Section 13.04

Applications

Part 1: Continuous Income Streams

Example 1: Find the total income over the next 10 years from a continuous income stream that has an annual flow of $12,000 (imagine rent).

Question: Is it reasonable to assume a continuous income stream?

Example 2: A small company models its monthly income with $f(t) = 10000\, e^{0.02\, t}$. How much income can the company expect in its first two years (t in years)?

Part 2: Present and Future Value

Present vs. future value

Let $f(t)$ be the rate of continuous income flow for k years earning interest at a rate r, compounded continuously. Then, the present value of the continuous income stream is:

$$\text{PV} = \int_0^k f(t)\, e^{-r\,t}\, d\,t$$

Example 3: A continuous income stream has an annual rate of flow of $f(t) = 9000\, e^{0.12\,t}$. Find the present value of this income stream for the next 10 years, if money is worth 6%, compounded continuously.

Let $f(t)$ be the rate of continuous income flow for k years earning interest at a rate r, compounded continuously. Then, the future value of the continuous income stream is:

$$\text{FV} = e^{r k} \int_0^k f(t) e^{-r t} \, d t$$

Example 4: Suppose that a continuous income stream has an annual rate of flow given by $f(t) = 5000 e^{-0.01 t}$ and suppose that money is worth 7% compounded continuously. Create the integral used to find:

a.) The total income for the next 5 years.

b.) The present value of the income stream for the next 5 years.

c.) The future value of the income stream 5 years from today.

Part 3: Consumer and Supplier's Surplus

Consumer's Surplus:
Suppose that the demand for a product is given by $D: p = f(x)$ and that the supply of the product is described by $S: p = g(x)$. The price p_1 where the functions intersect is the equilibrium price. As the demand curve shows, some consumers would have been willing to pay more for the product that p_1; this is called the consumer's surplus

The formula:

$$\text{CS} = \int_0^{x_1} D\,dx - p_1 x_1$$

Supplier's Surplus:
When a product is sold at the equilibrium price, some suppliers benefit as they would have been willing to sell at a lower price. We refer to this increased revenue as the supplier's surplus.

The formula:

$$\text{SS} = p_1 x_1 - \int_0^{x_1} S\,dx$$

Example 5: If demand is $D: \; p = \frac{100}{x+1}$ and supply is $S: \; p = x + 1$, and market equilibrium is reached when 9 units are supplied at $10 each, create the integral used to find:

a.) The consumer's surplus

b.) The supplier's surplus