponential Notation

You have already learned about exponential notation.

Here's an example: $2^5 = 2 \times 2 \times 2 \times 2 \times 2$

The repeated factor, 2, is called the base. The exponent, 5, tells you how many times the base appears as a repeated factor.

Now you will use a negative number as the base. For example:

$(-2)^5$

Here the base is –2 and the exponent is 5. So,

$$\begin{aligned}(-2)^5 &= (-2) \times (-2) \times (-2) \times (-2) \times (-2)\\ &= 4 \times (-2) \times (-2) \times (-2)\\ &= -8 \times (-2) \times (-2)\\ &= 16 \times (-2)\\ &= -32\end{aligned}$$

Caution! Be sure to use parentheses when the base is negative.

For example, don't confuse $(-2)^4$ with -2^4. The base in the expression $(-2)^4$ is –2. The base in the expression -2^4 is 2. These two expressions also have different values as shown below.

$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = +16$

$-2^4 = -(2 \times 2 \times 2 \times 2) = -16$

Recall the steps used to find the value of an exponential expression.

To find the value of an exponential expression:

- Identify the base.
- Identify the exponent.
- Write a repeated multiplication.
  (The exponent tells how many times to write the base.)
- Do the multiplication.

You may find these Examples useful while doing the homework for this section.

Example 16

16. What is $(-5)^4$?

To find the value of the exponential expression: $(-5)^4$

- *Identify the base.* -5
- *Identify the exponent.* 4
- *Write a repeated multiplication.* $(-5) \times (-5) \times (-5) \times (-5)$
- *Do the multiplication.*

$$\begin{aligned} &= 25 \times (-5) \times (-5) \\ &= -125 \times (-5) \\ &= 625 \end{aligned}$$

So, $(-5)^4 = 625$.

Example 17

17. What is $\left(-\frac{2}{5}\right)^3$

To find the value of the exponential expression: $\left(-\frac{2}{5}\right)^3$

- *Identify the base.* $-\frac{2}{5}$
- *Identify the exponent.* 3
- *Write a repeated multiplication.* $\left(-\frac{2}{5}\right) \times \left(-\frac{2}{5}\right) \times \left(-\frac{2}{5}\right)$
- *Do the multiplication.*

$$\begin{aligned} &= \frac{4}{25} \times \left(-\frac{2}{5}\right) \\ &= -\frac{8}{125} \end{aligned}$$

So, $\left(-\frac{2}{5}\right)^3 = -\frac{8}{125}$.

The following example shows how to use a calculator to evaluate an exponential expression with a negative base.

Example 18

18. What is $(-2)^5$?

To use a calculator to calculate $(-2)^5$:	*On the calculator, you'll see...*
• *Enter 2*	2
• *Press "+/–"*	–2
• *Press "y^x"*	–2
• *Enter 5*	5
• *Press "="*	–32

So, $(-2)^5 = -32$.

Order of Operations

You have seen how to use grouping symbols and the order of operations when you work with whole numbers. The same rules apply when you work with signed numbers.

As a review, here are the rules for the order of operations:

Do the operations in this order:

- First, do operations inside grouping symbols.
- Next, do exponents and square root operations.
- Next, do multiplications and divisions, as they appear from left to right.
- Finally, do additions and subtractions, as they appear from left to right.

Because parenthesis are often used when working with signed numbers, square brackets may be used as grouping symbols. For example, in the expression

$-3 + [7 + (-6)]$,

the square brackets show that 7 and -6 should be added first.

Example 19

You may find these Examples useful while doing the homework for this section.

19. Find: $6 - [3 + (-7)]^3 \div 4 \times (-2)$

To find the value of the expression: $6 - [3 + (-7)]^3 \div 4 \times (-2)$

Follow the order of operations.

• *Do operations inside brackets.*	$= 6 - (-4)^3 \div 4 \times (-2)$
• *Do exponents and square root operations.*	$= 6 - (-64) \div 4 \times (-2)$
• *Do multiplications and divisions, as they appear from left to right.*	$= 6 - (-16) \times (-2)$
	$= 6 - (+32)$
• *Do additions and subtractions, as they appear from left to right.*	$= -26$

So, $6 - [3 + (-7)]^3 \div 4 \times (-2) = -26$.

Example 20

20. Find: $(-12) \div [2 + (-4)] - 5 \times (-2)^3$

To find the value of the expression: $(-12) \div [2 + (-4)] - 5 \times (-2)^3$

Follow the order of operations.

• *Do operations inside brackets.*	$= (-12) \div (-2) - 5 \times (-2)^3$
• *Do exponents and square root operations.*	$= (-12) \div (-2) - 5 \times (-8)$
• *Do multiplications and divisions, as they appear from left to right.*	$= 6 - 5 \times (-8)$
	$= 6 - (-40)$
• *Do additions and subtractions, as they appear from left to right.*	$= 46$

So, $(-12) \div [2 + (-4)] - 5 \times (-2)^3 = 46$.

The following example shows how to use a calculator to evaluate an expression that contains brackets.

Example 21

21. Find [2 + (–3)] × 5.

To use a calculator to find [2 + (–3)] × 5: *On the calculator, you'll see…*

There are no brackets on a calculator, so you use parentheses instead.

• *Press "("*	*0*
• *Enter 2*	*2*
• *Press "+"*	*2*
• *Press "("*	*0*
• *Enter 3*	*3*
• *Press "+/–"*	*–3*
• *Press ")"*	*–3*
• *Press ")"*	*–1*
• *Press "×"*	*–1*
• *Enter 5*	*5*
• *Press "="*	*–5*

So, [2 + (–3)] × 5 = –5.

The Commutative Property

You already know how to use the Commutative Property when you work with whole numbers. Now you will see how it can be used when you work with signed numbers.

As a review, the Commutative Property of Addition and the Commutative Property of Multiplication are presented here with some examples.

The Commutative Property of Addition

When you add, in any order, the sum is the same.

For example,

(–74) + 87 = 13 and 87 + (–74) = 13.

The order of the addition does not change the sum.

So, by the Commutative Property of Addition

(–74) + 87 = 87 + (–74).

*To remember the **commutative** properties, think of commuting by car. When you **commute**, you move from one place to another.*

The Commutative Property of Multiplication

When you multiply, in any order, the product is the same.

For example,

13 × (–5) = –65 and (–5) × 13 = –65.

The order of the multiplication does not change the product.

So, by the Commutative Property of Multiplication

13 × (–5) = (–5) × 13.

Caution! You have seen that addition and multiplication are commutative. But be careful! Subtraction is **not** commutative. Here's an example.

$4 - (-3) = 7$ $(-3) - 4 = -7$

So, $4 - (-3) \neq (-3) - 4$.

Division is **not** commutative, either. Here's an example.

$6 \div (-1) = -6$ $(-1) \div 6 = -\frac{1}{6}$

So, $6 \div (-1) \neq (-1) \div 6$.

You may find these Examples useful while doing the homework for this section.

Example 22

22. Use the Commutative Property of Addition to find the missing number:

$(-18) + (-47) = \_\_\_ + (-18)$

Here's how to use the Commutative Property of Addition to find the missing number: $(-18) + (-47) = \_\_\_ + (-18)$

• *When you add numbers, in any order, the sum is the same.* $(-18) + (-47) = (\mathbf{-47}) + (-18)$

So, $(-18) + (-47) = (-47) + (-18)$.

Example 23

23. Use the Commutative Property of Multiplication to find the missing number:

$(-35) \times (14) = 14 \times \_\_\_$

Here's how to use the Commutative Property of Multiplication to find the missing number: $(-35) \times 14 = 14 \times \_\_\_$

• *When you multiply numbers, in any order, the product is the same.* $(-35) \times 14 = 14 \times (\mathbf{-35})$

So, $(-35) \times 14 = 14 \times (-35)$.

The Associative Property

You have seen how to use the Commutative Property when you work with signed numbers. The Associative Property also can be used when you work with signed numbers.

The Associative Property of Addition

When you add, regardless of how you group (or associate) the numbers, the sum is the same.

For example, look at the sum $110 + (-74) + 87$.

Here are two different ways to group the numbers:

$[110 + (-74)] + 87$
$= 36 + 87$
$= 123$

or

$110 + [(-74) + 87]$
$= 110 + 13$
$= 123$

Regardless of how the numbers are grouped, the sum is 123.

So, by the Associative Property of Addition

$[110 + (-74)] + 87 = 110 + [(-74) + 87]$.

To remember the ***associative*** *properties, think of* ***associating*** *with your friends. If two of your friends are together and you join them, it's the same as if one of your friends joins you and another friend.*

The Associative Property of Multiplication

When you multiply, regardless of how you group (or associate) the numbers, the product is the same.

For example, look at the product $(-7) \times 2 \times 15$. Here are two different ways to group the factors:

$$\begin{aligned} & [(-7) \times 2] \times 15 \\ = & [-14] \times 15 \\ = & -210 \end{aligned} \qquad \text{or} \qquad \begin{aligned} & (-7) \times [2 \times 15] \\ = & (-7) \times 30 \\ = & -210 \end{aligned}$$

Regardless of how the factors are grouped, the product is –210.

So, by the Associative Property of Multiplication

$$[(-7) \times 2] \times 15 = (-7) \times [2 \times 15].$$

Caution! You have seen that addition and multiplication are associative. But be careful! Subtraction is **not** associative. Here's an example.

$$\begin{aligned} & [6 - (-2)] - 7 \\ = & 8 - 7 \\ = & 1 \end{aligned} \qquad \begin{aligned} & 6 - [(-2) - 7] \\ = & 6 - (-9) \\ = & 15 \end{aligned}$$

So, $[6 - (-2)] - 7 \neq 6 - [(-2) - 7]$.

Also, division is **not** associative. Here's an example.

$$\begin{aligned} & [8 \div (-4)] \div 2 \\ = & (-2) \div 2 \\ = & -1 \end{aligned} \qquad \begin{aligned} & 8 \div [(-4) \div 2] \\ = & 8 \div (-2) \\ = & -4 \end{aligned}$$

So, $[8 \div (-4)] \div 2 \neq 8 \div [(-4) \div 2]$.

You may find these Examples useful while doing the homework for this section.

Example 24

24. Use the Associative Property of Addition to find the missing number:

$[(-43) + 2] + 88 = (-43) + [\_\_\_ + 88]$

Here's how to use the Associative Property of Addition to find the missing number: $[(-43) + 2] + 88 = (-43) + [\_\_ + 88]$

- *When you add numbers, regardless of how they are grouped, the sum is the same.* $[(-43) + 2] + 88 = (-43) + [\mathbf{2} + 88]$

So, $[(-43) + 2] + 88 = (-43) + [2 + 88]$.

25. Use the Associative Property of Multiplication to find the missing number: **Example 25**

$[(-3) \times 57] \times 39 = (-3) \times [57 \times \_\_\_]$

Here's how to use the Associative Property of Multiplication to find the missing number: $[(-3) \times 57] \times 39 = (-3) \times [57 \times \_\_\_]$

- *When you multiply numbers, regardless of how they are grouped, the product is the same.* $[(-3) \times 57] \times 39 = (-3) \times [57 \times \mathbf{39}]$

So, $[(-3) \times 57] \times 39 = (-3) \times [57 \times 39]$.

The Distributive Property

When you work with signed numbers, the distributive property can also be used.

The Distributive Property

*To remember the **distributive** property, think of **distributing** the number outside the parentheses to the numbers inside the parentheses.*

To multiply the sum of two numbers by a number, you can first add, then multiply. Or you can first multiply, then add.

For example,

$(-5) \times [(-6) + 2]$	or	$(-5) \times (-6) + (-5) \times 2$
$= (-5) \times (-4)$		$= 30 + (-10)$
$= 20$		$= 20$

So, by the Distributive Property, $(-5) \times [(-6) + 2] = (-5) \times (-6) + (-5) \times 2$.

You can also distribute on the right. Here's an example.

$[(-6) + 2] \times (-5)$	or	$(-6) \times (-5) + 2 \times (-5)$
$= [-4] \times (-5)$		$= 30 + (-10)$
$= 20$		$= 20$

So, by the Distributive Property, $[(-6) + 2] \times (-5) = (-6) \times (-5) + 2 \times (-5)$.

Caution! The distributive property involves **two different** operations, multiplication and addition (or multiplication and subtraction).

You can use the distributive property here:

$$\overset{\text{multiply}}{\downarrow}\quad\overset{\text{add}}{\downarrow}$$

$$4 \times [(-3) + 8] = 4 \times (-3) + 4 \times 8$$

You **cannot** use the distributive property here:

$$\overset{\text{multiply}}{\downarrow}\quad\overset{\text{multiply}}{\downarrow}$$

$$4 \times [(-3) \times 8]$$

You may find these Examples useful while doing the homework for this section.

Example 26

26. Use the Distributive Property to find the missing number:

$8 \times [3 + (-7)] = 8 \times$ ___ $+ 8 \times (-7)$

Here's how to use the Distributive Property to find the missing number: $8 \times [3 + (-7)] = 8 \times$ ___ $+ 8 \times (-7)$

- *When you multiply the sum of two numbers by a number, you can first add, then multiply.*

Or you can first do each multiplication, then add. $8 \times [3 + (-7)] = 8 \times \mathbf{3} + 8 \times (-7)$

So, $8 \times [3 + (-7)] = 8 \times 3 + 8 \times (-7)$.

Example 27

27. Use the Distributive Property to find the missing number:

$4 \times [(-3) +$ ___ $] = 4 \times (-3) + 4 \times (-2)$

Here's how to use the Distributive Property to find the missing number: $4 \times [(-3) +$ ___ $] = 4 \times (-3) + 4 \times (-2)$

- *When you multiply the sum of two numbers by a number, you can first add, then multiply.*

Or you can first do each multiplication, then add. $4 \times [(-3) + (\mathbf{-2})] = 4 \times (-3) + 4 \times (-2)$

So, $4 \times [(-3) + (-2)] = 4 \times (-3) + 4 \times (-2)$.

Working with Variables

You have already seen how the Distributive Property can be used to simplify an expression containing a variable and whole numbers.

For example,

$$\begin{aligned} 4x + 2x &= (4 + 2)x \\ &= 6x \end{aligned}$$

In this section, you will see how to use the Distributive Property when working with expressions that contain a variable and signed numbers.

Remember, when a number is multiplied by a letter, the multiplication sign, in this case "·", is often omitted. $4x + (-2x)$ *is the same as* $4 \cdot x + (-2) \cdot x$.

For example, here's how to use the Distributive Property to simplify this expression: $4x + (-2x)$.

- Use the Distributive Property. $= [4 + (-2)]x$
- Do the operation inside the parentheses (add). $= [2]x$
- Simplify. $= 2x$

So, $4x + (-2x) = 2x$.

So, to simplify an expression, such as $4x + (-2x)$, use the Distributive Property to add the numbers, 4 and -2, then multiply by x.

$4x + (-2x) = 2x$

Here's how to simplify certain expressions that contain a variable such as x:

- Combine (add or subtract) the terms with an "x."
- Combine (add or subtract) the terms without an "x."
- Write the answer.

Example 28

You may find these Examples useful while doing the homework for this section.

28. Simplify: $1 - 3x - 22 + 8x + 14$

To simplify the expression:	$1 - 3x - 22 + 8x + 14$
• *Add the terms **with** an "x."*	$-3x + 8x$
	$= (-3 + 8)x$
	$= 5x$
• *Add the terms **without** an "x."*	$= 1 - 22 + 14$
	$= -7$
• *Write the answer.*	$5x - 7$

So, $1 - 3x - 22 + 8x + 14 = 5x - 7$.

Example 29

29. Simplify: $4x - (-2x) + (-10) + 2 + (-5)$

To simplify the expression:	$4x - (-2x) + (-10) + 2 + (-5)$
• *Add the terms **with** an "x."*	$4x - (-2x)$
	$= 4x + (+2x)$
	$= 4x + 2x$
	$= (4 + 2)x$
	$= 6x$
• *Add the terms **without** an "x."*	$(-10) + 2 + (-5)$
	$= -8 + (-5)$
	$= -13$
• *Write the answer.*	$6x - 13$

So, $4x - (-2x) + (-10) + 2 + (-5) = 6x - 13$.

Explore

This Explore contains two investigations.

- ***Ups and Downs***
- ***2's and 4's***

Investigation 1: Ups and Downs

Here are three applications of signed numbers.

Changes in Stock Prices

When one discusses the value of a stock, +5 means the value of a stock went up 5 points, and –5 means the value of a stock went down 5 points.

Elevator

In riding an elevator, +5 means the elevator went up 5 floors, and –5 means the elevator went down 5 floors.

Elevation

When measuring the level of water in a river, +5 means a rise of 5 feet in the level of the water, and –5 means a drop of 5 feet in the level of the water.

1. Describe what $2 \cdot (-3)$ means in each application.

   a. Changes in Stock Prices

   b. Elevator Motion

   c. Elevation of Water in a River

2. Interpret the result of the multiplication in (1) above for each application. That is, what does –6 mean in each application?

   a. Changes in Stock Prices

   b. Elevator Motion

   c. Elevation of Water in a River

3. Describe what (–8) ÷ 4 means in each application.

   a. Changes in Stock Prices

   b. Elevator Motion

   c. Elevation of Water in a River

4. Interpret the result of the division in (3) above for each application. That is, what does –2 mean in each application?

   a. Changes in Stock Prices

   b. Elevator Motion

   c. Elevation of Water in a River

Investigation 2: 2's and 4's

Using each of the numbers 2, –2, 4, and –4, exactly one time, and using multiplication, division and grouping symbols, find ten different ways to get the value 1. One way has already been done for you. If you can come up with more than 10, use the extra space below.

1.	(2 · 4) ÷ [(–2) · (–4)]	6.	__________________
2.	__________________	7.	__________________
3.	__________________	8.	__________________
4.	__________________	9.	__________________
5.	__________________	10.	__________________

Homework

CONCEPT 1: MULTIPLYING AND DIVIDING

Multiplying Two Numbers with Different Signs

For help working these types of problems, go back to Examples 1–2 in the Explain section of this lesson.

1. Find $-5 \cdot 12$.
2. Find $-10 \cdot 18$.
3. Find $25 \cdot (-6)$.
4. Find $32 \cdot (-4)$.
5. Find $-45 \cdot 21$.
6. Find $-23 \cdot 17$.
7. Find $220 \cdot (-100)$.
8. Find $301 \cdot (-201)$.
9. Find $-2.1 \cdot 5.4$.
10. Find $-4.01 \cdot 23.2$.
11. Find $2.225 \cdot (-1.12)$.
12. Find $3.07 \cdot (-201.1)$.
13. Find $-\frac{2}{3} \cdot \frac{9}{14}$.
14. Find $-2\frac{1}{5} \cdot 1\frac{3}{22}$.
15. Find $\frac{4}{9} \cdot \left(-\frac{15}{16}\right)$.
16. Find $3\frac{2}{3} \cdot \left(-4\frac{1}{5}\right)$.
17. In the last football game, Trent carried the ball six times. Each time he lost 5 yards. What was Trent's total yardage for the game?
18. In the last football game, Hank carried the ball five times. Each time he lost 7 yards. What was Hank's total yardage for the game?
19. Hannah is playing golf. She scored 1 under par on the first 4 holes. What is Hannah's score relative to par after the fourth hole?
20. Holly is playing golf. She scored 2 under par on the first 3 holes. What is Holly's score relative to par after the third hole?
21. Bill and Shirley are playing a card game. Bill's score is three times Shirley's score. If Shirley's score is –12, find Bill's score.
22. Frank and Caroline are playing a board game. Frank's score is twice Caroline's score. If Caroline's score is –15, find Frank's score.
23. The boiling point of neon is approximately 7.2 times the boiling point of chlorine. If the boiling point of chlorine is –34.1°C, find the boiling point of neon.
24. The melting point of oxygen is approximately 5.6 times the melting point of mercury. If the melting point of mercury is –39°C, find the melting point of oxygen.

Multiplying Two Numbers with the Same Sign

For help working these types of problems, go back to Examples 3–4 in the Explain section of this lesson.

25. Find $(-5) \cdot (-14)$.

26. Find $23 \cdot 16$.

27. Find $(-12) \cdot (-21)$.

28. Find $(-37) \cdot (-83)$.

29. Find $124 \cdot 45$.

30. Find $(-2001) \cdot (-301)$.

31. Find $(-50) \cdot (-36)$.

32. Find $22 \cdot 222$.

33. Find $(-3.5) \cdot (-2.4)$.

34. Find $(-2.001) \cdot (-30.1)$.

35. Find $4.78 \cdot 3.01$.

36. Find $(-27.2) \cdot (-1.001)$.

37. Find $(-4.05) \cdot (-2.2)$.

38. Find $(36.1) \cdot (0.002)$.

39. Find $4\frac{5}{7} \cdot \frac{6}{11}$.

40. Find $\left(-\frac{7}{15}\right) \cdot \left(-\frac{5}{14}\right)$.

41. Find $\left(-1\frac{1}{3}\right) \cdot \left(-2\frac{1}{4}\right)$.

42. Find $\left(1\frac{7}{8}\right) \cdot \left(3\frac{1}{5}\right)$.

43. Find $\left(-12\frac{4}{5}\right) \cdot \left(-11\frac{2}{3}\right)$.

44. Find $7\frac{9}{16} \cdot 4\frac{4}{11}$.

45. Bob and Christina are playing a two-player board game. The directions for scoring are as follows:

    Players take turns drawing cards from the deck of cards (provided with the game). Each player gets to draw a card five times in a row. After each draw, Player A records as his score the number shown on the card. Player B records as his score the opposite of the number on the card.

    For example, when Bob draws a 6, Bob scores a 6 and Christina scores a –6. During the course of the game, Bob draws a –5 on three consecutive draws. As a result of the three draws, what does Christina record for her score?

46. Christina has just finished a set of five draws. She scored –7 on each of the five cards she drew. As a result of these five draws, what does Bob record for his score?

47. Bob draws a +4 three times in a row. As a result of these three draws, what does Bob record for his score?

48. Christina draws a +11 four times in a row. As a result of these four draws, what does Christina record for her score?

Multiplying More Than Two Signed Numbers

For help working these types of problems, go back to Examples 5–6 in the Explain section of this lesson.

49. Find $(-2) \cdot 5 \cdot 12$.

50. Find $3 \cdot (-6) \cdot (-13)$.

51. Find $(-4) \cdot (-8) \cdot (-15)$.

52. Find $3 \cdot (-6) \cdot 7 \cdot (-12)$.

53. Find $(-4) \cdot (-15) \cdot 18 \cdot 2$.

54. Find $23 \cdot (-6) \cdot (-12) \cdot (-5)$.

55. Find $14 \cdot 39 \cdot (-100) \cdot (-15) \cdot 5$.

56. Find $(-24) \cdot (-6) \cdot 11 \cdot (-16) \cdot 3$.

57. Find $2.5 \cdot (-1.7) \cdot 10$.

58. Find $(-3.12) \cdot (-4.4) \cdot 20$.

59. Find $(-1.021) \cdot (-0.02) \cdot (-100.1)$.

60. Find $3.71 \cdot 2.01 \cdot (-3.3) \cdot (-100)$.

61. Find $3\frac{1}{3} \cdot 4\frac{2}{5} \cdot \left(-1\frac{1}{4}\right)$.

62. Find $\left(-\frac{2}{5}\right) \cdot \left(-\frac{5}{6}\right) \cdot \left(-\frac{9}{14}\right)$.

63. Find $5\frac{4}{5} \cdot 2\frac{1}{3} \cdot \left(-1\frac{1}{2}\right)$.

64. Find $\left(-\frac{2}{5}\right) \cdot \left(-4\frac{2}{9}\right) \cdot 3\frac{3}{4} \cdot 1\frac{1}{3}$.

65. What would be the sign of a number you might multiply the following product by so that the result would be positive?
    $(-3)(6)(-7)(-10)$

66. What would be the sign of a number you might multiply the following product by so that the result would be negative?
    $(4)(-8)(-2.1)\left(3\frac{1}{2}\right)$

67. Sally keeps a bundle of one-dollar bills in her car for parking. She spends \$2 per day, three days per week for parking. What number represents the change in the bundle of one-dollar bills over a 5 week period? Express your answer as a negative number.

68. Halley makes 4 deposits a week to her savings account for 3 weeks. If each deposit is for \$34.76, what is the change in Halley's savings account balance?

69. Jason feeds his dog 3 dog biscuits four times per week. What is the change in the number of dog biscuits Jason has after a two-week period? Express your answer as a negative number.

70. Harold has a collection of rare coins. Over one four-week period, he collected 2 rare coins twice a week. What is the change in the number of coins Harold has in his collection?

71. The price of a certain stock decreased by $\$\frac{1}{8}$ three times a week for six weeks. Over this same six-week period, the price of the stock increased $\$\frac{3}{8}$ twice a week and remained unchanged twice a week. At the end of the six-week period, what is the net gain or loss in the price of the stock?

72. The price of a certain stock decreased by $\$\frac{3}{4}$ two times a week for four weeks. Over this same four-week period, the price of the stock increased $\$\frac{3}{8}$ three times a week and remained unchanged twice a week. At the end of the four-week period, what is the net gain or loss in the price of the stock?

Dividing Two Numbers with Different Signs

For help working these types of problems, go back to Examples 8–9 in the Explain section of this lesson.

73. Find $(-38) \div 2$.

74. Find $121 \div (-11)$.

75. Find $54 \div (-9)$.

76. Find $(-108) \div 4$.

77. Find $(-115) \div 5$.

78. Find $425 \div (-25)$.

79. Find $\frac{-425}{25}$.

80. Find $\frac{316}{-4}$.

81. Find $(-3.12) \div 0.4$.

82. Find $22.5 \div (-0.15)$.

83. Find $\frac{-45.1}{1.1}$.

84. Find $\frac{3.42}{-0.9}$.

85. Find $\left(-\frac{3}{5}\right) \div \frac{21}{25}$.

86. Find $\left(-3\frac{5}{9}\right) \div 2\frac{2}{3}$.

87. Find $\frac{4}{3} \div \left(-\frac{8}{21}\right)$.

88. Find $15\frac{3}{10} \div \left(-7\frac{4}{11}\right)$.

89. In his last football game, Kevin ran the ball six times for a net loss of 12 yards. What was his average yardage per play?

90. In his last football game, Jaime ran the ball five times for a net loss of 25 yards. What was his average yardage per play?

91. Steve and Caroline are playing a board game. Steve's score is four times Caroline's score. If Steve's score is –244, what is Caroline's score?

92. Jody and Fisher are playing a game of cards. Jody's score is three times Fisher's score. If Jody's score is –96, what is Fisher's score?

93. The price of stock A gained twice as much in one day as the price of stock B. If the price of stock A gained $\$\frac{3}{4}$, how much did the price of stock B gain?

94. The price of stock A lost twice as much in one day as the price of stock B. If the price of stock A lost $\$\frac{3}{4}$, how much did the price of stock B lose?

95. The boiling point of oxygen is approximately 1.2 times the boiling point of krypton. If the boiling point of oxygen is –180˚C, find the boiling point of krypton.

96. The melting point of nitrogen is approximately 5.4 times the melting point of mercury. If the melting point of nitrogen is –210˚C, find the melting point of oxygen. Round your answer to the nearest degree.

Dividing Two Numbers with the Same Sign

For help working these types of problems, go back to Examples 11–12 in the Explain section of this lesson.

97. Find $(-42) \div (-6)$.

98. Find $35 \div 5$.

99. Find $(-72) \div (-18)$.

100. Find $108 \div 12$.

101. Find $(-225) \div (-15)$.

102. Find $308 \div 22$.

103. Find $\frac{-950}{-50}$.

104. Find $\frac{1250}{25}$.

105. Find $4.2 \div 0.7$.

106. Find $(-3.15) \div (-2.1)$.

107. Find $0.036 \div 0.009$.

108. Find $(-24.3) \div (-0.81)$.

109. Find $\frac{5.511}{1.1}$.

110. Find $\frac{-36.5}{-6.25}$.

111. Find $\left(-\frac{7}{16}\right) \div \left(-\frac{21}{32}\right)$.

112. Find $\frac{9}{25} \div \frac{27}{75}$.

113. Find $\left(-2\frac{3}{8}\right) \div \left(-1\frac{3}{16}\right)$.

114. Find $5\frac{3}{5} \div 2\frac{2}{15}$.

115. Find $\left(-1\frac{3}{4}\right) \div \left(-2\frac{7}{8}\right)$.

116. Find $4\frac{2}{3} \div 5\frac{5}{6}$.

117. Steve and Wendy are playing a 2-player board game. For a portion of the game, the scoring is recorded as follows:

Player A draws a card from the deck of red cards. Player B draws a card from the deck of blue cards. Player A records as his score the number obtained by dividing the number on the red card by the number on the blue card.

Steve draws a red card with a –24 on it. Wendy draws a blue card with a –8 on it. What does Steve record as his score?

118. Wendy draws a red card with a –96 on it. Steve draws a blue card with a –24 on it. What does Wendy record as her score?

119. Steve draws a red card with a 3.4 on it. Wendy draws a blue card with a 0.2 on it. What does Steve record as his score?

120. Wendy draws a red card with a $\frac{3}{8}$ on it. Steve draws a blue card with a $\frac{3}{32}$ on it. What does Wendy record as her score?

Solving an Equation

For help working these types of problems, go back to Examples 13–15 in the Explain section of this lesson.

121. Solve this equation for x: $9x = -99$

122. Solve this equation for x: $-7x = 63$

123. Solve this equation for x: $14x = 112$

124. Solve this equation for x: $-12x = -180$

125. Solve this equation for x: $-6x = 96$

126. Solve this equation for x: $15x = -525$

127. Solve this equation for x: $\frac{x}{7} = -8$

128. Solve this equation for x: $\frac{x}{-4} = 11$

129. Solve this equation for x: $\frac{x}{12} = 13$

130. Solve this equation for x: $\frac{x}{-11} = -17$

131. Solve this equation for x: $4.2x = -8.4$

132. Solve this equation for x: $-3.75x = -1.05$

133. Solve this equation for x: $\frac{1}{5}x = -\frac{3}{10}$

134. Solve this equation for x: $-\frac{2}{3}x = -48$

135. Solve this equation for x: $-\frac{4}{7}x = 32$

136. Solve this equation for x: $-\frac{3}{4}x = -\frac{9}{16}$

137. In the last football game Greg played, he ran the ball five times. His net yardage was –15 yards. What was Greg's average yardage per play? One way to answer the question is to solve this equation for x: $5x = -15$.

138. In the last football game Jose played, he ran the ball six times. His net yardage was 32 yards. What was Jose's average yardage per play? One way to answer the question is to solve this equation for x: $6x = 32$.

139. Maya and Reese are playing a boardgame. Maya's score is three times Reese's. If Maya's score is –411, what is Reese's score? One way to answer the question is to solve this equation for x: $3x = -411$.

140. Jodi and Kevin are playing a board game. Jody's score is one-fourth Kevin's score. If Jody's score is –22, what is Kevin's score? One way to answer the question is to solve this equation for x: $\frac{1}{4}x = -22$.

141. The price of a certain stock, A, gained twice as much as the price of a certain stock, B. If the price ofstock A gained $\$\frac{1}{2}$, how much did the price of stock B gain? One way to answer the question is to solve this equation for x: $2x = \frac{1}{2}$.

142. The price of a certain stock, C, lost a fourth as much as the price of a certain stock, D. If the price of stock C lost $\$\frac{3}{8}$, what is the change in the price of stock D? One way to answer the question is to solve this equation for x: $\frac{1}{4}x = -\frac{3}{8}$.

143. The boiling point of sodium is –26.1 times the boiling point of chlorine. If the boiling point of sodium is 889°C, find the boiling point of chlorine. One way to answer the question is to solve this equation for x: $-26.1x = 889$. Round your answer to the nearest tenth of a degree.

144. The melting point of mercury is 5.42 times the melting point of bromine. If the melting point of mercury is –39°C, find the melting point of bromine. One way to answer the question is to solve this equation for x: $5.42x = -39$. Round your answer to the nearest tenth of a degree.

CONCEPT 2: COMBINING OPERATIONS

Exponential Notation

For help working these types of problems, go back to Examples 16–18 in the Explain section of this lesson.

145. What is $(-4)^3$?

146. What is $(-5)^2$?

147. What is $(-3)^4$?

148. What is -2^4?

149. What is $(-2)^5$?

150. What is -3^5?

151. What is $(-7)^3$?

152. What is $(-9)^4$?

153. What is $\left(-\frac{2}{7}\right)^2$?

154. What is $\left(-\frac{3}{5}\right)^3$?

155. What is $\left(-\frac{3}{4}\right)^5$?

156. What is $\left(-\frac{1}{2}\right)^6$?

157. What is $(-0.2)^3$?

158. What is $(-1.1)^4$?

159. What is $(-0.01)^2$?

160. What is -0.01^2?

In a beginning algebra class, an expression that is commonly used is x^2. To evaluate this expression for a given value of x, replace x with the given value and then find the value of the resulting expression.

For example, to find the value of x^2 when x is 5:

- Replace x with 5. $(5)^2$
- Find the value of $(5)^2$. $= 25$

So, when x is 5, the value of x^2 is 25.

161. Evaluate x^2 when x is -3.

162. Evaluate x^2 when x is $-\frac{2}{5}$.

163. You have seen that -3^2 and $(-3)^2$ are not equal. Is -3^4 equal to $(-3)^4$? Why or why not?

164. In general, if n is an even number, is -3^n equal to $(-3)^n$?

165. You have seen that -4^3 and $(-4)^3$ are equal. Is -4^5 equal to $(-4)^5$? Why or why not?

166. In general, if n is an odd number, is-4^n equal to $(-4)^n$?

167. Rhett has agreed to weed Bart's garden. Bart says he will pay Rhett \$2 the first day and then double the amount each successive day Rhett spends weeding the garden. On the 5th day, the change in Bart's gardening account can be represented by $-\$2^5$. What is this amount?

168. Brenda has agreed to address and stamp some envelopes for Gladys. Gladys says she will pay Brenda 2¢ for the first envelope and then double the amount for each successive envelope Brenda addresses and stamps. For the 10th envelope, the change in Gladys' payroll account can be represented by -2^{10}¢. What is this amount?

Order of Operations

For help working these types of problems, go back to Examples 19–21 in the Explain section of this lesson.

169. Find: $3 \cdot [4 + (-7)]$

170. Find: $(-15) \div [(-2) - 1]$

171. Find: $3 \cdot (-5) + (-21) \div (-3)$

172. Find: $(-18) \div 6 - (-34) \div 2$

173. Find: $[(-6) + 4]^3 + 28 \div (-7)$

174. Find: $(-18) \div 3 - [7 - (-1)]^2$

175. Find: $(-3) \cdot (-14) + 2 \cdot [(-5) + 3]$

176. Find: $(-75) \div 5 - (-3) \cdot [4 - (-2)]$

177. Find: $(-5)^2 + [(-7) + 2]^3 - 10 \cdot (-5)$

178. Find: $(-4)^3 - [3 - (-8)]^2 + (-15) \cdot (-7)$

179. Find $[10 + (-4)] \div [(-7) + 4]$

180. Find: $[20 - 32] \cdot [(-12) + 9]$

181. Find: $\frac{1}{2} + \left(-\frac{2}{3}\right) \cdot \frac{9}{16}$

182. Find: $\left(-\frac{7}{16}\right) \div \frac{21}{32} - \frac{7}{8}$

183. Find: $[2.4 + (-7.6)] \div 3.2$

184. Find: $5.3 \cdot [(-10.8) - (-11.9)]$

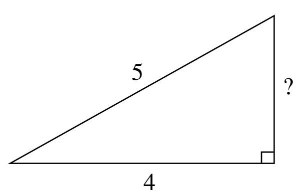
185. Marie and Julienne are discussing the expression $(-15) + 3 \cdot (-8)$. Marie says the value of the expression is 96. Julienne says the value of the expression is –39. Who is correct? To answer the question, use the order of operations.

186. Jonathan and Russell are discussing the expression $35 - (-12) \cdot (-3)$. Jonathan says the value of the expression is –141. Russell says the value of the expression is –1. Who is correct? To answer the question, use the order of operations.

187. Pedro and Paul are discussing the expression $(8 - 10)^3 - 6 \cdot (9 - 14)$. Pedro says the value of the expression is 22. Paul says the value of the expression is –38. Who is correct?

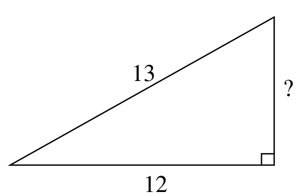
188. Mandy and Donna are discussing the expression $[(4 - 7)^3 + 6] \cdot (9 + 1)$. Mandy says the value of the expression is –210. Donna says the value of the expression is 330. Who is correct?

189. The following expression can be used to find the length of a side of a right triangle whose longest side is of length 5 inches and whose other side is of length 4 inches: $\sqrt{5^2 - 4^2}$.



Find the length of the side by evaluating this expression. Hint: The $\sqrt{}$ symbol acts like a grouping symbol.

190. The following expression can be used to find the length of a side of a right triangle whose longest side is of length 13 inches and whose other side is of length 12 inches: $\sqrt{13^2 - 12^2}$.



Find the length of the side by evaluating this expression. Hint: The $\sqrt{}$ symbol acts like a grouping symbol.

191. The following expression can be used to find the distance between two particular points: $\sqrt{(8 - 14)^2 + (12 - 4)^2}$. Find the value of this expression.

192. The following expression can be used to find the distance between two particular points: $\sqrt{(7 + 3)^2 + (-13 - 11)^2}$. Find the value of this expression.

The Commutative Property

For help working these types of problems, go back to Examples 22–23 in the Explain section of this lesson.

193. Use the Commutative Property of Addition to find the missing number:

    $5 + (-14) = (-14) + \_\_\_$.

194. Use the Commutative Property of Addition to find the missing number: $(-6) + 17 = 17 + \_\_\_$.

195. Use the Commutative Property of Addition to find the missing number: $3 + (-1) = \_\_\_ + 3$.

196. Use the Commutative Property of Addition to find the missing number: $(-13) + 25 = \_\_\_ + (-13)$.

197. Use the Commutative Property of Multiplication to find the missing number: $(-7) \cdot 2 = 2 \cdot \_\_\_$.

198. Use the Commutative Property of Multiplication to find the missing number: $6 \cdot (-11) = \_\_\_ \cdot 6$

199. Use the Commutative Property of Multiplication to find the missing number: $14 \cdot (-86) = (-86) \cdot \_\_\_$.

200. Use the Commutative Property of Multiplication to find the missing number: $(-22) \cdot 73 = \_\_\_ \cdot (-22)$.

201. True or false. The statement $(-5) + 7 = 7 + (-5)$ is an example of the Commutative Property of Addition.

202. True or false. The statement $7 + [(-6) + 2]= [7 + (-6)] + 2$ is an example of the Commutative Property of Addition.

203. True or false. The statement $(-5) + [2 +(-7)] = [2 + (-7)] + (-5)$ is an example of the Commutative Property of Addition.

204. True or false. The statement $(-5) \cdot [2 + (-7)] = (-5) \cdot [(-7) + 2]$ is an example of the Commutative Property of Addition.

205. True or false. The statement $(-11) + 8 = 8 + (-11)$ is an example of the Commutative Property of Multiplication.

206. True or false. The statement $(-11) \cdot [8 \cdot (-10)] = [(-11) \cdot 8] \cdot (-10)$ is an example of the Commutative Property of Multiplication.

207. True or false. The statement $(-11) \cdot [8 + (-10)] = [8 + (-10)] \cdot (-11)$ is an example of the Commutative Property of Multiplication.

208. True or false. The statement $(-11) \cdot 8 + (-10) = 8 \cdot (-11) + (-10)$ is an example of the Commutative Property of Multiplication.

209. Jason ripped his homework paper as he was taking it out of his folder. The small piece that was ripped off blew away. Fortunately, only one problem was ruined. It appeared as follows: $(-27) + 11 = 11 +$

    Jason knows that only one number was torn off. Use the Commutative Property of Addition to supply the missing number.

210. Rachel's roommate ripped a corner from Jefferson's homework paper to write down a phone number. Fortunately, only one problem was ruined. It appeared as follows: $13 \cdot (-5) = (-5) \cdot$

Rachel knows that only one number was torn off. Use the Commutative Property of Multiplication to supply the missing number.

You know that subtraction is not commutative. However, you know how to rewrite a subtraction as an addition. For example, you can rewrite $3 - 5$ as $3 + (-5)$. Then use the Commutative Property of Addition to get: $(-5) + 3$. So, $3 - 5 = -5 + 3$.

211. Rewrite $-7 - (-2)$ as an addition. Then rewrite the result using the Commutative Property of Addition.

212. Rewrite $-10 - 3$ as an addition. Then rewrite the result using the Commutative Property of Addition.

213. Sometimes the Commutative Property of Addition is helpful when you want to do "mental" math. For example, some people find it easier to add -5 to 13 than to add 13 to -5. Which do you find easier to compute mentally, $(-5) + 13$ or $13 + (-5)$? Why?

214. Which do you find easier to compute mentally, $45 + (-130)$ or $(-130) + 45$? Why?

215. Sometimes the Commutative Property of Multiplication is helpful when you want to do "mental" math. For example, some people find it easier to multiply (–7) by 4 than to multiply 4 by (–7). Which do you find easier to compute mentally, $(-7) \cdot 4$ or $4 \cdot (-7)$? Why?

216. Which do you find easier to compute mentally, $(-300) \cdot 22$ or $22 \cdot (-300)$?

The Associative Property

For help working these types of problems, go back to Examples 24–25 in the Explain section of this lesson.

217. Use the Associative Property of Addition to find the missing number: $[7 + (-2)] + 5 = \_\_\_ + [(-2) + 5]$

218. Use the Associative Property of Addition to find the missing number: $[(-8) + 11] + (-3) = \_\_\_ + [11 + (-3)]$

219. Use the Associative Property of Addition to find the missing number: $[(-6) + 9] + (-13) = -6 + [\_\_\_ + (-13)]$

220. Use the Associative Property of Addition to find the missing number: $[2 + 23] + (-17) = 2 + [23 + \_\_\_]$

221. Use the Associative Property of Multiplication to find the missing number: $[(-4) \cdot 3] \cdot (-2) = \_\_\_ \cdot [3 \cdot (-2)]$

222. Use the Associative Property of Multiplication to find the missing number: $[(-12) \cdot (-7)] \cdot 3 = (-12) \cdot [(-7) \cdot \_\_\_]$

223. Use the Associative Property of Multiplication to find the missing number: $[25 \cdot (-34)] \cdot 2 = \_\_\_ \cdot [(-34) \cdot 2]$

224. Use the Associative Property of Multiplication to find the missing number: $[9 \cdot (-81)] \cdot 2 = 9 \cdot [\_\_\_ \cdot 2]$

225. True or false. The statement $-1 + [(-2) + 3] = -1 + [3 + (-2)]$ is an example of the Associative Property of Addition.

226. True or false. The statement $(-3) + [11 + (-5)] = [(-3) + 11] + (-5)$ is an example of the Associative Property of Addition.

227. True or false. The statement $(-5) \cdot [(-9) + 11] = (-5) \cdot (-9) + (-5) \cdot 11$ is an example of the Associative Property of Addition.

228. True or false. The statement $[7 + (-23)] +12 = 7 + [(-23) + 12]$ is an example of the Associative Property of Addition.

229. True or false. The statement $(-19) \cdot (6 \cdot 3) = [(-19) \cdot 6] \cdot 3$ is an example of the Associative Property of Multiplication.

230. True or false. The statement $[4 \cdot (-6)] \cdot (-7) = (-7) \cdot [4 \cdot (-6)]$ is an example of the Associative Property of Multiplication.

231. True or false. The statement $[4 \cdot (-6)] \cdot (-7) = 4 \cdot [(-6) \cdot (-7)]$ is an example of the Associative Property of Multiplication.

232. True or false. The statement $6 \cdot [(-10) + 8] = 6 \cdot (-10) + 6 \cdot 8$ is an example of the Associative Property of Multiplication.

233. Susan missed class and got notes from a friend. For some reason her friend did not complete the last part of an example of the Associative Property of Addition. This is what Susan's friend had written down:

$[3 + (-15)] + 24 = 3 + [(-15) +$

Use the Associative Property of Addition to complete this example.

234. Chandler had to leave class early. Later, his friend was giving him an example of the Associative Property of Multiplication but his friend's notes were not complete. This is what was written down before his friend stopped taking notes:

$[(-8) \cdot 3] \cdot (-25) = (-8) \cdot [3 \cdot$

Use the Associative Property of Multiplication to complete this example.

235. Linda and Ellen are studying the Associative Property. Linda has written the following example:

$[(-9) + 4] + (-10) = (-10) + [(-9) + 4]$

Ellen says that this is not an example of the Associative Property of Addition but Linda disagrees. Who is correct and why?

236. Hal and Rupert are studying the Associative Property. Hal has written the following example:

$(-9) \cdot [(-7) \cdot 12] = [(-9) \cdot (-7)] \cdot 12$

Rupert says that this is not an example of the Associative Property of Multiplication but Hal disagrees. Who is correct and why?

237. Sometimes the Associative Property of Addition is helpful when you want to do "mental" math. For example, some people find it easier to find the value of $43 + [(-5) + 40]$ than to find the value of $[43 + (-5)] + 40$. Which do you find easier to compute mentally? Why?

238. Which do you find easier to compute mentally, $[(-28) + 18] + 77$ or $(-28) + [18 + 77]$? Why?

239. Sometimes the Associative Property of Multiplication is helpful when you want to do "mental" math. For example, some people find it easier to find the value of $(-11) \cdot [6 \cdot (-5)]$ than to find the value of $[(-11) \cdot 6] \cdot (-5)$. Which do you find easier to compute mentally? Why?

240. Which do you find easier to compute mentally, $18 \cdot [(-10) \cdot 3]$ or $[18 \cdot (-10)] \cdot 3$? Why?

The Distributive Property

For help working these types of problems, go back to Examples 26–27 in the Explain section of this lesson.

241. Use the Distributive Property to find the missing number: $4 \cdot [(-8) + 1] = \_\_\_ \cdot (-8) + 4 \cdot 1$

242. Use the Distributive Property to find the missing number: $4 \cdot [(-8) + 1] = 4 \cdot (-8) + \_\_\_ \cdot 1$

243. Use the Distributive Property to find the missing number: $6 \cdot [(-13) + 9] = 6 \cdot \_\_\_ + 6 \cdot 9$

244. Use the Distributive Property to find the missing number: $21 \cdot [7 + (-4)] = 21 \cdot 7 + 21 \cdot \_\_\_$

245. Use the Distributive Property to find the missing number: $[(-8) + 3] \cdot 11 = \_\_\_ \cdot 11 + 3 \cdot 11$

246. Use the Distributive Property to find the missing number: $[17 + (-5)] \cdot (-1) = 17 \cdot (-1) + (-5) \cdot \_\_\_$

247. Use the Distributive Property to find the missing number: $(-41) \cdot 2 + (-41) \cdot (-3) = \_\_\_ \cdot [2 + (-3)]$

248. Use the Distributive Property to find the missing number: $11 \cdot 7 + (-6) \cdot 7 = [11 + (-6)] \cdot \_\_\_$

249. True or false. The statement $(-4) \cdot (12 + 3) = (-4) \cdot 12 + (-4) \cdot 3$ is an example of the Distributive Property.

250. True or false. The statement $7 \cdot [(-14) + 4] = 7 \cdot (-10)$ is an example of the Distributive Property.

251. True or false. The statement $(-5) \cdot [19 + (-8)] = [19 + (-8)] \cdot (-5)$ is an example of the Distributive Property.

252. True or false. The statement $[(-21) + 14] \cdot (-7) = (-21) \cdot (-7) + 14 \cdot (-7)$ is an example of the Distributive Property.

253. True or false. The statement $[2 \cdot (-8)] \cdot 6 = 2 \cdot [(-8) \cdot 6]$ is an example of the Distributive Property.

254. True or false. The statement $[(-12) + 3] \cdot 9 = (-12) \cdot 9 + 3 \cdot 9$ is an example of the Distributive Property.

255. True or false. The statement $[(-17) \cdot 5] + (-12) = (-12) + [(-17) \cdot 5]$ is an example of the Distributive Property.

256. True or false. The statement $[(-1) + 2] \cdot [(-3) + 5] = [(-1) + 2] \cdot (-3) + [(-1) + 2] \cdot 5$ is an example of the Distributive Property.

257. Rhonda was doing her homework on the Distributive Property while she was watching her neighbor's puppy. The puppy tore off a piece of Rhonda's homework paper. Here is what remained:

$(-8) \cdot [12 + (-36)] = (-8) \cdot 12 + (-8) \cdot$

Use the Distributive Property to complete this problem.

258. Dan was looking over his homework while drinking a soda. Dan's son accidentally knocked the soda over onto Dan's homework paper. Fortunately, Dan could read all of his homework except the last problem. Now the last problem read:

$(-11) \cdot 4 + (-11) \cdot (-24) = (-11) \cdot [4 +$

Use the Distributive Property to complete the problem.

259. Kelly and Craig are studying the Distributive Property. Craig has written down the following completed example:

$5 \cdot [(-5) + 8] = 5(-5) + 8$

Kelly says something is wrong with the example but Craig disagrees. Who is correct and why?

260. Steven and Eric are studying the Distributive Property. Eric has written down the following example:

$(-12) \cdot [3 \cdot (-7)] = (-12)(3) \cdot (-12)(-7)$

Steven says something is wrong with the example but Eric disagrees. Who is correct and why?

Sometimes the Distributive Property is helpful when you want to do "mental" math. For example, to find the product $(-13) \cdot 25$ mentally:

• Rewrite 25 as 20 + 5.	$(-13) \cdot (20 + 5)$
• Apply the Distributive Property.	$= (-13) \cdot 20 + (-13) \cdot 5$
• Simplify.	$= -260 + (-65)$
	$= -325$

261. Use the idea above and the Distributive Property to find the following product mentally: $(-7) \cdot 52$.
Hint: Rewrite 52 as 50 + 2.

262. Use the idea above and the Distributive Property to find the following product mentally: $(-11) \cdot 71$.
Hint: Rewrite 71 as 70 + 1.

263. Use the idea above and the Distributive Property to find the following product mentally: $(-14) \cdot (-31)$

264. Use the idea above and the Distributive Property to find the following product mentally: $(-43) \cdot (-132)$

Working with Variables

For help working these types of problems, go back to Examples 28–29 in the Explain section of this lesson.

265. Simplify this expression by combining appropriate terms: $7 + 2x + (-3) + (-5x)$

266. Simplify this expression by combining appropriate terms: $-13x + 9 + (-19) + 8x$

267. Simplify this expression by combining appropriate terms: $17 + (-5x) + 8x + 4 + (-29)$

268. Simplify this expression by combining appropriate terms: $6x + (-18) + 9 + (-8x) + 7x$

269. Simplify this expression by combining appropriate terms: $-7x + (-6) + (-11x) + 22 + 13x + (-13)$

270. Simplify this expression by combining appropriate terms: $6 + 14x + 7 + (-8x) + (-32) + (-15x)$

271. Simplify this expression by combining appropriate terms: $6 + 10x - 8x$

272. Simplify this expression by combining appropriate terms: $2x + 7 - 5x$

273. Simplify this expression by combining appropriate terms: $-9 + 5x + 19 - 18x$

274. Simplify this expression by combining appropriate terms: $-3x + 17 - 14x - 11$

275. Simplify this expression by combining appropriate terms: $8 + 10x - 25x - 17$

276. Simplify this expression by combining appropriate terms: $22 - 3x + 7x - 9$

277. Simplify this expression by combining appropriate terms: $-16 + 8x - 4x + 8 - 5x$

278. Simplify this expression by combining appropriate terms: $-21x + 7 + 12x - 4 - 2x - 6$

279. Simplify this expression by combining appropriate terms: $-2x + 6 + 7x - 15 + 3x$

280. Simplify this expression by combining appropriate terms: $-x + 5x - 6 + 8 - 3x$

281. The sum of four consecutive signed integers can be expressed as follows: $x + x + 1 + x + 2 + x + 3$. Simplify this expression by combining appropriate terms.

282. The sum of three consecutive odd signed integers can be expressed as follows: $x + x + 2 + x + 4$. Simplify this expression by combining appropriate terms.

283. Simplify this expression by combining appropriate terms: $4x + 3x - x - 5$

284. Simplify this expression by combining appropriate terms: $6x + 3x + 4x - 2x - 7$

285. Simplify this expression by combining appropriate terms: $3.5x + 2.4x + 8.2 + 7.9$

286. Simplify this expression: $11x + 4x + 7 + 19 + 25$

287. One type of application that is often seen in a beginning algebra class are number problems. In such problems part of a statement such as "ten less than three times a number, less five…" is common. The following expression can be used to represent this statement: $3x - 10 - 5$. Simplify this expression.

288. One type of application that is often seen in a beginning algebra class are number problems. In such problems part of a statement such as "seven more than twice a number plus –9 less three times the number…" is common. The following expression can be used to represent this statement: $7 + 2x + (-9) - 3x$. Simplify this expression.

Evaluate

Take this Practice Test to prepare for the final quiz in the Evaluate module of this lesson on the computer.

Practice Test

1. Do each multiplication.

   a. $(-7) \times (9)$
   b. $(-9) \times (-7)$

2. Choose the expression that has a positive value.

   a. $3.1 \times 15 \times (-2.5)$
   b. $-6 \times (-4.2) \times 24$
   c. $-2.4 \times (-32) \times (-5.5)$

3. Do each division.

   a. $15 \div (-3)$
   b. $(-24) \div (-6)$

4. Solve this equation for x: $13x = -91$

5. Find the value of each exponential expression.

   a. $(-4)^3$
   b. $(-4)^2$

6. Use the order of operations to find the value of this expression.
   $8 + (-5) \times [(-10) + 24 \div 4]$

7. Fill in the numbers that correctly illustrate the Distributive Property.
   $11 \times [25 + (-8)] = 11 \times$ ____ + ____ $\times (-8)$

8. Do this addition and subtraction: $17 - 35 + 7x + 13 - 4x$

Topic F4 Cumulative Review

These problems cover the material from this and previous topics. You may wish to do these problems to check your understanding of the material before you move on to the next topic, or to review for a test.

1. Use exponential notation to write $4 \times 4 \times 4 \times 7 \times 7 \times 7 \times 7$.
2. What is the base and what is the exponent of the expression: 13^{11}?
3. Find the value of the expression 1^{333}.
4. What is 13 squared?
5. What is the square root of 169?
6. If a square has an area 225 square inches, find the length of a side.
7. What is 5 cubed?
8. Find the value of $\{[(16 - 2) \div 7] \cdot 3\} \cdot (-5)$.
9. Find the missing number: $\frac{3}{16} = \frac{?}{64}$.
10. Do the subtraction: 12.73 – 6.91.
11. True or False? 1101.01 > 1101.001
12. Choose the fraction below that is equivalent to $\frac{3}{8}$:

    $\frac{21}{8}$ $\frac{21}{56}$ $\frac{3}{56}$ $\frac{9}{16}$
13. Round this decimal to the nearest hundredth: 0.3496.
14. Combine the terms with an "x" and combine the terms with a "y":

    $\frac{3}{5}x + \frac{1}{5}y + \frac{3}{5}y + \frac{1}{5}x$
15. True or False? $\frac{3}{17} \geq \frac{3}{17}$
16. What is the reciprocal of $\frac{3}{17}$?
17. Find the least common denominator of $\frac{3}{7}$ and $\frac{5}{8}$.
18. Round this number to the nearest thousand: 436, 499.
19. What number is 20% of 20?
20. A receipe calls for $\frac{2}{3}$ cup of milk. Ed is making 5 times what the receipe calls for.

    How many cups of milk does Ed need?
21. Find $-7 \cdot 9$.
22. Find $43 \cdot (-12)$.
23. Find $\left(-7\frac{1}{3}\right)\left(-9\frac{1}{5}\right)$.

24. Find $\left(-\frac{1}{8}\right) \cdot \left(-\frac{3}{7}\right) \cdot \left(-\frac{5}{12}\right)$.

25. Find $-\frac{336}{6}$.

26. Find $\left(-\frac{8}{23}\right) \div \left(-\frac{69}{13}\right)$.

27. Find $8\frac{4}{7} \div 2\frac{3}{14}$.

28. Solve for x: $-\frac{3}{4}x = -\frac{7}{16}$.

29. What is $(-5)^3$?

30. What is $\left(-\frac{3}{5}\right)^2$?

31. Find $(-60) \div 5 - (-18) \div 3$?

32. Find the value of the expression: $\sqrt{(7-7)^2 + (-3+15)^2}$.

33. Use the Commutative Property of Addition to find the missing number:
$(-7) + (-18) = \_\_\_\_ + (-7)$

34. Simplify this expression by combining appropriate terms:
$-12x + (-7) + (-8x) + 9 + (-13)$

35. Fill in the numbers that illustrate the Distributive Property:
$19 \times [8 + (-3)] = 19 \times \_\_\_\_ + \_\_\_\_ \times (-3)$