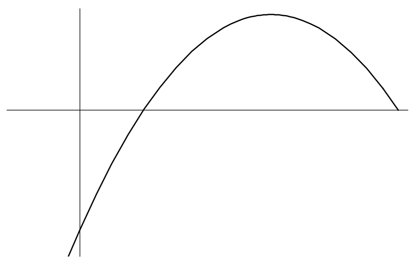
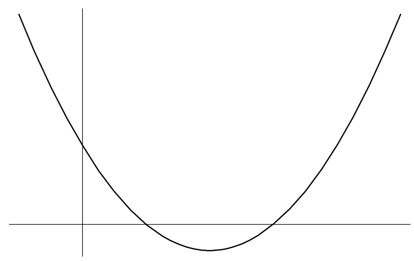
Section 10.02

Concavity

Part 1: The gist of concavity

In considering concavity, we need to look at whether the slopes of the tangents to a curve are increase, decreasing, or remaining constant.





On the left we see the slopes are __________ ______ and on the right the slopes are ________________ . But, changes in the slopes are measured by the derivative of the "slope function" or the second derivative of the original function.

When the slopes are increasing we say that the curve is __________________ and when the slopes are decreasing we say that the curve is __________________. We call the points where the concavity changes ___________________________.

In summary:

If $f'' > 0$ we would say that f is concave up and if $f'' < 0$ we would say that f is concave down.

Example 1: Given a graph of *f*, when is the second derivative positive, negative, or zero?

Definition: We call critical points where a function changes concavity, points of inflection (notice that these points have an *x* and *y* coordinate).

Part 2: Curve sketching with concavity

Example 2: Sketch $y = x^3 - x^2$

Example 3: Sketch $y = x^4 - 16x^2$

The second derivative test can be used to determine whether a point is a maximum or minimum. Basically, if the derivative is zero, then the curve potentially has a maximum or minimum. If it is a max, the curve must be concave down and if it is a min, the curve must be concave up. (Why?). Anyway, you would find the zeros of the derivative and then test the second derivative at those points. If the second derivative was negative, you would have a max, if it was positive, you would havea min. If the second derivative was zero or undefined, you would try something else.

However, the second derivative test is lame. It is lame because you can get all the info about maximums and minimums from the sign diagram of the first derivative. So, if you want to use it - great. If you don't - then figure out the sign diagrams.

So, what is the second derivative good for - finding the points of inflection. These have application to the points of diminishing marginal return etc.

Example 4: Sketch $y = 3x^5 - 20x^3$ given that $y' = 15x^2(x+2)(x-2)$

Example 5: Sketch $y = x^{1/3}(x-4)$ given that $y' = \frac{4\,(x-1)}{3\,x^{2/3}}$ and $y'' = \frac{4\,(x+2)}{9\,x^{5/3}}$