

LESSON 8.2 – RATIONAL EXPRESSIONS II





OVERVIEW

Here is what you'll learn in this lesson:

Negative Exponents

- a. Notation
- b. Scientific notation

Multiplying and Dividing

- a. Reducing a rational expression of the form $\frac{a-b}{b-a}$
- b. Multiplying rational expressions
- c. Dividing rational expressions
- d. Simplifying a complex fraction

Adding and Subtracting

- a. Finding the least common denominator of rational expressions
- b. Adding rational expressions with different denominators
- c. Subtracting rational expressions with different denominators
- d. Simplifying a complex fraction

Rational expressions are fractions in which the numerator and denominator are polynomials.

In this lesson, you will learn more about how to multiply, divide, add, and subtract rational expressions. You will also learn about negative exponents.



EXPLAIN

NEGATIVE EXPONENTS

Summary

Negative Exponents

You have seen how to work with exponents that are positive integers or 0. Now you will learn about negative integer exponents.

For example, you have previously found $\frac{3^5}{3^7}$ by canceling:

$$\frac{3^5}{3^7} = \frac{\overset{1}{3} \cdot \overset{1}{3} \cdot \overset{1}{3} \cdot \overset{1}{3} \cdot \overset{1}{3}}{\underset{1}{3} \cdot \underset{1}{3} \cdot \underset{1}{3} \cdot \underset{1}{3} \cdot \underset{1}{3} \cdot \underset{1}{3} \cdot \underset{1}{3}} = \frac{1}{3^2}$$

You can also find $\frac{3^5}{3^7}$ by subtracting exponents:

$$\frac{3^5}{3^7} = 3^{5-7} = 3^{-2}$$

$$\text{So, } 3^{-2} = \frac{1}{3^2}.$$

In general:

$$x^{-n} = \frac{1}{x^n}$$

Here, $x \neq 0$ and n is a nonnegative integer.

You can rewrite an expression with a negative exponent as a fraction by following these steps:

1. Write the numerator as 1.
2. Write the denominator as the original expression, except change the negative exponent to a positive exponent.
3. Do the multiplication in the denominator.

For example, to find 5^{-2} :

1. Write the numerator as 1.
2. Write the denominator as the original expression, except change the negative exponent to a positive exponent.

$$\frac{1}{5^2}$$

3. Do the multiplication in the denominator.

$$= \frac{1}{25}$$

$$\text{So, } 5^{-2} = \frac{1}{25}.$$

Even though the 5 is raised to a negative power, the result is a positive number.

Why does this work?

Well, since $4^{-3} = \frac{1}{4^3}$, then

$$\begin{aligned}\frac{1}{4^{-3}} &= \frac{1}{\frac{1}{4^3}} \\ &= 1 \div \frac{1}{4^3} \\ &= 1 \cdot 4^3 \\ &= 64\end{aligned}$$

Notice that the 2^{-4} in the numerator became the 2^4 in the denominator and that the 5^{-3} in the denominator became the 5^3 in the numerator.

Similarly, here's how to simplify an expression raised to a negative exponent that is in the denominator of a fraction:

1. Take the expression out of the denominator and change the negative exponent to a positive exponent.
2. Do the multiplication.

For example, to find $\frac{1}{4^{-3}}$:

1. Take the expression out of the denominator and change the negative exponent to a positive exponent. 4^3
2. Do the multiplication. $= 64$

In general:

$$\frac{1}{x^{-n}} = x^n$$

Here, $x \neq 0$ and n is a nonnegative integer.

Properties of Negative Exponents

The basic properties of nonnegative exponents also hold for negative exponents. The table below summarizes some of these properties.

Property	Positive Integer Exponents	Negative Integer Exponents
Multiplication	$3^2 \cdot 3^4 = 3^{2+4} = 3^6$	$3^{-2} \cdot 3^{-4} = 3^{(-2)+(-4)} = 3^{-6}$
Division	$\frac{4^7}{4^5} = 4^{7-5} = 4^2$	$\frac{4^{-7}}{4^{-5}} = 4^{(-7)-(-5)} = 4^{-7+5} = 4^{-2}$
Power of a Power	$(5^2)^3 = 5^{2 \cdot 3} = 5^6$	$(5^{-2})^{-3} = 5^{(-2)(-3)} = 5^6$
Power of a Product	$(5 \cdot 7)^3 = 5^3 \cdot 7^3$	$(5 \cdot 7)^{-3} = 5^{-3} \cdot 7^{-3}$
Power of a Quotient	$\left(\frac{3}{5}\right)^4 = \frac{3^4}{5^4}$	$\left(\frac{3}{5}\right)^{-4} = \frac{3^{-4}}{5^{-4}}$

Some other properties that hold for negative exponents are given below.

One property involving negative exponents shows how to rewrite a fraction in which the numerator and denominator are each raised to a negative power.

For example, here's one way to rewrite $\frac{2^{-4}}{5^{-3}}$ using only positive exponents:

$$\begin{aligned}\frac{2^{-4}}{5^{-3}} &= \frac{\frac{1}{2^4}}{\frac{1}{5^3}} \\ &= \frac{5^3}{2^4} \\ &= \frac{125}{16}\end{aligned}$$

In general:

$$\frac{x^{-m}}{y^{-n}} = \frac{y^n}{x^m}$$

Here, $x \neq 0$, $y \neq 0$, and m and n are nonnegative integers.

Another property involving negative exponents shows how to rewrite a fraction that is raised to a negative power.

For example, here's one way to simplify $\left(\frac{2}{3}\right)^{-4}$:

$$\begin{aligned}\left(\frac{2}{3}\right)^{-4} &= \frac{2^{-4}}{3^{-4}} \\ &= \frac{3^4}{2^4} \\ &= \left(\frac{3}{2}\right)^4\end{aligned}$$

In general:

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$$

Here, $x \neq 0$, $y \neq 0$, and n is a nonnegative integer.

Simplifying Expressions with Negative Exponents

You can combine different properties to simplify more complicated expressions containing negative exponents.

For example, to rewrite $(3r^4s^{-5}t)^{-2}$ using only positive exponents:

1. Use the Power of a Product Property. $(3r^4s^{-5}t)^{-2} = (3)^{-2} \cdot (r^4)^{-2} \cdot (s^{-5})^{-2} \cdot (t)^{-2}$
2. Use the Power of a Power Property. $= 3^{-2} \cdot r^{-8} \cdot s^{10} \cdot t^{-2}$
3. Rewrite the negative exponents as positive exponents. $= \frac{1}{3^2} \cdot \frac{1}{r^8} \cdot s^{10} \cdot \frac{1}{t^2}$
4. Simplify. $= \frac{s^{10}}{9r^8t^2}$

$$\text{So, } (3r^4s^{-5}t)^{-2} = \frac{s^{10}}{9r^8t^2}.$$

As another example, rewrite $\frac{m^3n^{-6}}{6m^{-2}n^3}$ using only positive exponents:

1. Since $\frac{x^{-m}}{y^{-n}} = \frac{y^n}{x^m}$, rewrite $\frac{n^{-6}}{m^{-2}}$ as $\frac{m^2}{n^6}$. $\frac{m^3n^{-6}}{6m^{-2}n^3} = \frac{m^3 \cdot m^2}{6n^6 \cdot n^3}$
2. Use the Multiplication Property of Exponents. $= \frac{m^5}{6n^9}$

$$\text{So, } \frac{m^3n^{-6}}{6m^{-2}n^3} = \frac{m^5}{6n^9}.$$

When a fraction is raised to a negative power, flip the fraction and change the negative power to positive.

As a third example, to simplify $\frac{73}{3^{-2} + 4^{-3}}$:

1. Rewrite using positive exponents.	$\frac{73}{3^{-2} + 4^{-3}} = \frac{73}{\frac{1}{3^2} + \frac{1}{4^3}}$
2. Multiply to eliminate the exponents.	$= \frac{73}{\frac{1}{9} + \frac{1}{64}}$
3. Rewrite each fraction in the denominator with the LCD, 576.	$= \frac{73}{\frac{64}{576} + \frac{9}{576}}$
4. Add the fractions in the denominator.	$= \frac{73}{\frac{73}{576}}$
5. Simplify.	$= 73 \div \frac{73}{576}$ $= 73 \cdot \frac{576}{73}$ $= 576$

So, $\frac{73}{3^{-2} + 4^{-3}} = 576$.

Scientific Notation

Now that you know how to work with negative exponents as well as positive exponents, you can learn how to use scientific notation. Scientific notation is a shorthand that is often used to write very small or very large numbers.

For example here are some very small and very large numbers:

- the gross national debt at the end of 1994 was \$4,643,700,000,000
- a number called Planck's constant that relates the energy of a photon to its frequency is 0.000000000000004135 electron volt seconds
- the speed of light is 299,792,500 meters per second

You can rewrite these numbers in scientific notation.

A number written in scientific notation is the product of a number between 1 and 10 and an integer power of 10. To write a number in scientific notation:

1. Move the decimal point to the left or to the right until you have a number between 1 and 10.
2. Multiply this number by a power of 10. To find the power, count the number of places you moved the decimal point.
 - If you moved the decimal point to the left, the power is positive.
 - If you moved the decimal point to the right, the power is negative.

Remember the decimal point is at the end of a whole number. For example, the number 239 can be written like this: 239.0

For example, to write \$4,643,700,000,000, in scientific notation:

1. Move the decimal point to the left.

$$4,643700000000$$

12 11 10 9 8 7 6 5 4 3 2 1

2. Multiply by the appropriate power of 10.

$$4.6437 \times 10^{12}$$

So, \$4,643,700,000,000 = 4.6437×10^{12} dollars.

As another example. To write 0.00000000000004135 eVsec (electron volt seconds), in scientific notation:

1. Move the decimal point to the right.

$$000000000000004.135$$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

2. Multiply by the appropriate power of 10.

$$4.135 \times 10^{-15}$$

So, 0.00000000000004135 eVsec = 4.135×10^{-15} eVsec.

You can also reverse this process to write a number given in scientific notation in expanded form.

To write a number in expanded form you must perform the multiplication indicated by the power of 10 in scientific notation. So:

1. Move the decimal point the number of places indicated by the power of 10.
 - If the power is positive, move the decimal point to the right.
 - If the power is negative, move the decimal point to the left.
2. Fill in additional zeros as needed.

For example, to write $2.997925 \times 10^8 \frac{\text{m}}{\text{s}}$, in expanded form:

1. Move the decimal point to the right 8 places.

$$2997925$$

1 2 3 4 5 6 7 8

2. Fill in additional zeros.

$$299792500.$$

1 2 3 4 5 6 7 8

So, $2.997925 \times 10^8 \frac{\text{m}}{\text{s}} = 299,792,500 \frac{\text{m}}{\text{s}}$.

You moved the decimal point 12 places to the left, so multiply by 10^{12} .

Here's a hint to help you know if you've moved the decimal point in the correct direction. If the power is positive, the number should be big. If the power is negative, the number should be less than 1.

c. $\frac{1}{a^6}, \frac{1}{c^3}$
 d. $\frac{125b^{21}}{a^6c^3}$

a. y^3, x^4

b. $\frac{3y^6}{x^6}$

b. $\frac{17}{\frac{1}{8} + \frac{1}{9}}$

c. $\frac{17}{\frac{9}{72} + \frac{8}{72}}$


d. $\frac{17}{\frac{17}{72}}$

e. $17 \div \frac{17}{72}$

$17 \cdot \frac{72}{17}$

72

b. 4.5318×10^7

a. 

b. 0.0000968

Sample Problems

1. Rewrite $(5a^{-2}b^7c^{-1})^3$ using only positive exponents.

a. Use the Power of a Product Property. $= (5)^3 \cdot (a^{-2})^3 \cdot (b^7)^3 \cdot (c^{-1})^3$

b. Use the Power of a Power Property. $= 125 \cdot a^{-6} \cdot b^{21} \cdot c^{-3}$

c. Rewrite using only positive exponents. $= 125 \cdot \underline{\hspace{1cm}} \cdot b^{21} \cdot \underline{\hspace{1cm}}$

d. Simplify. $= \underline{\hspace{2cm}}$

2. Rewrite $\frac{3x^{-4}y^3}{x^2y^{-3}}$ using only positive exponents.

a. Use $\frac{x^{-m}}{y^{-n}} = \frac{y^n}{x^m}$ to $\frac{3x^{-4}y^3}{x^2y^{-3}} = \frac{3 \underline{\hspace{1cm}} \cdot y^3}{x^2 \cdot \underline{\hspace{1cm}}}$
 rewrite $\frac{x^{-4}}{y^{-3}}$.

b. Use the Multiplication Property of Exponents. $= \underline{\hspace{2cm}}$

3. Simplify this expression: $\frac{17}{2^{-3} + 3^{-2}}$

a. Rewrite using only positive exponents. $= \frac{17}{\frac{1}{2^3} + \frac{1}{3^2}}$

b. Multiply to eliminate the exponents. $= \underline{\hspace{2cm}}$

c. Rewrite each fraction in the denominator with the LCD. $= \underline{\hspace{2cm}}$

d. Add the fractions in the denominator. $= \underline{\hspace{2cm}}$

e. Simplify. $= \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$

4. Write 45,318,000 in scientific notation.

a. Move the decimal point. 4.5318000

b. Multiply by the appropriate power of 10. $= \underline{\hspace{2cm}}$

5. Write 9.68×10^{-5} in expanded form.

a. Move the decimal point. $\underline{\hspace{2cm}}$

b. Fill in additional zeros. $= \underline{\hspace{2cm}}$

MULTIPLYING AND DIVIDING

Summary

Reducing a Rational Expression of the Form $\frac{a-b}{b-a}$

Look at this rational expression: $\frac{7-x}{x-7}$

Here, the numerator and denominator are already factored, and at first glance the rational expression may appear to be in lowest terms.

But notice that the numerator and denominator are the same except for their signs. To reduce this rational expression further:

1. Factor -1 out of **either** the numerator or the denominator.
Here, the numerator is factored.

$$\frac{7-x}{x-7} = \frac{-1(-7+x)}{x-7}$$

2. Rewrite $(-7+x)$ as $(x-7)$.

$$= \frac{-1(x-7)}{x-7}$$

3. Cancel common factors.

$$= \frac{-1(\cancel{x-7})}{\cancel{x-7}}$$

$$= -1$$

So, $\frac{7-x}{x-7} = -1$.

In general:

$$\frac{a-b}{b-a} = -1$$

Here, $a \neq b$.

Here's another example. To reduce $\frac{x^2+6x-27}{9-x^2}$ to lowest terms:

1. Factor the numerator and denominator.

$$= \frac{(x-3)(x+9)}{(3-x)(3+x)}$$

In the numerator, notice $(x-3)$.

In the denominator, notice $(3-x)$.

2. Recall that $\frac{x-3}{3-x} = -1$.

$$= -1 \cdot \frac{(x+9)}{(3+x)}$$

So, you can rewrite the expression.

$$= -\frac{x+9}{3+x}$$

Multiplying Rational Expressions

You have learned how to multiply rational expressions. Here is an example.

$$\begin{aligned} \text{Find } \frac{x^2 + 2x - 3}{x^2 + 5x + 6} \cdot \frac{x^2 + x - 12}{x^2 - 4x + 3}: \\ & 1. \text{ Factor the numerators and denominators.} & = \frac{(x-1)(x+3)}{(x+2)(x+3)} \cdot \frac{(x+4)(x-3)}{(x-1)(x-3)} \\ & 2. \text{ Cancel pairs of factors common to the numerators and denominators.} & = \frac{\overset{1}{\cancel{(x-1)}} \overset{1}{\cancel{(x+3)}}}{(x+2)\overset{1}{\cancel{(x+3)}}} \cdot \frac{(x+4)\overset{1}{\cancel{(x-3)}}}{\overset{1}{\cancel{(x-1)}} \overset{1}{\cancel{(x-3)}}} \\ & 3. \text{ Multiply the numerators. Multiply the denominators.} & = \frac{x+4}{x+2} \end{aligned}$$

So, $\frac{x^2 + 2x - 3}{x^2 + 5x + 6} \cdot \frac{x^2 + x - 12}{x^2 - 4x + 3} = \frac{x+4}{x+2}$.

Dividing Rational Expressions

You have learned how to divide rational expressions by inverting the divisor and then multiplying. Here is an example.

$$\begin{aligned} \text{Find } \frac{2x^2 + 5x - 3}{x^2 - 6x + 5} \div \frac{2x^2 + x - 1}{x^2 - 4x - 5}: \\ & 1. \text{ Invert the second fraction and change } \div \text{ to } \cdot \text{. Then multiply.} & = \frac{2x^2 + 5x - 3}{x^2 - 6x + 5} \cdot \frac{x^2 - 4x - 5}{2x^2 + x - 1} \\ & 2. \text{ Factor the numerators and denominators.} & = \frac{(2x-1)(x+3)}{(x-1)(x-5)} \cdot \frac{(x+1)(x-5)}{(2x-1)(x+1)} \\ & 3. \text{ Cancel pairs of factors common to the numerators and denominators.} & = \frac{\overset{1}{\cancel{(2x-1)}} \overset{1}{\cancel{(x+3)}}}{(x-1)\overset{1}{\cancel{(x-5)}}} \cdot \frac{\overset{1}{\cancel{(x+1)}} \overset{1}{\cancel{(x-5)}}}{\overset{1}{\cancel{(2x-1)}} \overset{1}{\cancel{(x+1)}}} \\ & 4. \text{ Multiply the numerators. Multiply the denominators.} & = \frac{x+3}{x-1} \end{aligned}$$

So, $\frac{2x^2 + 5x - 3}{x^2 - 6x + 5} \div \frac{2x^2 + x - 1}{x^2 - 4x - 5} = \frac{x+3}{x-1}$.

Simplifying a Complex Fraction

A fraction that contains other fractions is called a complex fraction. To simplify a complex fraction, start by rewriting the complex fraction as a division problem. Then invert the second fraction, and multiply.

$$\begin{aligned} \text{For example, to simplify the complex fraction } \frac{\frac{x^2 - 4}{x^2 - 8x + 15}}{\frac{12x + 24}{3x - 15}}: \\ & 1. \text{ Rewrite the complex fraction as a division problem.} & = \frac{x^2 - 4}{x^2 - 8x + 15} \div \frac{12x + 24}{3x - 15} \end{aligned}$$

2. Divide by inverting the second fraction and multiplying. $= \frac{x^2 - 4}{x^2 - 8x + 15} \cdot \frac{3x - 15}{12x + 24}$
3. Factor the numerators and denominators. $= \frac{(x+2)(x-2)}{(x-5)(x-3)} \cdot \frac{3(x-5)}{12(x+2)}$
 $= \frac{(x+2)(x-2)}{(x-5)(x-3)} \cdot \frac{3(x-5)}{2 \cdot 2 \cdot 3(x+2)}$
4. Cancel pairs of factors common to the numerators and denominators. $= \frac{\overset{1}{(x+2)}\overset{1}{(x-2)}}{\underset{1}{(x-5)}\underset{1}{(x-3)}} \cdot \frac{\overset{1}{3}\overset{1}{\cancel{(x-5)}}}{2 \cdot 2 \cdot \underset{1}{2}\underset{1}{\cancel{(x+2)}}$
5. Multiply the numerators. Multiply the denominators. $= \frac{x-2}{4(x-3)}$

So, $\frac{\frac{x^2 - 4}{12x + 24}}{3x - 15} = \frac{x - 2}{4(x - 3)}$.

Sample Problems

1. Reduce to lowest terms: $\frac{9-x}{x-9}$
- a. Factor -1 out of the numerator. $= \frac{-1(-9+x)}{x-9}$
- b. Rewrite $(-9+x)$ as $(x-9)$. $= \underline{\hspace{2cm}}$
- c. Cancel common factors. $= \underline{\hspace{2cm}}$
2. Reduce to lowest terms: $\frac{x^2 - 9x + 20}{16 - x^2}$
- a. Factor the numerator and denominator. $= \frac{(x-4)(x-5)}{(4-x)(4+x)}$
- b. Recall that $\frac{x-4}{4-x} = -1$. $= \underline{\hspace{2cm}} \cdot \underline{\hspace{2cm}}$
So rewrite the expression.
- c. Multiply. $= \underline{\hspace{2cm}}$

Answers to Sample Problems

b. $-\frac{1(x-9)}{x-9}$

c. -1

b. $-1, \frac{x-5}{4+x}$

c. $-\frac{x-5}{4+x}$

Answers to Sample Problems

a. $x + 5, x + 1$

b. $x \cdot \frac{x-4}{(x+1)}$

c. $\frac{x(x-4)}{x+1}$

b. $x + 2, x + 2$

c. $\frac{x}{(x+2)} \cdot \frac{(x+4)}{(x+2)}$

d. $\frac{x(x+4)}{(x+2)^2}$

b. $\frac{9}{2x+4}$

c. $\frac{x(x+2)}{3 \cdot x} \cdot \frac{3 \cdot 3}{2(x+2)}$

d. $\frac{1}{1} \cdot \frac{3}{2}$

e. $\frac{3}{2}$

3. Find: $\frac{x^3 + 2x^2 - 15x}{x^2 + 8x + 15} \cdot \frac{x^2 - x - 12}{x^2 - 2x - 3}$

a. Factor the numerators and denominators. $= \frac{x(x+5)(x-3)}{(\quad)(x+3)} \cdot \frac{(x+3)(x-4)}{(\quad)(x-3)}$

b. Cancel common factors. $= \underline{\hspace{2cm}}$

c. Multiply the numerators. $= \underline{\hspace{2cm}}$
Multiply the denominators.

4. Find: $\frac{x^2 - 3x}{x^2 + x - 2} \div \frac{x^2 - x - 6}{x^2 + 3x - 4}$

a. Invert the second fraction and change \div to \cdot . $= \frac{x^2 - 3x}{x^2 + x - 2} \cdot \frac{x^2 + 3x - 4}{x^2 - x - 6}$

b. Factor the numerators and denominators. $= \frac{x(x-3)}{(x-1)(\quad)} \cdot \frac{(x-1)(x+4)}{(x-3)(\quad)}$

c. Cancel common factors. $= \underline{\hspace{2cm}}$

d. Multiply the numerators. $= \underline{\hspace{2cm}}$
Multiply the denominators.

5. Simplify this complex fraction: $\frac{\frac{x^2 + 2x}{3x}}{\frac{2x + 4}{9}}$

a. Rewrite as a division problem. $= \frac{x^2 + 2x}{3x} \div \frac{2x + 4}{9}$

b. Divide. (Invert the second fraction and multiply.) $= \frac{x^2 + 2x}{3x} \cdot \underline{\hspace{2cm}}$

c. Factor the numerators and denominators. $= \underline{\hspace{2cm}}$

d. Cancel pairs of factors common to the numerator and denominator. $= \underline{\hspace{2cm}}$

e. Multiply the numerators. $= \underline{\hspace{2cm}}$
Multiply the denominators.

ADDING AND SUBTRACTING

Summary

Finding the Least Common Denominator (LCD) of Two or More Rational Expressions

In order to add or subtract rational expressions with different denominators, you need to find the least common denominator (LCD) of the rational expressions. This LCD is the least common multiple (LCM) of the polynomials in the denominators. You can find the LCM of a collection of polynomials in the same way you find the LCM of a collection of numbers.

To find the LCM of a collection of polynomials:

1. Factor each polynomial.
2. List each factor the greatest number of times it appears in any factorization.
3. Find the product of these factors. This is the LCM.

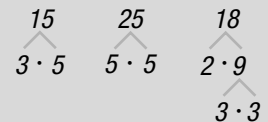
For example, to find the LCM of $x^2 - 4$, $x^2 + x - 6$, and $x^2 + 6x + 9$:

1. Factor each polynomial.
 $x^2 - 4 = (x + 2)(x - 2)$
 $x^2 + x - 6 = (x + 3)(x - 2)$
 $x^2 + 6x + 9 = (x + 3)(x + 3)$
2. List each factor the greatest number of times it appears in any factorization. $(x + 2)$, $(x - 2)$, $(x + 3)$, $(x + 3)$
3. Find the product of these factors. $(x + 2)(x - 2)(x + 3)(x + 3)$

So, the LCM of $x^2 - 4$, $x^2 + x - 6$, and $x^2 + 6x + 9$ is $(x + 2)(x - 2)(x + 3)(x + 3)$.

Remember, to find the LCM of a collection of numbers:

1. Find the prime factorization of each number.



2. List each prime factor the greatest number of times it appears in any factorization. $2, 3, 3, 5, 5$
3. Find the product of these factors.
This is the LCM.
 $2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 = 450$

The LCM of a collection of polynomials is usually left in factored form. You don't have to do the multiplication.

Remember, to add fractions with different denominators:

- Factor each denominator into its prime factors.

$$\frac{5}{6} + \frac{7}{15} = \frac{5}{2 \cdot 3} + \frac{7}{3 \cdot 5}$$

- Find the LCD of both fractions. This is the LCM of the denominators.

$$\text{LCD} = 2 \cdot 3 \cdot 5$$

- Rewrite each fraction with this LCD.

$$= \frac{5 \cdot 5}{2 \cdot 3 \cdot 5} + \frac{7 \cdot 2}{3 \cdot 5 \cdot 2}$$

- Add the numerators. Keep the denominator the same.

$$= \frac{25 + 14}{30}$$

$$= \frac{39}{30}$$

- Factor and reduce the fraction to lowest terms as appropriate.

$$= \frac{1}{3} \cdot \frac{13}{2 \cdot 5}$$

$$= \frac{13}{10}$$

$$\text{So, } \frac{5}{6} + \frac{7}{15} = \frac{13}{10}.$$

In the third step you may have been tempted to cancel common factors. But this would get you back to where you started. Remember to add first, then factor and cancel common factors.

Adding Rational Expressions with Different Denominators

You can add rational expressions with different denominators in much the same way you add fractions with different denominators. The key idea is to write the rational expressions with the same denominator.

To add rational expressions with different denominators:

- Factor each denominator.
- Find the LCM of the denominators. This is the LCD of the rational expressions.
- Rewrite each algebraic fraction with this LCD.
- Add the numerators. The denominator stays the same.
- Factor and reduce the rational expression to lowest terms as appropriate.

For example, to find $\frac{3}{m^3n} + \frac{5}{m^2n^2}$:

- Factor each denominator:
$$= \frac{3}{m \cdot m \cdot m \cdot n} + \frac{5}{m \cdot m \cdot n \cdot n}$$

- Find the LCD of the rational expressions.
$$\text{LCD} = m \cdot m \cdot m \cdot n \cdot n$$

- Rewrite each algebraic fraction with the LCD, $m \cdot m \cdot m \cdot n \cdot n$.
$$= \frac{3 \cdot n}{m \cdot m \cdot m \cdot n \cdot n} + \frac{5 \cdot m}{m \cdot m \cdot n \cdot n \cdot m}$$

- Add the numerators. The denominator stays the same.
$$= \frac{3n + 5m}{m^3n^2}$$

This rational expression is in lowest terms.

$$\text{So, } \frac{3}{m^3n} + \frac{5}{m^2n^2} = \frac{3n + 5m}{m^3n^2}.$$

As another example, to find $\frac{x+2}{x^2-4x} + \frac{x+2}{x^2-16}$:

- Factor each denominator:
$$= \frac{x+2}{x(x-4)} + \frac{x+2}{(x+4)(x-4)}$$

- Find the LCD of the rational expressions.
$$\text{LCD} = x(x+4)(x-4)$$

- Rewrite each algebraic fraction with the LCD, $x(x-4)(x+4)$.
$$= \frac{(x+2)(x+4)}{x(x-4)(x+4)} + \frac{(x+2)x}{(x+4)(x-4)x}$$

- Add the numerators. The denominator stays the same.
$$= \frac{x^2 + 6x + 8 + x^2 + 2x}{x(x+4)(x-4)}$$

$$= \frac{2x^2 + 8x + 8}{x(x+4)(x-4)}$$

- Factor and reduce the rational expression to lowest terms.
$$= \frac{2(x+2)(x+2)}{x(x+4)(x-4)}$$

$$\text{So, } \frac{x+2}{x^2-4x} + \frac{x+2}{x^2-16} = \frac{2(x+2)(x+2)}{x(x+4)(x-4)}.$$

Subtracting Rational Expressions with Different Denominators

You can subtract rational expressions with different denominators in much the same way you subtract fractions with different denominators. The key idea is to write the rational expressions with the same denominator.

To subtract rational expressions with different denominators:

1. Factor each denominator.
2. Find the LCM of the denominators. This is the LCD of the rational expressions.
3. Rewrite each algebraic fraction with this LCD.
4. Subtract the numerators. The denominator stays the same.
5. Factor and reduce the rational expression to lowest terms as appropriate.

For example, to find $\frac{4}{ab} - \frac{7}{a^3b^2}$:

1. Factor each denominator:
$$= \frac{4}{a \cdot b} - \frac{7}{a \cdot a \cdot a \cdot b \cdot b}$$
2. Find the LCD of the rational expressions.
$$\text{LCD} = a \cdot a \cdot a \cdot b \cdot b$$
3. Rewrite each algebraic fraction with the LCD, $a \cdot a \cdot a \cdot b \cdot b$.
$$= \frac{4 \cdot a \cdot a \cdot b}{a \cdot b \cdot a \cdot a \cdot b} - \frac{7}{a \cdot a \cdot a \cdot b \cdot b}$$
4. Subtract the numerators. The denominator stays the same.
$$= \frac{4a^2b - 7}{a^3b^2}$$

This rational expression is in lowest terms.

$$\text{So, } \frac{4}{ab} - \frac{7}{a^3b^2} = \frac{4a^2b - 7}{a^3b^2}.$$

As another example, to find $\frac{10}{x^2 + x - 6} - \frac{3}{x - 2}$:

1. Factor each denominator:
$$= \frac{10}{(x - 2)(x + 3)} - \frac{3}{x - 2}$$
2. Find the LCD of the rational expressions.
$$\text{LCD} = (x - 2)(x + 3)$$
3. Rewrite each algebraic fraction with the LCD $(x - 2)(x + 3)$.
$$= \frac{10}{(x - 2)(x + 3)} - \frac{3(x + 3)}{(x - 2)(x + 3)}$$
4. Subtract the numerators. The denominator stays the same.
$$= \frac{10 - 3(x + 3)}{(x - 2)(x + 3)}$$
$$= \frac{10 - 3x - 9}{(x - 2)(x + 3)}$$
$$= \frac{1 - 3x}{(x - 2)(x + 3)}$$

This rational expression is in lowest terms.

$$\text{So, } \frac{10}{x^2 + x - 6} - \frac{3}{x - 2} = \frac{1 - 3x}{(x - 2)(x + 3)}.$$

Remember, to subtract fractions with different denominators:

1. Factor each denominator into its prime factors.

$$\frac{5}{6} - \frac{7}{15} = \frac{5}{2 \cdot 3} - \frac{7}{3 \cdot 5}$$

2. Find the LCD of both fractions. This is the LCM of the denominators.

$$\text{LCD} = 2 \cdot 3 \cdot 5$$

3. Rewrite each fraction with this LCD.

$$= \frac{5 \cdot 5}{2 \cdot 3 \cdot 5} - \frac{7 \cdot 2}{3 \cdot 5 \cdot 2}$$

4. Subtract the numerators. Keep the denominator the same.

$$= \frac{25 - 14}{30}$$

$$= \frac{11}{30}$$

$$\text{So, } \frac{5}{6} - \frac{7}{15} = \frac{11}{30}.$$

In the third step you may have been tempted to cancel common factors.

But this would get you back to where you started. Remember to subtract first, then factor and cancel common factors.

Simplifying a Complex Fraction

Recall that a fraction that contains other fractions is called a complex fraction. Here's one way to simplify certain complex fractions:

1. Perform any addition or subtraction in the numerator or denominator.
2. Rewrite the complex fraction as a division problem.
3. Divide.

For example, to find $\frac{\frac{3}{x+1} + 5}{\frac{3}{x+1}}$:

1. Perform the addition in the numerator.

The least common denominator

of $\frac{3}{x+1}$ and 5 is $(x+1)$.

$$= \frac{\frac{3}{x+1} + \frac{5(x+1)}{x+1}}{\frac{3}{x+1}}$$

$$= \frac{\frac{3 + 5(x+1)}{x+1}}{\frac{3}{x+1}}$$

$$= \frac{\frac{3 + 5x + 5}{x+1}}{\frac{3}{x+1}}$$

$$= \frac{\frac{8 + 5x}{x+1}}{\frac{3}{x+1}}$$

2. Rewrite the complex fraction as a division problem.

$$= \frac{8 + 5x}{x+1} \div \frac{3}{x+1}$$

3. To divide, invert the second fraction and replace \div with \cdot .

$$= \frac{8 + 5x}{x+1} \cdot \frac{x+1}{3}$$

4. Cancel common factors.

$$= \frac{8 + 5x}{3}$$

$$\text{So, } \frac{\frac{3}{x+1} + 5}{\frac{3}{x+1}} = \frac{8 + 5x}{3}.$$

Sample Problems

1. Find the LCM of $x^2 + 7x + 12$, $x^2 - 9$, and $x^2 + 8x + 16$.

a. Factor each polynomial.

$$x^2 + 7x + 12 = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}})$$

$$x^2 - 9 = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}})$$

$$x^2 + 8x + 16 = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}})$$

b. List each factor the greatest number of times it appears in any factorization.

_____, _____, _____, _____

c. Find the product of the factors in the list. This is the LCM.

(____)(____)(____)(____)

2. Find: $\frac{2}{x^2 + 2x - 15} + \frac{x}{x^2 - 9}$

a. Factor each denominator.

$$= \frac{2}{(x+5)(x-3)} + \frac{x}{(x+3)(x-3)}$$

b. Find the LCD of the rational expressions.

$$\text{LCD} = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}})(\underline{\hspace{2cm}})$$

c. Rewrite each rational expression using this LCD.

$$= \frac{2(\underline{\hspace{2cm}})}{(x+5)(x-3)(\underline{\hspace{2cm}})} + \frac{x(\underline{\hspace{2cm}})}{(x+3)(x-3)(\underline{\hspace{2cm}})}$$

d. Add the numerators. The denominator stays the same.

$$= \frac{2(\underline{\hspace{2cm}}) + x(\underline{\hspace{2cm}})}{(x+5)(x+3)(x-3)}$$

$$= \frac{2x + 6 + x^2 + 5x}{(x+5)(x+3)(x-3)}$$

$$= \underline{\hspace{4cm}}$$

3. Find: $\frac{x}{x^2 - 6x + 9} - \frac{3}{x^2 - 9}$

a. Factor each denominator.

$$= \frac{x}{(x-3)(x-3)} - \frac{3}{(x+3)(x-3)}$$

b. Find the LCD of the rational expressions.

$$\text{LCD} = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}})(\underline{\hspace{2cm}})$$

c. Rewrite each rational expression using this LCD.

$$= \frac{x(\underline{\hspace{2cm}})}{(x-3)(x-3)(\underline{\hspace{2cm}})} - \frac{3(\underline{\hspace{2cm}})}{(x+3)(x-3)(\underline{\hspace{2cm}})}$$

d. Subtract the numerators. The denominator stays the same.

$$= \frac{x(\underline{\hspace{2cm}}) - 3(\underline{\hspace{2cm}})}{\underline{\hspace{4cm}}}$$

$$= \underline{\hspace{4cm}}$$

Answers to Sample Problems

a. $x + 3$, $x + 4$ (in either order)
 $x + 3$, $x - 3$ (in either order)
 $x + 4$, $x + 4$ (in either order)

b. $x + 3$, $x - 3$, $x + 4$, $x + 4$
 (in any order)

c. $(x + 3)(x - 3)(x + 4)(x + 4)$
 (in any order)

b. $x + 5$, $x + 3$, $x - 3$ (in any order)

c. top: $x + 3$, $x + 5$
 bottom: $x + 3$, $x + 5$

d. $x + 3$, $x + 5$

$$\frac{x^2 + 7x + 6}{(x+5)(x+3)(x-3)} \text{ or}$$

$$\frac{(x+1)(x+6)}{(x+5)(x+3)(x-3)}$$

b. $x - 3$, $x - 3$, $x + 3$ (in any order)

c. top: $x + 3$, $x - 3$
 bottom: $x + 3$, $x - 3$

d. top: $x + 3$, $x - 3$
 bottom: $(x - 3)(x - 3)(x + 3)$

$$\frac{x^2 + 3x - 3x + 9}{(x-3)(x-3)(x+3)} \text{ or}$$

$$\frac{x^2 + 9}{(x-3)(x-3)(x+3)}$$

Answers to Sample Problems

b. $z + 5, z - 5$

c. top: $z + 5, z - 5$
bottom: $z + 5, z - 5$

d. top: $z + 5, -1, z - 5$
bottom: $(z + 5)(z - 5)$

$$\frac{2z + 10 - z + 5 - 10}{(z + 5)(z - 5)}$$

$$\frac{z + 5}{(z + 5)(z - 5)}$$

e. $\frac{1}{z - 5}$

a. xy^2

b. $\frac{2y}{xy^2}, \frac{1}{xy^2}$

c. $\frac{2y + 1}{xy^2}$

d. $3 \div \frac{2y + 1}{xy^2}$

e. $\frac{3xy^2}{2y + 1}$

4. Find: $\frac{2}{z - 5} + \frac{-1}{z + 5} - \frac{10}{z^2 - 25}$

a. Factor each denominator. $= \frac{2}{z - 5} + \frac{-1}{z + 5} - \frac{10}{(z + 5)(z - 5)}$

b. Find the LCD of the rational expressions. LCD = $(\quad)(\quad)$

c. Rewrite each rational expression using this LCD. $= \frac{2(\quad)}{(z - 5)(\quad)} + \frac{-1(\quad)}{(z + 5)(\quad)} - \frac{10}{(z + 5)(z - 5)}$

d. Add or subtract the numerators. The denominator stays the same. $= \frac{2(\quad) + (\quad)(\quad) - 10}{\quad}$

$= \underline{\hspace{2cm}}$

e. Factor and reduce the rational expression to lowest terms. $= \underline{\hspace{2cm}}$

5. Simplify this complex fraction: $\frac{3}{\frac{2}{xy} + \frac{1}{xy^2}}$

a. To add the fractions in the denominator, find the LCD. LCD = $\underline{\hspace{2cm}}$

b. Rewrite each fraction using this LCD. $= \frac{3}{\quad + \quad}$

c. Add the fractions in the denominator. $= \frac{3}{\quad}$

d. Rewrite the complex fraction as a division problem. $= \underline{\hspace{2cm}}$

e. Divide. $= \underline{\hspace{2cm}}$



HOMWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.



Explain

Negative Exponents

- Find: 5^{-3}
- Find: $7^{-2} \cdot 7^3$
- Find: $\frac{50}{10^{-2} + 5^{-2}}$

In problems 4 through 9, simplify each expression. Use only positive exponents in your answers.

- $\left(\frac{2m}{n}\right)^{-3}$
- $\frac{(2m)^{-3}}{n^3}$
- $(n^4p^{-3})^5$
- $(2s^3t)^{-5} \cdot (2st^3)^6$
- $\frac{(3uv^4)(9u^3v)^{-2}}{(3u^5v^{-4})^{-3}}$
- All matter is made up of tiny particles called atoms. Through experimentation, it has been found that the diameters of atoms range from 1×10^{-8} cm to 5×10^{-8} cm. Rewrite each of these numbers in expanded form.
- The monthly payment E on a loan of amount P can be computed by using the formula below, where r is the monthly interest rate, and n is the number of months for which the loan is made. Find the monthly payment on a \$15,000 loan for 4 years (48 months) if the monthly interest rate is 1%.

$$E = \frac{Pr}{1 - (1 + r)^{-n}}$$

11. Write $\left(\frac{3x^4y}{z^3}\right)^{-2}$ using only positive exponents.

- Suppose $x = 3$ and $y = 5$.
 - Is $x^{-1}y^{-1} = \frac{1}{x} \cdot \frac{1}{y}$?
 - Is $x^{-1}y^{-1} = \frac{1}{xy}$?
 - Is $x^{-1} + y^{-1} = \frac{1}{x} + \frac{1}{y}$?
 - Is $x^{-1} + y^{-1} = \frac{1}{x+y}$?

Multiplying and Dividing

- Reduce to lowest terms: $\frac{y-13}{13-y}$
- Reduce to lowest terms: $\frac{y^2 + y - 20}{16 - y^2}$
- Find: $\frac{6-x}{x+7} \cdot \frac{x^2+7x}{x-6}$
- $\frac{a^2 - 5a - 14}{a^2 - 2a - 35} \cdot \frac{a^2 + 6a + 5}{a^2 - a - 6}$
- Find: $\frac{3x^2 - 7x + 2}{x^2 + 4x - 12} \cdot \frac{2x^2 + 12x}{3x^2 + 2x - 1}$
- Find: $\frac{(x+3)(x-2)}{x^2 - 4} \div \frac{(x+3)(x-4)}{x^2 - 16}$
- Find: $\frac{y^2 + 15y + 56}{8y^2 - 10y + 3} \div \frac{y+7}{4y-3}$
- Find: $\frac{x(x-9)}{x^2 + 3x + 2} \div \frac{x^2 - 81}{x(x+2)}$
- Find: $\frac{z^2 + 8z + 15}{z^2 - 16} \div \frac{z^2 + 10z + 25}{z+4}$

In Problems 22, 23 and 24, simplify the complex fractions. Reduce your answer to lowest terms.

22.
$$\frac{\frac{x^2 - 13x + 42}{x^2 - 4}}{\frac{3x - 18}{3x - 6}}$$

23.
$$\frac{\frac{2z^2 - 4z}{3z^3 + 6z^2}}{\frac{z-2}{z+2}}$$

24.
$$\frac{\frac{2w^2 - 4w}{2w^2 - 2}}{\frac{w^2 - 5w + 6}{w^2 - 2w - 3}}$$

Adding and Subtracting

25. Find the LCM of $x^2 + 10x + 25$ and $x^2 - 3x - 40$.

26. Find the LCM of x , $x^2 - 16$, and $x^2 + 7x + 12$.

27. Find: $\frac{5}{xy^3} + \frac{14}{x^2y^2}$

28. Find: $\frac{-1}{x^2 + 3x + 2} + \frac{2}{x^2 + 2x}$

29. Find: $\frac{4}{xy^2z} - \frac{3}{xyz^2}$

30. Find: $\frac{8}{x^2 + 14x + 49} - \frac{4}{x^2 - 49}$

31. Find: $\frac{-1}{y+2} + \frac{2}{y-2} - \frac{4}{y^2-4}$

32. Find: $\frac{1}{x^2 + 8x} - \frac{3}{x^2 - 64} + \frac{2}{x^2 + 6x - 16}$

33. Simplify the left side of the equation below to show it equals $\frac{1}{n}$.

Then use the equation to find two fractions with 1 in the numerator whose difference is $\frac{1}{5}$. (Hint: let $n = 5$).

$$\frac{1}{n-1} - \frac{1}{n(n-1)} = \frac{1}{n}$$

34. Optometrists use the formula below to find the strength to be used for the lenses of glasses. Simplify the right side of this formula, then find the value of P that corresponds to $a = 12$ and $b = 0.3$.

$$P = \frac{1}{a} + \frac{1}{b}$$

35. The total resistance, R , of a circuit that consists of two resistors connected in parallel with resistance R_1 and R_2 is given by the formula below. Simplify this formula, then find the resistance, R , if $R_1 = 3$ ohms and $R_2 = 4$ ohms.

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

36. Simplify this complex fraction: $\frac{1}{\frac{1}{a} + \frac{1}{ab^2}}$



Practice Problems

Here are some additional practice problems for you to try.

Negative Exponents

- Find: 2^{-3}
- Find: 4^{-2}
- Find: 3^{-4}
- Find: 5^{-2}
- Find: $\frac{1}{3^{-4}}$
- Find: $\frac{1}{5^{-3}}$
- Find: $4^{-7} \cdot 4^5$
- Find: $2^8 \cdot 2^{-5}$
- Find: $5^{-9} \cdot 5^6$
- Find: $\frac{1}{3^{-2} + 2^{-3}}$
- Find: $\frac{3}{4^{-3} + 5^{-2}}$
- Find: $\frac{2}{4^{-2} + 7^{-1}}$
- Rewrite using only positive exponents: $(a^4b^6)^{-1}$
- Rewrite using only positive exponents: $(x^3y^5)^{-2}$
- Rewrite using only positive exponents: $(m^6n^3p)^{-4}$
- Rewrite using only positive exponents: $(x^{-5}b^2)^5$
- Rewrite using only positive exponents: $(a^{-3}b^7)^4$
- Rewrite using only positive exponents: $(x^{-6}yz^{-3})^5$
- Rewrite using only positive exponents: $\left(\frac{c^4d^{-5}}{a^2}\right)^{-2}$
- Rewrite using only positive exponents: $\left(\frac{x^3w^{-2}}{y^4}\right)^{-3}$
- Rewrite using only positive exponents: $\left(\frac{m^{-4}n^5}{p^{-3}}\right)^{-4}$
- Rewrite using only positive exponents: $(3x^3y)^{-2} \cdot (3x^2y^{-1}z^3)^4$

- Rewrite using only positive exponents: $(4a^4b^2)^4 \cdot (4a^{-3}bc^2)^{-3}$
- Rewrite using only positive exponents: $(5x^{-3}y^{-1}z)^{-3} \cdot (5xz^{-5})^4$
- Write in scientific notation: 0.000057
- Write in scientific notation: 148,000,000
- The following number is written in scientific notation. Write it in expanded form: 4.3×10^6
- The following number is written in scientific notation. Write it in expanded form: 1.785×10^{-4}

Multiplying and Dividing

- Reduce to lowest terms: $\frac{x-5}{5-x}$
- Reduce to lowest terms: $\frac{x-3}{3-x}$
- Reduce to lowest terms: $\frac{x^2-2x-35}{x^2-25}$
- Reduce to lowest terms: $\frac{x^2+2x-24}{16-x^2}$
- Reduce to lowest terms: $\frac{x^2-8x+7}{49-x^2}$
- Reduce to lowest terms: $\frac{x^2-11x+30}{x^2-36}$
- Reduce to lowest terms: $\frac{x^2-8x-9}{81-x^2}$
- Find: $\frac{x^2+9x+14}{x^2+2x-15} \cdot \frac{x^2-4x+3}{x^2+6x-7}$
- Find: $\frac{x^2-7x+12}{x^2+2x-15} \cdot \frac{x^2-x-30}{x^2-3x-18}$
- Find: $\frac{x^2+5x+6}{x^2-5x-6} \cdot \frac{x^2-10x+24}{x^2-x-12}$

39. Find: $\frac{3x^2 - 6x}{x + 1} \cdot \frac{x - 1}{2 - x}$

40. Find: $\frac{5x^2 - 25x}{x - 3} \cdot \frac{x + 3}{5 - x}$

41. Find: $\frac{2x^2 - 6x}{x - 5} \cdot \frac{x + 5}{3 - x}$

42. Find: $\frac{x^2 + 2x - 15}{x^2 - 25} \cdot \frac{x^2 + 3x + 2}{x^2 - 2x - 3}$

43. Find: $\frac{x^2 + 4x - 45}{x^2 - 81} \cdot \frac{x^2 - 4x - 45}{x^2 - 7x + 10}$

44. Find: $\frac{x^2 + 10x + 21}{x^2 - 2x - 15} \cdot \frac{x^2 - x - 20}{x^2 - 16}$

45. Find: $\frac{x^2 + 2x - 35}{x^2 + x - 90} \div \frac{x^2 + 10x + 21}{x^2 + x - 90}$

46. Find: $\frac{x^2 + 2x - 35}{x^2 + x - 90} \div \frac{x^2 + 10x + 21}{x^2 + x - 90}$

47. Find: $\frac{x^2 - 2x - 3}{x^2 + 4x - 5} \div \frac{x^2 - 10x + 21}{x^2 + 4x - 5}$

48. Find: $\frac{5x - 25}{x^2 - 49} \div \frac{x^2 - 9x + 20}{x^2 - 11x + 28}$

49. Find: $\frac{x^2 + 7x + 6}{x^2 - 5x - 6} \div \frac{3x + 18}{x^2 + 5x - 66}$

50. Find: $\frac{4x - 16}{x^2 - 36} \div \frac{x^2 - x - 12}{x^2 + 9x + 18}$

51. Simplify the complex fraction below. Write your answer in lowest terms.

$$\frac{\frac{x^2 + 3x - 70}{x^2 - 49}}{\frac{x^2 + 9x - 10}{3x^2 - 3x}}$$

52. Simplify the complex fraction below. Write your answer in lowest terms.

$$\frac{\frac{x^2 - 4x - 45}{4x^2 + 20x}}{x^2 + 2x - 99}$$

53. Simplify the complex fraction below. Write your answer in lowest terms.

$$\frac{\frac{x^2 - 8x - 33}{x^2 - 9}}{x^2 - 9x - 22}$$

54. Simplify the complex fraction below. Write your answer in lowest terms.

$$\frac{\frac{x^2 - 7x + 6}{x^2 + 8x + 12}}{\frac{3x^2 - 3x}{x^2 - 36}}$$

55. Simplify the complex fraction below. Write your answer in lowest terms.

$$\frac{\frac{x^2 + 2x - 15}{5x^2 + 15x}}{x^2 - 2x - 35}$$

56. Simplify the complex fraction below. Write your answer in lowest terms.

$$\frac{\frac{x^2 + x - 20}{x^2 + 5x + 4}}{2x^2 + 10x}$$

Adding and Subtracting

57. Find the LCM of $x^2 + 7x + 12$ and $x^2 - 3x - 28$.

58. Find the LCM of $x^2 + 11x + 28$ and $x^2 + 2x - 8$.

59. Find the LCM of $x^2 + 4x$, $x^2 + 3x - 4$, and $x^2 - 2x + 1$.

60. Find the LCM of $x^2 - 6x$, $x^2 - 5x - 6$, and $x^2 + 2x + 1$.

61. Find: $\frac{4}{9a} + \frac{2}{3b}$

62. Find: $\frac{3}{8m} + \frac{7}{10n}$

63. Find: $\frac{2}{3x} + \frac{2}{6y}$

64. Find: $\frac{4}{x^2 - 4x - 12} + \frac{3}{x - 6}$

65. Find: $\frac{2}{x + 10} + \frac{5}{x^2 + 5x - 50}$

66. Find: $\frac{5}{x^2 + 4x - 21} + \frac{1}{x + 7}$

67. Find: $\frac{x}{x + 1} + \frac{x + 4}{x - 4}$

68. Find: $\frac{3x}{x + 3} + \frac{x + 2}{x - 7}$

69. Find: $\frac{5x}{x - 3} + \frac{x + 1}{x + 2}$

70. Find: $\frac{x + 2}{x^2 + 7x + 12} + \frac{x - 1}{x^2 + x - 12}$

71. Find: $\frac{x + 4}{x^2 + 5x + 6} + \frac{x - 3}{x^2 - 3x - 10}$

72. Find: $\frac{x - 5}{x^2 - 6x + 8} + \frac{x + 3}{x^2 + 2x - 8}$

73. Find: $\frac{4}{m^2n} - \frac{1}{mn^2}$

74. Find: $\frac{5}{abc} - \frac{7}{b^2}$

75. Find: $\frac{3}{xyz} - \frac{2}{x^2}$

76. Find: $\frac{7x}{x+6} - \frac{5}{x-1}$

77. Find: $\frac{3x}{x-8} - \frac{11}{x+3}$

78. Find: $\frac{8x}{x-7} - \frac{3}{x+1}$

79. Find: $\frac{x+2}{x^2+6x+5} - \frac{x+3}{x^2+4x-5}$

80. Find: $\frac{x+5}{x^2-8x+12} - \frac{x-1}{x^2-3x-18}$

81. Find: $\frac{x+4}{x^2+x-2} - \frac{x+1}{x^2+2x-3}$

82. Simplify this complex fraction: $\frac{\frac{4}{x} + \frac{1}{y}}{\frac{3}{x} - \frac{5}{y}}$

83. Simplify this complex fraction: $\frac{\frac{1}{x+1} + \frac{2}{x}}{\frac{4}{x} - \frac{3}{x+1}}$

84. Simplify this complex fraction: $\frac{\frac{7}{x} - \frac{3}{y}}{\frac{1}{x} + \frac{2}{y}}$

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

- Find: $\left(\frac{1}{2}\right)^{-3}$
- Fill in the blanks below by writing the numbers in either scientific notation or expanded form.

$73901 = \underline{\hspace{2cm}} \times 10^4$

$0.00004003 = 4.003 \times 10^{\underline{\hspace{1cm}}}$

$2.081 \times 10^2 = \underline{\hspace{2cm}}$

$9.019 \times 10^{-5} = \underline{\hspace{2cm}}$
- Rewrite $\frac{a^{-8}b^{13}}{a^{-2}b^{-2}}$ using only positive exponents.
- Rewrite $\frac{(2xy^5)(8x^2y)^{-3}}{(2x^4y^{-5})^{-2}}$ using only positive exponents.
- Reduce to lowest terms: $\frac{2x^3 - 8x}{4x - 2x^2}$
- Multiply and reduce to lowest terms:
 - $\frac{x+3}{x^2-4} \cdot \frac{x^2-10x+16}{x-8}$
 - $\frac{x^2+x-2}{x^3-6x^2} \cdot \frac{2x^2-14x+12}{x+2}$
- Divide and reduce your result to lowest terms:
 - $\frac{3x^2-75}{x^2-10x+25} \div \frac{-x-5}{x-5}$
 - $\frac{y^2+5y+4}{y^2+y-30} \div \frac{y^2+2y+1}{y-5}$
- Simplify this complex fraction: $\frac{\frac{4x+12}{5x-5}}{\frac{2x^2-18}{x^2-2x+1}}$
- Find the LCM of x , $x^2 + 12x + 35$, and $x^2 - 25$.
- Add and reduce your answer to lowest terms.

$$\frac{4}{b^2+4b+3} + \frac{3}{b^2+3b+2}$$
- Subtract and reduce your answer to lowest terms.

$$\frac{3y}{y^2+7y+10} - \frac{2y}{y^2+6y+8}$$
- Simplify this complex fraction: $\frac{\frac{3}{5}}{\frac{1}{x}+3}$