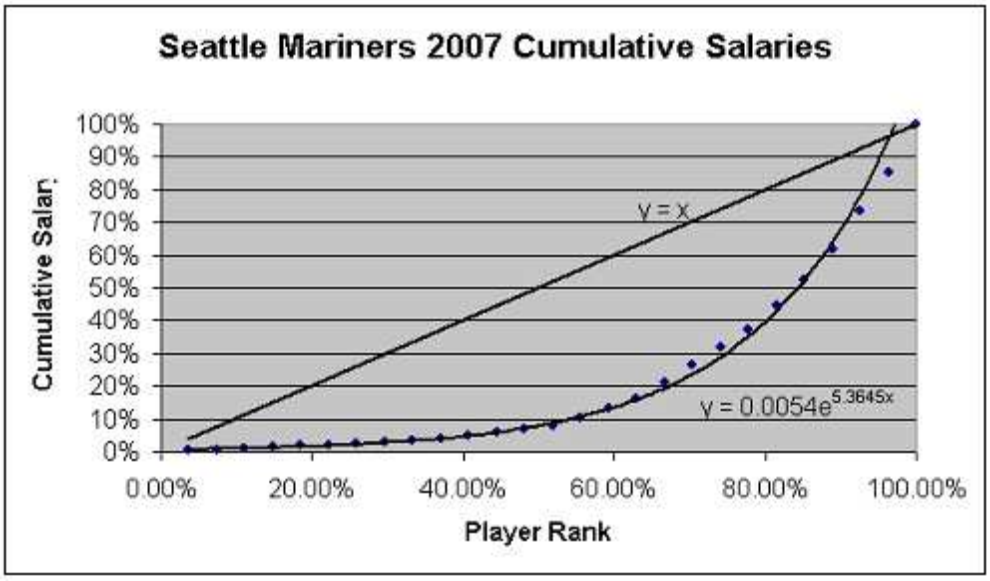
Section 13.03

Area between curves

Part 1: The Lorenz Curve

In economics, the Lorenz curve is used to represent the inequality of income distribution among different groups in the population of a country. The curve is constructed by plotting the cumulative percent of families at or below a given income level and the cumulative percent of total personal income received by these families.

The curve below shows the Lorenz curve $L(x)$ for the 27 players on the Seattle Mariner roster (as listed in USA Today).



Part 2: Area between curves

Example 1: Find the area between $y = x^2 + 2$ and $y = -x^2$ on $[0, 2]$

Example 2: Find the area bounded between $y = x^2$ and $y = \sqrt{x}$

Part 3: The Gini coefficient

Definition: The Gini Coefficient
We measure income distribution through the Gini coefficient which is defined as:

$$\frac{\text{area between } y=x \text{ and } L(x)}{\text{area below } y=x} = \frac{\int_0^1 [x - L(x)]\, dx}{1/2}$$

$$= 2\int_0^1 [x - L(x)]\, dx$$

Example 3: The Lorenz curve for income distribution in the US in 1950 and 1970 are given. Find and compare the Gini coefficients.

$$1950: \; y = 0.925\, x^{1.891}$$

$$1970: \; y = 0.920\, x^{1.783}$$

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This problem doesn't make
sense and the Lorenz curves
do not go thru (1,1). Use
different curves from the
text for accuracy.
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Question: Which is better (an ethical question): a small or large Gini coefficient?

Part 4: Average value of a function

Definition: Average value of a function
The average value of a continuous function $y = f(x)$ on $[a, b]$ is:

$$\text{average value} = f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Example 4: The cost to produce x units of a product is $C(x) = x^2 + 400x + 2000$.

a.) Find the average cost of producing 1000 units (the cost per unit)

b.) Find the average value of the cost function on [0, 1000] (the cost per shipment).