Limits at Infinity

Section 9.2

Part 1: Graphical limits at infinity

■ Example 1: A Review

(a.)

(b.)

(c.)

Part 2: Limits at infinity algebraically

■ Example 2: Simple Algebraic Examples

(a.) Find $\lim_{x\to\pm\infty} \frac{1}{x}$

(b.) Find $\lim_{x\to\pm\infty} c$

(c.) Find $\lim_{x\to\pm\infty} x$

■ Example 3: Find the following limits analytically

(a.) Find $\lim_{x\to-\infty} \frac{3}{x+2}$

(b.) Find $\lim_{x\to\infty} \frac{4x^2+2}{x^2-7}$

(c.) Find $\lim_{x\to-\infty} \frac{5x^3-4x}{2-3x^3}$

(d.) Find $\lim_{x\to-\infty} \frac{3x^6-2x}{4x^2+7x}$

■ Example 4: Sales application

The sales volume S (in \$1000's) is related to advertising expenditures d (also in \$1000's) according to $S(d) = \frac{35d}{7+d}$. What would happen to sales if there was an infinite advertising budget?

Part 3: Continuity at a point

Definition: Continuity at a point

The function f is continuous at c if $\lim_{x \to c} f(x) = f(c)$.

If the LHS DNE and/or the RHS is undefined and/or the LHS ≠ RHS, then we say that the function is discontinous.

Examples of discontinuity

(a.) A function where the limit DNE

(b.) A function that is undefined at a point

(c.) A function where the limit does not equal the function value at a point

■ Example 5: If or when are the following functions discontinuous?

(a.) $f(x) = \frac{x^2-4}{x-2}$

(b.) $g(x) = \frac{x^2+5x-6}{x+1}$

(c.) $h(x) = \begin{cases} 2, & x \le 0 \\ x+2, & x > 0 \end{cases}$

(d.) $f(x) = \begin{cases} x^2+1, & x \le 1 \\ 2x^2-1, & x > 1 \end{cases}$

■ Example 4 revisited: Sales application

When is the sales function $S(d) = \frac{35\,d}{7+d}$ discontinuous? What does this mean in context?

■ Real world examples of piecewise defined functions

- cell phones and calling cards

- income tax

- postage rates

- utility rates (electricity, natural gas, water, etc)

- rental cars