LESSON 7.2 – FACTORING POLYNOMIALS II

OVERVIEW

Here's what you'll learn in this lesson:

Trinomials I

a. Factoring trinomials of the form $x^2 + bx + c$; $x^2 + bxy + cy^2$

Trinomials II

a. Factoring trinomials of the form $ax^2 + bx + c$, $a \neq 1$, by trial-and-error

b. Factoring trinomials of the form $ax^2 + bx + c$, $a \neq 1$, by grouping

c. Solving quadratic equations by factoring

You have already learned how to factor certain polynomials by finding the greatest common factor (GCF) and by grouping.

In this lesson, you will learn techniques for factoring trinomials. Then you will see how to use factoring to solve certain equations.

EXPLAIN

TRINOMIALS I

Summary

Factoring Polynomials of the Form $x^2 + bx + c$

One way to factor a polynomial of the form $x^2 + bx + c$ as a product of binomials is to use the FOIL method, but work backwards. Here's an example.

The product of the first terms is x^2

$$x^2 - 3x - 4 = (x \quad)(x \quad)$$

The product of the last terms is -4

$$x^2 - 3x - 4 = (x \quad)(x \quad)$$

Try all the possible factorizations for which the product of the first terms is x^2 and the product of the last terms is -4. Since the product of the last terms is negative, one of the last terms is positive and the other is negative. Use the FOIL method to find factors whose "inner" and "outer" products add together to make $-3x$.

1. Make a chart of the possibilities for the binomial factors. These are shown in the table.

possible factorizations
$(x + \mathbf{4})(x - \mathbf{1})$
$(x - \mathbf{4})(x + \mathbf{1})$
$(x + \mathbf{2})(x - \mathbf{2})$

2. Use the FOIL method to multiply the possible factorizations you listed in step (1). These are shown in the table.

possible factorizations		
$(x + \mathbf{4})(x - \mathbf{1})$	=	$x^2 + 3x - 4$
$(x - \mathbf{4})(x + \mathbf{1})$	=	$x^2 - 3x - 4$
$(x + \mathbf{2})(x - \mathbf{2})$	=	$x^2 - 4$

3. Find the factorization that gives the original polynomial, $x^2 - 3x - 4$. In the second row you see that $x^2 - 3x - 4 = (x - 4)(x + 1)$.

   So the factorization is: $x^2 - 3x - 4 = (x - 4)(x + 1)$.

Answers to Sample Problems

b. $x^2 + 3x + 2$

c. $(x + 1)(x + 2)$ (in either order)

a. 6

3

Sample Problems

1. Factor: $x^2 + 3x + 2$

☑ a. List all the possible factorizations where:

- the product of the first terms is x^2
- the product of the last terms is $+2$

Since the product of the last terms is positive, both of the last terms are positive or both are negative.

possible factorizations
$(x + 1)(x + 2)$
$(x - 1)(x - 2)$

☐ b. Multiply the possible factorizations. Identify the factorization that gives the middle term $+3x$.

possible factorizations
$(x + 1)(x + 2) =$ ____________
$(x - 1)(x - 2) = x^2 - 3x + 2$

☐ c. Write the correct factorization. $x^2 + 3x + 2 =$ ________________

2. Factor: $x^2 - 7x + 12$

☐ a. List all the possible factorizations where:

- the product of the first terms is x^2
- the product of the last terms is $+12$

Since the product of the last terms is positive, both of the last terms are positive or both are negative.

possible factorizations
$(x + 1)(x + 12)$
$(x - 1)(x - 12)$
$(x + 2)(x + 6)$
$(x - 2)(x -$ __$)$
$(x +$ __$)(x + 4)$
$(x - 3)(x - 4)$

☐ b. Multiply the possible factorizations. Identify the factorization that gives the middle term $-7x$.

possible factorizations		
$(x+1)(x+12)$	=	$x^2+13x+12$
$(x-1)(x-12)$	=	____________
$(x+2)(x+6)$	=	____________
$(x-2)(x-$ ___$)$	=	$x^2-8x+12$
$(x+$ ___$)(x+4)$	=	$x^2+7x+12$
$(x-3)(x-4)$	=	____________

☐ c. Write the correct factorization. $x^2-7x+12 =$ ____________

3. Factor: x^2+x-2

☐ a. List all the possible factorizations where:
- the product of the first terms is x^2
- the product of the last terms is -2

Since the product of the last terms is negative, one of the last terms is positive and the other is negative.

possible factorizations
$(x+1)(x-2)$
$(x-1)($____$)$

☐ b. Multiply the possible factorizations.
Identify the factorization that gives the middle term $+1x$.

possible factorizations		
$(x+1)(x-2)$	=	____________
$(x-1)(\quad)$	=	____________

☐ c. Write the correct factorization. $x^2+x-2 =$ ____________.

Answers to Sample Problems

b. $x^2-13x+12$

$x^2+8x+12$

6

3

$x^2-7x+12$

c. $(x-3)(x-4)$ *(in either order)*

a. $x+2$

b. x^2-x-2

$x+2$, x^2+x-2

c. $(x-1)(x+2)$ *(in either order)*

4. Factor: $x^2 + 2x - 2$

☐ a. List all the possible factorizations where:
- the product of the first terms is x^2
- the product of the last terms is -2

Since the product of the last terms is negative, one of the last terms is positive and the other is negative.

possible factorizations
$(x + 1)(x - 2)$
$(x - 1)(\_\_\_\_\_)$

a. $x + 2$

☐ b. Multiply the possible factorizations.
Identify the factorization that gives the middle term +2x.

possible factorizations
$(x + 1)(x - 2) = \_\_\_\_\_\_\_\_\_\_$
$(x - 1)(\_\_\_\_\_) = \_\_\_\_\_\_\_\_\_\_$

b. $x^2 - x - 2$

$x + 2$, $x^2 + x - 2$

☑ c. Write the correct factorization. Neither of the possible factorizations gives the original polynomial, $x^2 + 2x - 2$. So, $x^2 + 2x - 2$ cannot be factored using integers.

Answers to Sample Problems

TRINOMIALS II

Summary

Factoring Polynomials of the Form $ax^2 + bx + c$ by Trial and Error

You have learned how to factor trinomials of the form $x^2 + bx + c$, where b and c are integers. Notice that the coefficient of x^2 is 1.

Now you will see how to factor trinomials of the form $ax^2 + bx + c$, where a, b, and c are integers. Notice that the coefficient of x^2 can be an integer other than 1.

One way to factor a trinomial of the form $ax^2 + bx + c$ as a product of binomials is by trial and error. Here's an example.

Factor the trinomial $3x^2 - 14x - 5$ using trial and error. Notice that any factorization of this trinomial must look like this:

$$3x^2 - 14x - 5 = (\mathbf{?}x \quad \mathbf{?})(\mathbf{?}x \quad \mathbf{?})$$

The product of the x-terms must be $3x^2$ and the product of the constants must be -5. Since the product of the constants is negative, one of the constants is positive and the other is negative.

1. Make a chart of the possibilities for the x-terms in the binomial factors and possibilities for the constant terms in the binomial factors. These are shown in the table below.

x-terms	constants
$3x$, x	1, –5
$3x$, x	5, –1
$3x$, x	–1, 5
$3x$, x	–5, 1

2. Use the values from step (1) to list possible factorizations. These are shown in the table below.

x-terms	constants	possible factorizations
$3x$, x	1, –5	$(3x + 1)(x - 5)$
$3x$, x	5, –1	$(3x + 5)(x - 1)$
$3x$, x	–1, 5	$(3x - 1)(x + 5)$
$3x$, x	–5, 1	$(3x - 5)(x + 1)$

3. Use the FOIL method to do the multiplication of the possible factorizations you listed in step (2). These are shown in the table below.

x-terms	constants	possible factorizations
$3x$, x	1, –5	$(3x + 1)(x - 5) = 3x^2 - 14x - 5$
$3x$, x	5, –1	$(3x + 5)(x - 1) = 3x^2 + 2x - 5$
$3x$, x	–1, 5	$(3x - 1)(x + 5) = 3x^2 + 14x - 5$
$3x$, x	–5, 1	$(3x - 5)(x + 1) = 3x^2 - 2x - 5$

4. Find the factorization that equals the original polynomial, $3x^2 - 14x - 5$. You can see that the shaded row is $3x^2 - 14x - 5$. So the factorization is: $3x^2 - 14x - 5 = (3x + 1)(x - 5)$

Here's another example. Factor the trinomial $15x^2 - 16x + 4$ using trial and error. Notice that any factorization of this trinomial must look like this:

$15x^2 - 16x + 4 = (\mathbf{?}x \quad \mathbf{?})(\mathbf{?}x \quad \mathbf{?})$

The product of the x-terms must be $15x^2$ and the product of the constant terms must be +4. Since the product of the last terms is positive, both of the last terms are positive or both are negative.

1. Make a chart of the possibilities for the x-terms in the binomial factors and possibilities for the constant terms in the binomial factors. These are shown in the table below.

x-terms	constants
x, 15x	1, 4
x, $15x$	2, 2
x, $15x$	4, 1
x, $15x$	–1, –4
x, $15x$	–2, –2
x, $15x$	–4, –1
$3x$, $5x$	1, 4
$3x$, $5x$	2, 2
$3x$, $5x$	4, 1
$3x$, $5x$	–1, –4
$3x$, $5x$	–2, –2
$3x$, $5x$	–4, –1

2. Use the values from step (1) to list possible factorizations. These are shown in the table that follows.

x-terms	constants	possible factorizations
x, $15x$	1, 4	$(x + 1)(15x + 4)$
x, $15x$	2, 2	$(x + 2)(15x + 2)$
x, $15x$	4, 1	$(x + 4)(15x + 1)$
x, $15x$	–1, – 4	$(x - 1)(15x - 4)$
x, $15x$	–2, –2	$(x - 2)(15x - 2)$
x, $15x$	–4, –1	$(x - 4)(15x - 1)$
$3x$, $5x$	1, 4	$(3x + 1)(5x + 4)$
$3x$, $5x$	2, 2	$(3x + 2)(5x + 2)$
3x, 5x	4, 1	$(3x + 4)(5x + 1)$
$3x$, $5x$	–1, –4	$(3x - 1)(5x - 4)$
$3x$, $5x$	–2, –2	$(3x - 2)(5x - 2)$
$3x$, $5x$	–4, –1	$(3x - 4)(5x - 1)$

3. Use the FOIL method to do the multiplication of the possible factorizations you listed in step (2). These are shown in the table.

x-terms	constants	possible factorizations
x, $15x$	1, 4	$(x + 1)(15x + 4) = 15x^2 + 19x + 4$
x, $15x$	2, 2	$(x + 2)(15x + 2) = 15x^2 + 32x + 4$
x, $15x$	4, 1	$(x + 4)(15x + 1) = 15x^2 + 61x + 4$
x, $15x$	–1, –4	$(x - 1)(15x - 4) = 15x^2 - 19x + 4$
x, $15x$	–2, –2	$(x - 2)(15x - 2) = 15x^2 - 32x + 4$
x, $15x$	–4, –1	$(x - 4)(15x - 1) = 15x^2 - 61x + 4$
$3x$, $5x$	1, 4	$(3x + 1)(5x + 4) = 15x^2 + 17x + 4$
3x, $5x$	2, 2	$(3x + 2)(5x + 2) = 15x^2 + 16x + 4$
$3x$, $5x$	4, 1	$(3x + 4)(5x + 1) = 15x^2 + 23x + 4$
$3x$, $5x$	–1, –4	$(3x - 1)(5x - 4) = 15x^2 - 17x + 4$
$3x$, $5x$	–2, –2	$(3x - 2)(5x - 2) = 15x^2 - 16x + 4$
$3x$, $5x$	–4, –1	$(3x - 4)(5x - 1) = 15x^2 - 23x + 4$

4. Find the factorization that equals the original polynomial, $15x^2 - 16x + 4$. You can see that the shaded row is $15x^2 - 16x + 4$. So the factorization is:
$15x^2 - 16x + 4 = (3x - 2)(5x - 2)$

Here's another example. Factor the trinomial $3x^2 - 8x - 5$ using trial and error. Notice that any factorization of this trinomial must look like this:
$3x^2 - 8x - 5 = (\mathbf{?}x \quad \mathbf{?})(\mathbf{?}x \quad \mathbf{?})$

The product of the x-terms must be $3x^2$ and the product of the constants must be –5. Since the product of the constants is negative, one of the constants is positive and the other is negative.

1. Make a chart of the possibilities for the x-terms in the binomial factors and possibilities for the constant terms in the binomial factors. These are shown in the table below.

x-terms	constants
$3x$, x	1, –5
$3x$, x	5, –1
$3x$, x	–1, 5
$3x$, x	–5, 1

2. Use the values from step (1) to list possible factorizations. These are shown in the table below.

x-terms	constants	possible factorizations
$3x$, x	1, –5	$(3x + 1)(x - 5)$
$3x$, x	5, –1	$(3x + 5)(x - 1)$
$3x$, x	–1, 5	$(3x - 1)(x + 5)$
$3x$, x	–5, 1	$(3x - 5)(x + 1)$

3. Use the FOIL method to do the multiplication of the possible factorizations you listed in step (2). These are shown in the table below.

x-terms	constants	possible factorizations
$3x$, x	1, –5	$(3x + 1)(x - 5) = 3x^2 - 14x - 5$
$3x$, x	5, –1	$(3x + 5)(x - 1) = 3x^2 + 2x - 5$
$3x$, x	–1, 5	$(3x - 1)(x + 5) = 3x^2 + 14x - 5$
$3x$, x	–5, 1	$(3x - 5)(x + 1) = 3x^2 - 2x - 5$

4. Find the factorization that equals the original polynomial, $3x^2 - 8x - 5$. You can see that no row is $3x^2 - 8x - 5$. So, $3x^2 - 8x - 5$ cannot be factored using integers.

Factoring Polynomials of the Form $ax^2 + bx + c$ by Grouping

Another way to factor a trinomial of the form $ax^2 + bx + c$ is by grouping.

Remember how to multiply binomials using the FOIL method.

$$\begin{aligned}(x + 2)(3x + 1) &= 3x^2 + x + 6x + 2 \\ &= 3x^2 + 7x + 2\end{aligned}$$

To factor $3x^2 + 7x + 2$, we go the other way. We first write $3x^2 + 7x + 2$ using two x-terms, like this:

$$3x^2 + x + 6x + 2$$

Now, factor $3x^2 + x + 6x + 2$ by grouping:

1. Factor each term.

$$3x^2 = 3 \cdot x \cdot x$$
$$x = x$$
$$6x = 2 \cdot 3 \cdot x$$
$$2 = 2$$

2. Group terms with common factors. $= (3x^2 + x) + (6x + 2)$

3. Factor out the GCF in each grouping. $= x(3x + 1) + 2(3x + 1)$

4. Factor out the binomial GCF of the polynomial. $= (3x + 1)(x + 2)$

5. Check your answer. Is $(3x + 1)(x + 2) = 3x^2 + 7x + 2$?

   Is $3x^2 + 7x + 2 = 3x^2 + 7x + 2$? Yes.

In order to use grouping to factor this trinomial, you had to find two integers whose sum was 7 and whose product was 6.

To factor a trinomial of the form $ax^2 + bx + c$, you need to find two integers whose sum is b and whose product is ac. Then you can split the x-term into two terms and factor by grouping.

For example, to factor $6x^2 + 7x + 2$ by grouping:

1. Make a chart of possible pairs of integers product is $6 \cdot 2 = 12$.

possibilities	product	sum
1, 12	12	13
2, 6	12	8
3, 4	12	7

Notice that the chart doesn't include negative factors of 12. Can you see why not? Since the product of the two numbers has to be +12, if one factor is negative, both would have to be negative. But since the sum of the integers needs to be +7, a positive number, you know both factors can't be negative.

2. Identify the numbers that work. Here, the last choice works since $3 + 4 = 7$ and $3 \cdot 4 = 12$.

3. Rewrite the trinomial. $6x^2 + 7x + 2 = 6x^2 + 3x + 4x + 2$

4. Group the terms. $= (6x^2 + 3x) + (4x + 2)$

5. Factor out the GCF in each grouping. $= 3x(2x + 1) + 2(2x + 1)$

6. Factor out the binomial GCF of the polynomial. $= (2x + 1)(3x + 2)$

7. Check your answer. Is $(2x + 1)(3x + 2) = 6x^2 + 7x + 2$?

   Is $6x^2 + 4x + 3x + 2 = 6x^2 + 7x + 2$? Yes.

Solving Quadratic Equations of the Form $ax^2 + bx + c = 0$ by Factoring

You can use what you have learned about factoring to solve some quadratic equations.

A quadratic (or second-degree) equation in one variable is an equation that can be written in this form:

$$ax^2 + bx + c = 0$$

This is called standard form. Here, a, b, and c are real numbers, and $a \neq 0$. The terms on the left side of the equation are in descending order by degree. The right side of the equation is zero.

If the left side of a quadratic equation in standard form can be factored, then you can solve the quadratic equation by factoring. To solve such an equation, you'll use a property called the Zero Product Property, which states the following: if P and Q are polynomials and if $P \cdot Q = 0$, then $P = 0$ or $Q = 0$ or both P and Q are 0.

Here's how to solve a quadratic equation in standard form when the left side can be factored:

1. Make sure the equation is in standard form.
2. Factor the left side.
3. Use the Zero Product Property. Set each factor equal to zero.
4. Finish solving for x.
5. Check your answer.

For example, to solve the equation $x^2 = 4x$:

1. Write the equation in standard form. $\quad x^2 - 4x = 0$
2. Factor the left side. $\quad x(x - 4) = 0$
3. Use the Zero Product Property to set each factor equal to zero. $\quad x = 0 \quad \text{or} \quad x - 4 = 0$
4. Finish solving for x. $\quad x = 0 \quad \text{or} \quad x = 4$
5. Check your answer.

Check $x = 0$:	Check $x = 4$:
Is $0^2 = 4(0)$?	Is $4^2 = 4(4)$?
Is $0 = 0$? Yes.	Is $16 = 16$? Yes.

So, both 0 and 4 are valid solutions of the equation $x^2 = 4x$.

Sample Problems

1. Use trial and error to factor the polynomial $35x^2 + 73x + 6$.

   ☐ a. Write possible x-terms whose product is $35x^2$ and write possible constant terms whose product is 6.

   ☐ b. List the possible factorizations.

   ☐ c. Multiply the possible factorizations.

x-terms	constants	possible factorizations
x, $35x$	1, 6	$(x + 1)(35x + 6) = 35x^2 + 41x + 6$
x, $35x$	2, __	(_____)(_____) = _______________
x, $35x$	3, 2	$(x + 3)(35x + 2) = 35x^2 + 107x + 6$
x, $35x$	6, __	(_____)(_____) = _______________
x, $35x$	__, –6	$(x - 1)(35x - 6) = 35x^2 - 41x + 6$
x, $35x$	–2, __	(_____)(_____) = _______________
x, $35x$	–3, –2	$(x - 3)(35x - 2) = 35x^2 - 107x + 6$
x, $35x$	–6, __	(_____)(_____) = _______________
$5x$, $7x$	1, 6	$(5x + 1)(7x + 6) =$ _______________
$5x$, $7x$	2, __	(_____)(_____) $= 35x^2 + 29x + 6$
$5x$, $7x$	3, 2	$(5x + 3)(7x + 2) = 35x^2 + 31x + 6$
$5x$, $7x$	6, __	$(5x + 6)(7x + 1) =$ _______________
$5x$, $7x$	–1, __	(_____)(_____) = _______________
$5x$, $7x$	__, –3	(_____)(_____) = _______________
$5x$, $7x$	–3, –2	$(5x - 3)(7x - 2) = 35x^2 - 31x + 6$
$5x$, $7x$	__, –1	(_____)(_____) = _______________

   ☐ d. Write the correct factorization. $35x^2 + 73x + 6 =$ _____________ (“in either order”)

2. Use trial and error to factor $4x^2 - 4x - 15$.

   ☐ a. Write possible x-terms whose product is $4x^2$ and the possible constant terms whose product is –15.

   ☐ b. List the possible factorizations.

   ☐ c. Multiply the possible factorizations.

Answers to Sample Problems

a., b., c.

3, $(x + 2)(35x + 3) = 35x^2 + 73x + 6$

1, $(x + 6)(35x + 1) = 35x^2 + 211x + 6$

–1

–3, $(x - 2)(35x - 3) = 35x^2 - 73x + 6$

–1, $(x - 6)(35x - 1) = 35x^2 - 211x + 6$

$35x^2 + 37x + 6$

3, $(5x + 2)(7x + 3)$

1, $35x^2 + 47x + 6$

–6, $(5x - 1)(7x - 6) = 35x^2 - 37x + 6$

–2, $(5x - 2)(7x - 3) = 35x^2 - 29x + 6$

–6, $(5x - 6)(7x - 1) = 35x^2 - 47x + 6$

$(x + 2)(35x + 3)$

x-terms	constants	possible factorizations
x, $4x$	1, –15	$(x+1)(4x-15) = 4x^2 - 11x - 15$
$2x$, $2x$	1, –15	$(2x+1)(2x-15) = 4x^2 - 28x - 15$
x, $4x$	3, –5	$(x+3)(4x-5) = 4x^2 + 7x - 15$
$2x$, $2x$	3, –5	$(2x+3)(2x-5) = 4x^2 - 4x - 15$
x, $4x$	5, –3	$(x+5)(4x-3) = 4x^2 + 17x - 15$
$2x$, $2x$	5, –3	$(2x+5)(2x-3) = 4x^2 + 4x - 15$
x, $4x$	15, ___	(_____)(_____) = ______________
$2x$, $2x$	15, ___	(_____)(_____) = ______________
x, $4x$	–1, ___	(_____)(_____) = ______________
$2x$, $2x$	–1, ___	(_____)(_____) = ______________
x, $4x$	___, 5	(_____)(_____) = ______________
$2x$, $2x$	___, 5	(_____)(_____) = ______________
x, $4x$	___, 3	(_____)(_____) = ______________
$2x$, $2x$	___, 3	(_____)(_____) = ______________
x, $4x$	–15, ___	(_____)(_____) = ______________
$2x$, $2x$	–15, ___	(_____)(_____) = ______________

☐ d. Write the correct factorization. $4x^2 - 4x - 15 =$ (_________)(_________)

3. Use grouping to factor $6x^2 + 11x + 4$.

✔ a. Make a chart of pairs of integers whose product is $6 \cdot 4 = 24$.

possibilities	product	sum
1, 24	24	25
2, 12	24	14
3, 8	24	11
4, 6	24	10

✔ b. Identify the two integers whose product is 24 and whose sum is 11. The two integers are 3 and 8.

✔ c. Rewrite the trinomial by splitting the x-term. $6x^2 + 11x + 4 = 6x^2 + 3x + 8x + 4$

☐ d. Group the terms. $=$ (_________) + (_________)

☐ e. Factor out the GCF in each grouping. $=$ ___ $(2x+1)$ + ___ $(2x+1)$

☐ f. Factor out the binomial GCF of the polynomial. $= (2x+1)$(__________)

Answers to Sample Problems

a., b., c.

–1, x +15, 4x – 1, 4x² + 59x –15

–1, 2x +15, 2x – 1, 4x² + 28x – 15

15, x – 1, 4x + 15, 4x² + 11x –15

15, 2x – 1, 2x + 15, 4x² + 28x –15

–3, x – 3, 4x + 5, 4x² – 7x –15

–3, 2x – 3, 2x + 5, 4x² + 4x –15

–5, x – 5, 4x + 3, 4x² – 17x –15

–5, 2x – 5, 2x + 3, 4x² – 4x –15

1, x – 15, 4x + 1, 4x² – 59x –15

1, 2x – 15, 2x + 1, 4x² – 28x –15

d. 2x – 5, 2x + 3 (in either order)

d. $6x^2 + 3x$, $8x + 4$

e. $3x$, 4

f. $(3x + 4)$

☐ g. Check your answer.

4. Use grouping to factor $3x^2 - 4x - 15$.

☐ a. Make a chart of pairs of integers whose product is $3 \cdot (-15) = -45$.

possibilities	product	sum
–1, 45	–45	44
–3, 15	–45	___
–5, ___	–45	___
1, ___	–45	___
___, –15	–45	___
___, ___	–45	___

☐ b. Identify the two integers whose product is –45 and whose sum is –4. The two integers are ______ and ______.

☐ c. Rewrite the trinomial by splitting the x-term. $3x^2 - 4x - 15 = 3x^2 +$ ___x + ___$x - 15$

☐ d. Group the terms. = (_________) + (_________)

☐ e. Factor out the GCF in each grouping. = __(______) + __(_______)

☐ f. Factor out the binomial GCF of the polynomial. = (_________)(_________)

☐ g. Check your answer.

5. Solve this quadratic equation for x by factoring: $8x^2 = 26x + 45$

✔ a. Write the equation in standard form. $8x^2 - 26x - 45 = 0$

☐ b. Factor the left side. (_______)(_______) = 0

☐ c. Use the Zero Product Property. $4x + 5 =$ ___ or $2x - 9 =$ ___

☐ d. Finish solving for x. $4x = -5$ or $2x = 9$

$x =$ ___ or $x =$ ___

☐ e. Check your answer.

Answers to Sample Problems

g. $(2x + 1)(3x + 4)$
$= 2x(3x) + 2x(4) + 1(3x) + 1(4)$
$= 6x^2 + 8x + 3x + 4$
$= 6x^2 + 11x + 4$

a. 12
9, 4
–45, –44
3, –12
5, –9, –4

b. 5, –9 (in either order)

c. 5, –9 (in either order)

d. $(3x^2 - 9x) + (5x - 15)$
or $(3x^2 + 5x) - (9x + 15)$

e. $3x(x - 3) + 5(x - 3)$
or $x(3x + 5) - 3(3x + 5)$

f. $(x - 3)(3x + 5)$ in either order

g. $(x - 3)(3x + 5)$
$= 3x^2 + 5x - 9x - 15$
$= 3x^2 - 4x - 15$

b. $4x + 5$, $2x - 9$ (in either order)

c. 0, 0

d. $-\frac{5}{4}, \frac{9}{2}$

e. Is $8\left(-\frac{5}{4}\right)^2 = 26\left(-\frac{5}{4}\right) + 45$?

Is $8\left(\frac{25}{16}\right) = 26\left(-\frac{5}{4}\right) + 45$?

Is $\frac{25}{2} = -\frac{65}{2} + \frac{90}{2}$?

Is $\frac{25}{2} = \frac{25}{2}$? Yes.

Is $8\left(\frac{9}{2}\right)^2 = 26\left(\frac{9}{2}\right) + 45$?

Is $8\left(\frac{81}{4}\right) = 26\left(\frac{9}{2}\right) + 45$?

Is $162 = 117 + 45$?

Is $162 = 162$? Yes.

Answers to Sample Problems

c. $3x$

a. $\frac{1}{5} \cdot x \cdot x \cdot y$

$\frac{1}{5} \cdot 3 \cdot x \cdot y$

b. $\frac{1}{5}xy$

c. $\frac{1}{5}xy(x-3)$

EXPLORE

Sample Problems

On the computer you used overlapping circles to help find the GCF of a collection of monomials. You used a table to help factor polynomials. Below are some additional problems.

1. Use overlapping circles to find the GCF of $3x$ and $-9xy^3$.

   ☑ a. Factor each monomial.

   $3x = 3 \cdot x$

   $-9xy^3 = -1 \cdot 3 \cdot 3 \cdot x \cdot y \cdot y \cdot y$

   ☑ b. Write the factorizations in the overlapping circles.

   $1 \cdot \quad 3 \cdot x \cdot (-1) \cdot 3 \cdot y \cdot y \cdot y$

   ☐ c. Find the GCF from the overlapping circles. GCF = ________

2. Factor: $\frac{1}{5}x^2y - \frac{3}{5}xy$

   ☐ a. Factor each monomial.

   $\frac{1}{5}x^2y =$ ________

   $\frac{3}{5}xy =$ ________

   ☐ b. Find the GCF of $\frac{1}{5}x^2y$ and $\frac{3}{5}xy$. GCF = ________

   ☐ c. Factor the polynomial $\frac{1}{5}x^2y - \frac{3}{5}xy$. (________)(________)

3. Find the GCF of the polynomials below.

A: $22x^2z + 22yz$

B: $11x^3 + 11xy$

C: $2x^2 + 2y$

☐ a. Factor each polynomial.

$22x^2z + 22yz = 2 \cdot 11 \cdot z(x^2 + y)$

$11x^3 + 11xy =$ __________

$2x^2 + 2y =$ __________

☐ b. Finish writing the factorizations in the overlapping circles.

4. A trinomial with a missing constant term has been partially factored in the table below. Complete the table and write the polynomial and its factorization.

☐ a. What times x gives $7x$? Use this to fill in box *a*.

☐ b. What times $3x$ gives $–9x$? Use this to fill in box b.

☐ c. Multiply boxes a and b. Use this to fill in box c.

☐ d. Write the polynomial and its factorization.

	x	b
$3x$	$3x^2$	$–9x$
a	$7x$	c

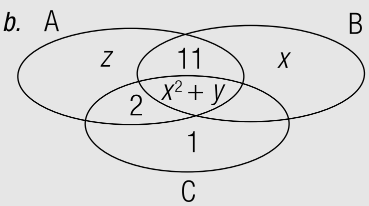
(________) = (________) + (________)

Answers to Sample Problems

a. $11 \cdot x \cdot (x^2 + y)$

$2 \cdot (x^2 + y)$

b.



a., b., c.

	x	$–3$
$3x$	$3x^2$	$–9x$
7	$7x$	$–21$

d.

$3x^2 – 2x – 21 = (3x + 7)(x – 3)$

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain

Trinomials I

1. Factor: $x^2 + 7x + 12$
2. Factor: $y^2 + 9y + 18$
3. Factor: $x^2 + 12x + 35$
4. Factor: $z^2 + 10z + 16$
5. Factor: $x^2 - 5x - 24$
6. Factor: $a^2 - 15a - 16$
7. Factor: $x^2 - x - 6$
8. Factor: $x^2 + 10x - 11$
9. Factor: $x^2 - 4x - 21$
10. Factor: $y^2 + 3y - 40$
11. Factor: $x^2 + 35x - 36$
12. Factor: $a^2 - 9a + 14$

Trinomials II

13. Factor: $2x^2 + 11x + 5$
14. Factor: $3x^2 + 13x + 4$
15. Factor: $4y^2 - 8y - 21$
16. Factor: $3z^2 - 17z + 20$
17. Factor: $15a^2 - 30a + 15$
18. Solve for x by factoring: $6x^2 = 63 - 13x$
19. Solve for x by factoring: $25x^2 + 5x = 2$
20. Factor: $4x^2 - 12x + 9$
21. Factor: $13x^2 + 37x + 22$
22. Factor: $x^2 - a^2$
23. Factor: $x^2 + 2xy + y^2$
24. Factor: $x^4 - 2ax^2 + a^2$

Explore

25. Circle the monomial(s) below that might appear in the factorization of

    $3x^3y^2 + 2x^2y - 3xy$

    $3x^2y$ $2x^2y$ xy $3x$

26. If the GCF of the terms of a polynomial is $4x^2y^3$, which of the monomials below could be terms in the polynomial?

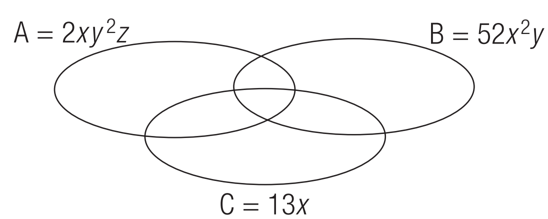
    $4xy^3$ $8x^3y^4$ $4x^2y^3$ $4x^2$

27. Factor this polynomial using overlapping circles: $\frac{x^2y}{2} - \frac{2y}{4}$

28. A trinomial with a missing constant term has been partially factored in the table below. Complete the table and write the polynomial and its factorization.

	x	
$5x$	$5x^2$	$-10x$
	$4x$	

29. Complete the diagram below to find the GCF of the polynomials A, B, and C.



30. Factor this polynomial using overlapping circles:

    $\frac{1}{2}x^2y^2 + \frac{3}{2}x^3y^3 - 3x^2y$

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Trinomials I

1. Factor: $x^2 + 5x + 4$
2. Factor: $x^2 + 6x + 5$
3. Factor: $x^2 + 15x + 14$
4. Factor: $x^2 + 11x + 10$
5. Factor: $x^2 + 8x + 15$
6. Factor: $x^2 + 9x + 18$
7. Factor: $x^2 + 7x + 12$
8. Factor: $x^2 - 13x + 30$
9. Factor: $x^2 - 8x + 12$
10. Factor: $x^2 - 7x + 10$
11. Factor: $x^2 - 15x + 44$
12. Factor: $x^2 - 11x + 30$
13. Factor: $x^2 - 10x + 21$
14. Factor: $x^2 - 6x - 27$
15. Factor: $x^2 - 7x - 30$
16. Factor: $x^2 - 5x - 14$
17. Factor: $x^2 + 4x - 21$
18. Factor: $x^2 + 10x - 24$
19. Factor: $x^2 + 5x - 36$
20. Factor: $x^2 + 2x - 15$
21. Factor: $x^2 - 7x - 18$
22. Factor: $x^2 + 9x - 36$
23. Factor: $x^2 - 4x - 21$
24. Factor: $x^2 + 10x + 24$
25. Factor: $x^2 - 2x - 63$
26. Factor: $x^2 + 9x - 22$
27. Factor: $x^2 - 7x - 60$
28. Factor: $x^2 - 6x - 91$

Trinomials II

29. Factor: $2x^2 + 7x + 5$
30. Factor: $2x^2 + 9x + 9$
31. Factor: $3x^2 - 19x - 14$
32. Factor: $2x^2 - 3x - 20$
33. Factor: $2x^2 - x - 28$
34. Factor: $3x^2 + 16x - 35$
35. Factor: $2x^2 + 5x - 12$
36. Factor: $2x^2 + 9x - 5$
37. Factor: $2x^2 + 13x + 15$
38. Factor: $2x^2 + 15x + 28$
39. Factor: $3x^2 + 11x + 6$
40. Factor: $12x^2 - 7x + 1$
41. Factor: $10x^2 - 9x + 2$
42. Factor: $6x^2 - 5x + 1$
43. Factor: $6x^2 - 11x - 10$
44. Factor: $9x^2 - 18x - 7$
45. Factor: $8x^2 - 2x - 3$
46. Factor: $6x^2 + 13x - 28$
47. Factor: $9x^2 - 3x - 20$
48. Factor: $4x^2 - 4x - 15$
49. Factor: $36x^2 + 13x + 1$
50. Factor: $30x^2 + 11x + 1$
51. Factor: $5x^2 + 14xy - 3y^2$
52. Factor: $4x^2 - 7xy - 2y^2$
53. Factor: $3x^2 - 5xy - 2y^2$
54. Factor: $6x^2 + xy - 12y^2$
55. Factor: $9x^2 - 3xy - 2y^2$
56. Factor: $4x^2 - 4xy - 3y^2$

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Factor: $x^2 - 10x + 24$

2. Circle the statement(s) below that are true.

   $x^2 + 2x - 1 = (x - 1)(x - 1)$

   $x^2 + 2x - 1 = (x + 2)(x - 1)$

   $x^2 + 2x - 1 = (x - 1)(x + 1)$

   $x^2 + 2x - 1 = (x + 1)(x + 1)$

   $x^2 + 2x - 1$ cannot be factored using integers

3. Factor: $t^2 - 16t - 17$

4. Factor: $r^2 + 10rt + 25t^2$

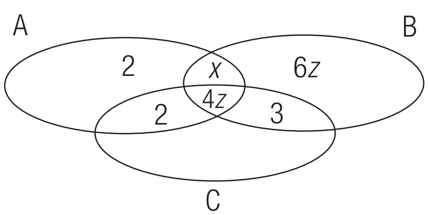
5. Factor: $5x^2 + 8x - 4$

6. Factor: $27v^2 - 57v + 28$

7. Factor: $4x^2 + 57x + 108$

8. Solve for x by factoring: $7x^2 - 5x - 12 = 0$

9. The overlapping circles contain the factors of three monomials, A, B, and C.
   Circle the true statements below.



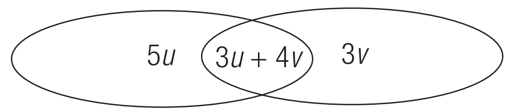
   Two factors of C are z and 2.

   B = $72xz$

   The GCF of A and B is x.

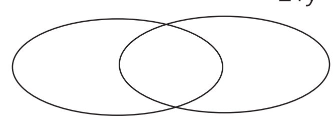
   The GCF of A, B, and C is $4z$.

10. The overlapping circles contain the factors of two binomials, A and B. Their GCF is $(3u + 4v)$. What are A and B?



11. The polynomial $14xy + 21y - 6x^2 - 9x$ can be grouped as two binomials: $(14xy - 6x^2) + (21y - 9x)$. Find the GCF of the two binomials by factoring the polynomial using the overlapping circles below.

$14xy - 6x^2$ $\qquad$ $21y - 9x$



12. Finish factoring the trinomial $6x^2 - 7xy - 3y^2$ using the table below.

		y
$2x$	$6x^2$	
		$-3y^2$