Test 1
Dusty Wilson
Math 107

Name: KEY

Young men should prove theorems, old men should write books.

Godfrey Harold "GH" Hardy (1877 - 1947)
English mathematician

No work = no credit

Warm-ups (1 pt each): $2+3=$ 5 $7\times8=$ 56 $2^3=$ 8

1. Based on the quote by GH Hardy (upper right corner), how should Dusty spend his year? Use complete sentences.

Write Books

2. Find the winner of the election using the Borda count method. *(Show your work.)*

Voters	4	1	9	8	5
1st	A	B	C	A	C
2nd	B	A	D	B	D
3rd	D	D	A	C	B
4th	C	C	B	D	A

A - 16 + 3 + 18 + 32 + 5
B - 12 + 4 + 9 + 24 + 10
C - 4 + 1 + 36 + 16 + 20
D - 8 + 2 + 27 + 8 + 15

3. Find the winner of the election using the method of pairwise comparisons. *(Show your work.)*

Voters	8	4	6	2	1
1st	A	D	D	C	E
2nd	B	B	B	A	A
3rd	C	A	E	B	D
4th	D	C	C	D	B
5th	E	E	A	E	C

21

(A) ∪ B B ∪ C C ∪ D D ∪ E
(A) ∪ C B ∪ D C ∪ E
(A) ∪ D B ∪ E
(A) ∪ E A WINS

4. a) Find the winner of the straw poll under the plurality-with-elimination method. *(Show your work.)*

Voters	4	5	6	2
1st	A	D	C	A
2nd	B	C	A	C
3rd	D	A	B	B
4th	C	B	D	D

17

	A	B	C	D
R1	8	0	6	5
R2	8	X	6	5
R3	8	X	11	X

C wins

b) In the official vote, everyone votes the same as in the straw poll except for the two voters in the last column of the table – they switch their votes and move C ahead of A in their ballots. Find the winner of the official vote under the plurality-with-elimination method. *(Show your work.)*

Voters	4	5	8
1st	A	D	C
2nd	B	C	A
3rd	D	A	B
4th	C	B	D

	A	B	C	D
R1	4	0	8	5
R2	4	X	8	5
R3	X	X	8	9

D wins

c) The results of (a) and (b) show that the plurality-with-elimination method violates one of the fairness criteria. Which one?

II Monotonicity criterion.

5. An election is held among six candidates (A, B, C, D, E and F). There are 57 voters. Using the method of pairwise comparisons, A, B, and C win 2 pairwise comparisons each. D wins 4 pairwise comparisons, E wins no pairwise comparisons, and F wins whatever is left. Circle the following that is true and ***explain why you chose it.***

D is a Condorcet candidate.

F is a Condorcet candidate.

There is no Condorcet candidate.

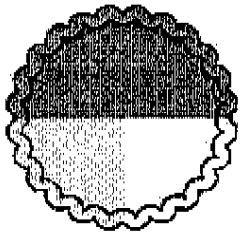
Cannot be determined from the information given.

None of the above

A ∪ B	B ∪ C	C ∪ D	D ∪ E	E ∪ F
C	D	E	F	
D	E	F		
E	F			
F				

15 head to heads.

(7.) (5 pts) Carli, Brenda, and Dale want to divide fairly the chocolate-strawberry-banana cake shown below.



chocolate
strawberry
banana

Piece A

strawberry
banana

- The total cost of the cake was $12.00.
- Carli values the chocolate portion five times as much as the banana and the strawberry four times as much as banana.
- Brenda values strawberry and banana equally, but values the chocolate portion three times more than the strawberry or banana portions.
- Dale values the strawberry portion twice as much as he values the chocolate. Further, he values the chocolate three times as much as he values the banana portion.

Piece A is a fair share in the eyes of one of the players. Find this player and justify your answer.

Carli

B - 1 → 1.20

C - 5

S - 4 → 4.80

$1.20/pleasure

Brenda

B - 1 → 2.40

S - 1 → 2.40

C - 3

$2.40/pleasure

Dale

B - 1 → 1.20

C - 3

S - 6 → 7.20

$1.20/pleasure

Piece A.

.5(1.20) + .5(4.80)

.6 + 2.40

$3.00

.5(2.40) + .5(2.40)

1.20 + 1.20

$ 2.40

.5(1.20) + .5(7.20)

.60 + 3.60

$4.20

Piece A is a fair share to Dale because it is worth at least $4 to him.

8. Four partners are dividing a piece of land using the ***lone-divider method***. The following table (missing some values) represents some of the information of their individual preferences.

	S_1	S_2	S_3	S_4
Alonzo	70000	$50,000	$50,000	$70,000
Brenda	$40,000	$80,000	$55,000	60000
Camille	$110,000	$40,000	$30,000	60000
Donald	$60,000	60000	60000	60000

a) Fill in the missing values of the table.

b) Who was the Divider?

Donald

c) What would be a fair division of the pieces of land?

A – S_1, S_4 C – S_1, S_4

B – S_2, S_4

A – S_1 B – S_2

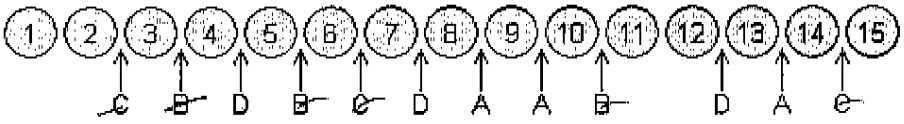
C – S_4 D – S_3

d) Is this the only possible division? If so, state why. If not, give an alternate fair division.

A – S_4 B – S_2

C – S_1 D – S_3

9. Four players agree to divide the following 15 items using the method of markers.



a) Which items go to player A?

14, 15

b) Which items go to player B?

4, 5

c) Which items go to player C?

1, 2

d) Which items go to player D?

8, 9, 10, 11, 12

e) Which items are left over?

3, 6, 7, 13

10. Using the method of sealed bids, Andre, Bea, Chad, and Dave are dividing a house, cabin, and boat.

	Andre	Bea	Chad	Dave
House	180,000	(200,000)	190,000	185,000
Cabin	(60,000)	50,000	40,000	55,000
Boat	16,000	12,000	(18,000)	10,000
Total	256000	262000	248000	250000
FS	64000	65500	62000	62500

Determine the final settlement of this fair-division.

A – Cabin & 4000 + 6000

B – House & pays 134500 − 6000

C – Boat & 44000 + 6000

D – 62500 + 6000

Surplus
250000
− 64000
− 65500
− 44000
− 62500
24000

11. Five players agree to divide a cake fairly using the last diminisher method. The players play in the following order: Anne first, Betty second, Cindy third, Doris fourth, and Ellen last. Suppose that there are no diminishers in round 1, Cindy and Ellen are the only diminishers in round 2, everyone diminishes in round 3, and there are no diminishers in round 4.

R_1 – Anne.

a) Which player starts round 2?

Betty

b) Which player gets a piece at the end of round 2?

Ellen

c) Which player gets a piece at the end of round 3?

Doris

d) Which player gets a piece at the end of round 4?

Betty

e) Which player starts round 5?

Cindy.