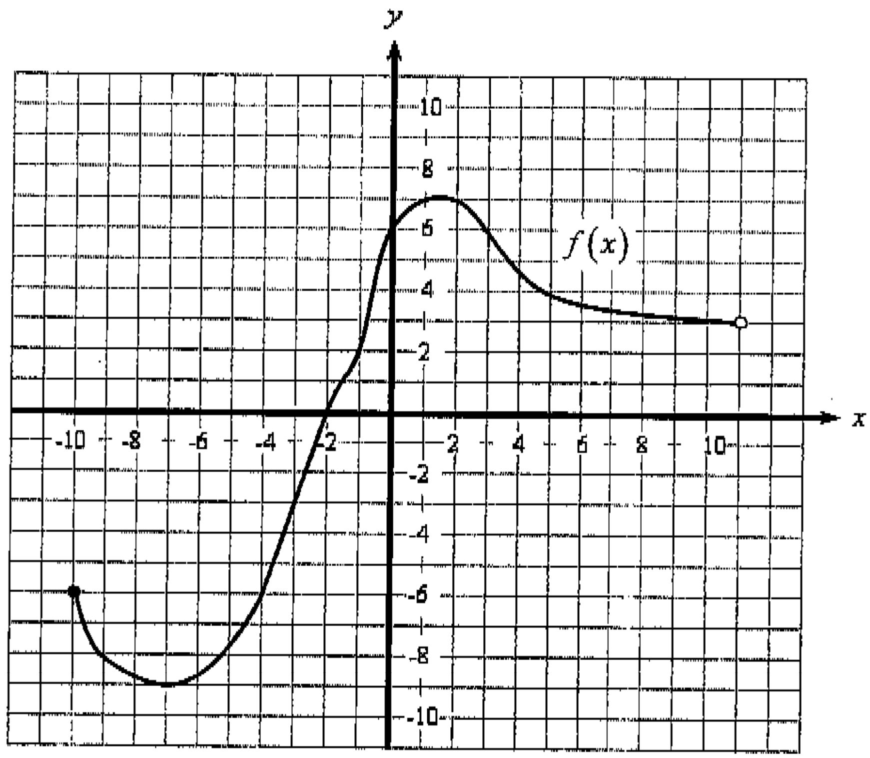
Spring 2005: Math 112 **Functions and Graphs: 2** **Name:** ____________

Consider the complete graph of $f(x)$ that is given below.



Use the graph to answer the following questions.

a.) $f(x) = -3$ when ________

b.) $f(x) = 3$ when ________

c.) $f(x) = -6$ when ________

d.) $f(x) = 7$ when ________

e.) $f(x) = 9$ when ________

f.) $f(-7) + f(-1) =$ ________

g.) The max of $f(x)$ takes place when $x =$ ___

h.) The min of $f(x)$ takes place when $x =$ ___

i.) $f(f(-1)) =$ ________

j.) $f(f(-7)) =$ ________

* Hand-out.

9.1a
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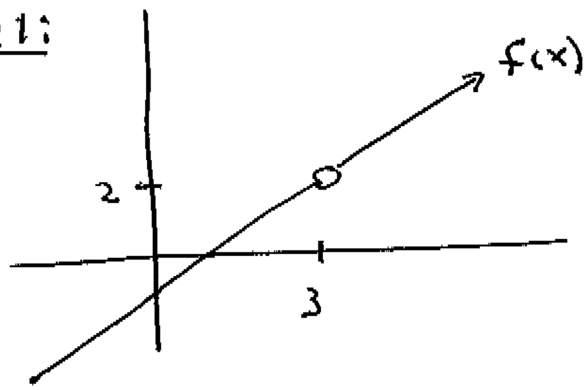
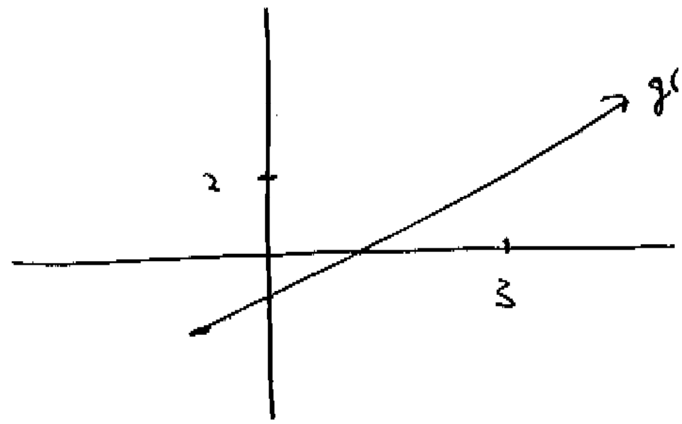
Graphical Limits.

Ex1:





the limit of f(x) as x approaches 3 is 2.

the limit of g(x) as x approaches 3 is 2.

Defn: The Limit.

Let $f(x)$ be a fct defined on an open interval containing c, except perhaps at $x=c$,

$$\text{then} \quad \lim_{x \to c} f(x) = L$$

if we can make values of $f(x)$ as close to L as we desire by choosing values of x sufficiently close to c.

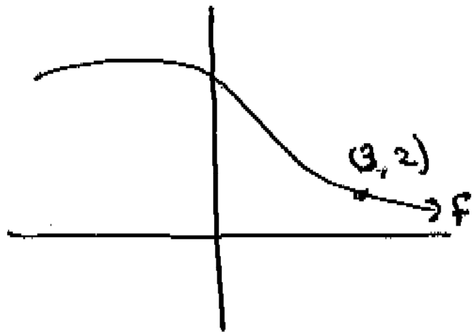
If the values of $f(x)$ do not approach a single finite L, the limit does not exist.

We read. $\lim_{x \to c} f(x) = L$ as, "the limit...

9, lk
2/4

Ex 2: Find the following...

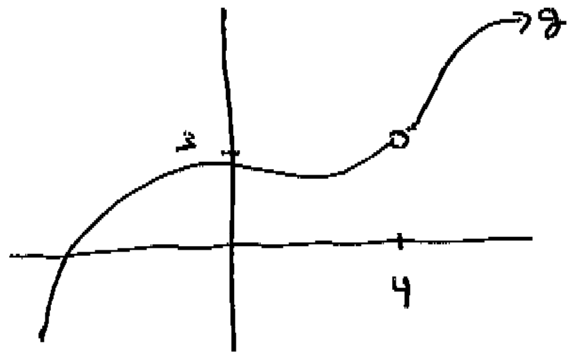
a)



i) $f(3)$

ii) $\lim_{x \to 3} f(x)$.

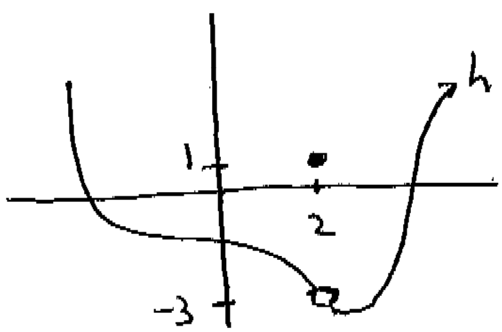
b)



i) $g(4)$

ii) $\lim_{x \to 4} g(x)$

c)



i) $h(2)$

ii) $\lim_{x \to 2} h(x)$

9,h
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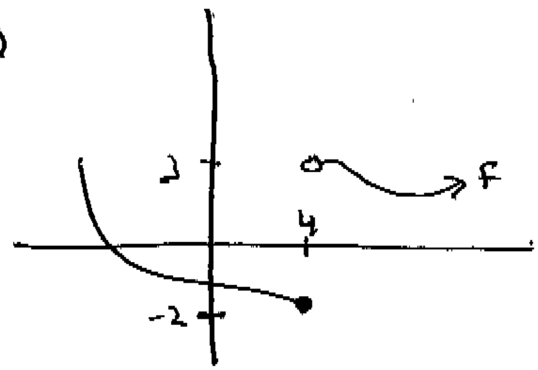
Ex 3: Find the following...

a)



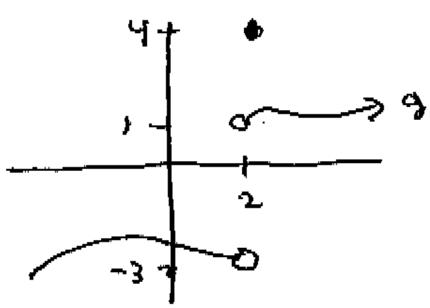
i) $f(4)$

ii) $\lim_{x \to 4} f(x)$

iii) $\lim_{x \to 4^+} f(x)$

iv) $\lim_{x \to 4^-} f(x)$.

b)



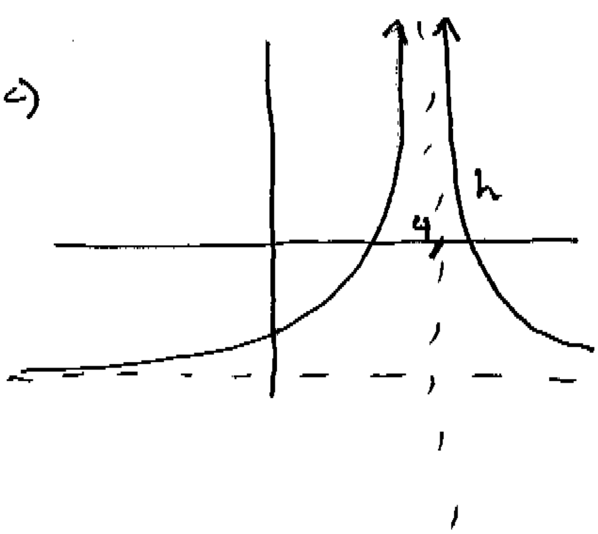
i) $g(2)$

ii) $\lim_{x \to 2} g(x)$

iii) $\lim_{x \to 2^+} g(x)$

iv) $\lim_{x \to 2^-} g(x)$

c)



i) $h(4)$

ii) $\lim_{x \to 4} h(x)$

iii) $\lim_{x \to 4^+} h(x)$

iv) $\lim_{x \to 4^-} h(x)$.

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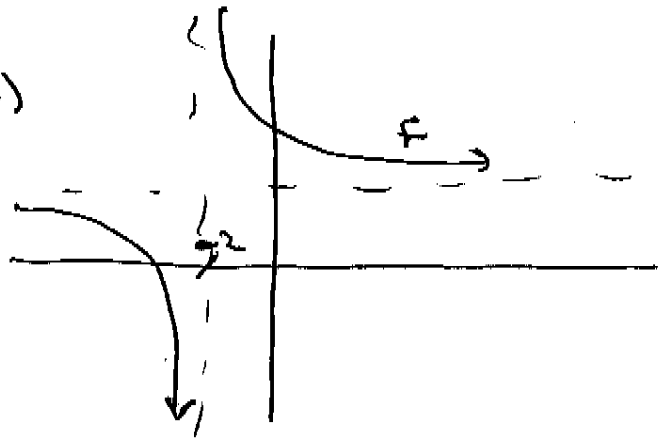
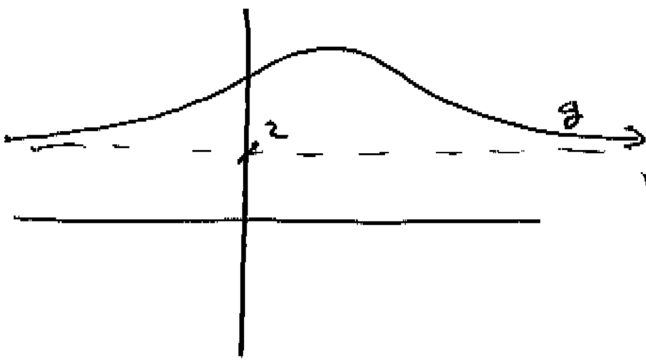
d)



i) $\lim_{x \to -2^+} f(x)$

ii) $\lim_{x \to -2^-} f(x)$

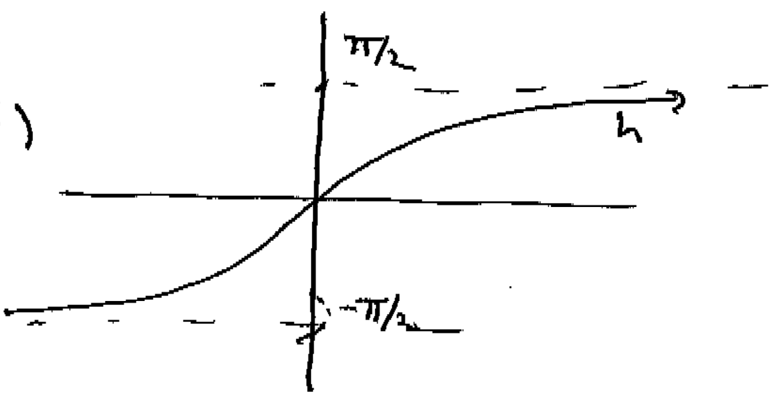
e)



i) $\lim_{x \to \infty} g(x)$

ii) $\lim_{x \to \infty} g(x)$

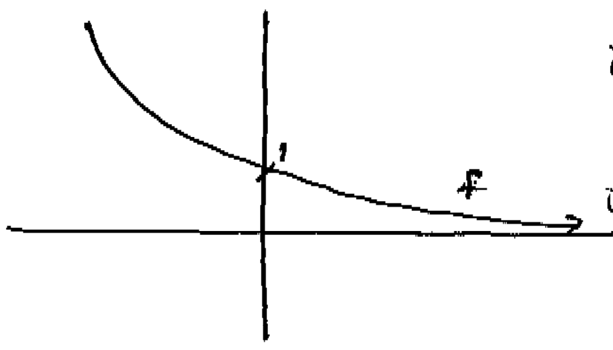
f)



i) $\lim_{x \to \infty} h(x)$

ii) $\lim_{x \to -\infty} h(x)$

g)



i) $\lim_{x \to \infty} f(x)$

ii) $\lim_{x \to -\infty} f(x)$.

9.1b
1/3

9.1: Limits Algebraically

Ex1: $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

Do these graphically ...

Ex2: $\lim_{x \to -1} (4x^3 - 2x^2 + 2)$

Ex3: $\lim_{x \to 3} f(x)$ where $f(x) = \begin{cases} 10 - 2x, & x < 3 \\ x^2 - x, & x \geq 3. \end{cases}$

Properties of Limits

If k is constant, $\lim_{x \to c} f(x) = L$, and $\lim_{x \to c} g(x) = M$

then.

I) $\lim_{x \to c} k = k$

II) $\lim_{x \to c} x = c$

III) $\lim_{x \to c} [f(x) \pm g(x)] = L \pm M$

IV) $\lim_{x \to c} (f \cdot g)(x) = L \cdot M$

V) $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}$ if $M \neq 0$.

VI) $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)} = \sqrt[n]{L}$ provided that $L > 0$ when n is even.

$\boxed{\begin{matrix} 9.1b \\ 2/3 \end{matrix}}$

Revisit examples 1 – 3 algebraically ...

<u>Ex2 rev</u>: $\lim_{x \to -1} (4x^3 - 2x^2 + 2)$

Recall that $\lim_{x \to c} f(x) = L$ iff $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) = L$

<u>Ex 3 rev</u>: $\lim_{x \to 3} f(x)$ where $f(x) = \begin{cases} 10 - 2x, & x < 3 \\ x^2 - x, & x \geq 3 \end{cases}$

<u>Ex1 rev</u>: Can we find $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$ using our limit laws? (not yet).

<u>Ex4</u>: $\lim_{x \to -1/3} \frac{1 - 3x}{9x^2 + 1}$

Examples 2 and 4 can be generalized to say if f & g are polynomials, then

a) $\lim_{x \to c} f(x) = f(c)$

b) $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{f(c)}{g(c)}$ when $g(c) \neq 0$.

9.1b
3/3

Ex 5: $\lim_{x \to 4} \dfrac{x^2-16}{x-4}$

Ex 6: $\lim_{x \to 7} \dfrac{x^2-8x+7}{x^2-6x-7}$

Ex 7: $\lim_{x \to 2} \dfrac{x^2+6x+9}{x-2}$

Ex 8: $\lim_{x \to 1} \dfrac{x^2-1}{x^2-2x+1}$

Conclusion: evaluating limits of rational functions where the denominator approaches zero.

a) If the numerator does not approach zero, then the limit D.N.E.

b) If the ~~d~~ numerator approaches zero, simplify and try again.

Ex 9: $\lim_{x \to -1} f(x)$, where $f(x) = \begin{cases} x^2 + \frac{4}{x}, & x \le -1 \\ 3x^3 - x - 1, & x > -1. \end{cases}$

Ex 10: Suppose that the cost c of removing p percent of the pollution from an industrial plant is modeled by: $c(p) = \dfrac{730{,}000}{100-p} - 7300.$

a) Find $\lim_{p \to 80} c(p)$

b) Find $\lim_{p \to 100^-} c(p)$

c) Can all the pollution be removed?