LESSON 6.3 – POLYNOMIAL OPERATIONS II

OVERVIEW

Here's what you'll learn in this lesson:

Multiplying Binomials

a. Multiplying binomials by the "FOIL" method

b. Perfect squares, product of the sum and difference of two terms

Multiplying and Dividing

a. Multiplying a polynomial by a polynomial

b. Dividing a polynomial by a polynomial

Polynomials can be used to solve many types of problems. Some people might use a polynomial to create a household budget. A structural engineer might use a polynomial to find the wind force on a large building. Or, an automobile company might use a polynomial to find the average cost of manufacturing an airbag.

In this lesson, you will learn more about Polynomial Operations. You will multiply and divide polynomials which have more than one term.

EXPLAIN

MULTIPLYING BINOMIALS

Summary

The multiplication of a binomial by a binomial can be simplified by using the "FOIL" method or by using patterns.

Using the FOIL Method to Multiply Two Binomials

The FOIL method can be used to multiply any two binomials. The letters in the word "FOIL" show you the order in which to multiply.

The general format is:

$$(a+b)(c+d) = a \cdot c + a \cdot d + b \cdot c + b \cdot d$$

$$\text{F} + \text{O} + \text{I} + \text{L}$$

$$\text{First} + \text{Outer} + \text{Inner} + \text{Last}$$

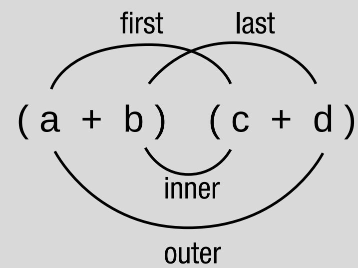
To multiply two binomials using the FOIL method:

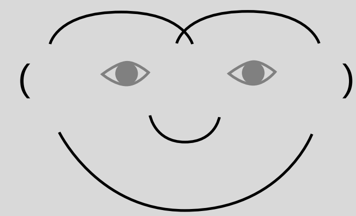
1. Multiply the First terms of the binomials.
2. Multiply the Outer terms (the terms next to the outer parentheses).
3. Multiply the Inner terms (the terms next to the inner parentheses).
4. Multiply the Last terms.
5. Add the terms. Be sure to combine like terms.

For example, to find: $(x-2)(x+5)$

1. Multiply the First terms: $x \cdot x$
2. Multiply the Outer terms: $x \cdot 5$
3. Multiply the Inner terms: $-2 \cdot x$
4. Multiply the Last terms: $-2 \cdot 5$
5. Add the terms. $x^2 + 5x - 2x - 10$

   $= x^2 + 3x - 10$





This picture may help you remember how to use the FOIL method. Notice how the connecting lines form a face: F and L make the eyebrows, O makes the smile and I the nose.

Using Patterns to Multiply Two Binomials

Patterns can be used to find certain binomial products.

In general, when you square a binomial you can use one of these patterns:

$(a + b)^2 = (a + b)(a + b) = a^2 + 2ba + b^2$

$(a - b)^2 = (a - b)(a - b) = a^2 - 2ba + b^2$

These products are called perfect square trinomials.

Another binomial product that follows a pattern has the form:

$(a + b)(a - b) = a^2 - b^2$

This product is called a difference of two squares.

If you forget the patterns, you can always use the FOIL method to figure them out:

F + O + I + L

$(a + b)(a + b)$
$= a \cdot a + b \cdot a + b \cdot a + b \cdot b$
$= a^2 + 2ba + b^2$

$(a - b)(a - b)$
$= a \cdot a - b \cdot a - b \cdot a + (-b) \cdot (-b)$
$= a^2 - 2ba + b^2$

$(a + b)(a - b)$
$= a \cdot a - b \cdot a + b \cdot a - b \cdot b$
$= a^2 - b^2$

To use a pattern to find the product of two binomials:

1. Determine which pattern to use.
2. Determine which values to substitute for *a* and *b* in the pattern.
3. Substitute the values into the pattern.
4. Simplify.

For example, to find $(3x^2 + 4)(3x^2 + 4)$:

1. Determine which pattern to use.
   - ✓ $(a + b)(a + b) = a^2 + 2ba + b^2$
   - ___ $(a - b)(a - b) = a^2 - 2ba + b^2$
   - ___ $(a + b)(a - b) = a^2 - b^2$
2. Determine the values to substitute for *a* and *b*. $a = 3x^2$, $b = 4$
3. Substitute $3x^2$ for *a* and 4 for *b*.
   $(a + b)^2 = a^2 + 2ba + b^2$
   $(3x^2 + 4)^2 = (3x^2)^2 + 2 \cdot 4 \cdot 3x^2 + 4^2$
4. Simplify. $= 9x^4 + 24x^2 + 16$

Sample Problems

Answers to Sample Problems

1. Use the FOIL method to find: $(x - 6y)(3x + 2y)$

- ☑ a. Multiply the First terms. $x \cdot 3x$
- ☐ b. Multiply the Outer terms. $x \cdot$ ____ — *b. 2y*
- ☐ c. Multiply the Inner terms. ____ $\cdot\ 3x$ — *c. –6y*
- ☐ d. Multiply the Last terms. ____ $\cdot$ ____ — *d. –6y, 2y*
- ☐ e. Add the terms, combining like terms. = ____________ — *e. $3x^2 - 16xy - 12y^2$ (in any order)*

2. Use a pattern to find: $(t - 7)^2$

- ☑ a. Determine which pattern to use.
  - ___ $(a + b)^2 = a^2 + 2ba + b^2$
  - ✓ $(a - b)^2 = a^2 - 2ba + b^2$
  - ___ $(a + b)(a - b) = a^2 - b^2$
- ☐ b. Determine the values to substitute for a and b. $a = t$, $b =$ ____ — *b. 7*
- ☐ c. Substitute these values. $(t - 7)^2 = t^2 - 2 \cdot$ ___ $\cdot\ t +$ ___ — *c. 7; 7^2 or 49*
- ☐ d. Simplify. = ____________ — *d. $t^2 - 14t + 49$ (in any order)*

3. Use a pattern to find: $(x + 5y)(x - 5y)$

- ☐ a. Determine which pattern to use. — *a. $(a + b)(a - b) = a^2 - b^2$*
  - ___ $(a + b)^2 = a^2 + 2ba + b^2$
  - ___ $(a - b)^2 = a^2 - 2ba + b^2$
  - ___ $(a + b)(a - b) = a^2 - b^2$
- ☑ b. Determine which values to substitute for a and b. $a = x$, $b = 5y$
- ☐ c. Substitute these values. $(x + 5y)(x - 5y) =$ ____________ — *c. $x^2 - (5y)^2$*
- ☐ d. Simplify. = ________ — *d. $x^2 - 25y^2$*

MULTIPYING AND DIVIDING

Summary

Multiplying Two Polynomials When Each Has More Than One Term

In general, to multiply a polynomial by a polynomial when each has more than one term:

$(a + b)(c + d + e)$

$= a(c + d + e) + b(c + d + e)$

$= ac + ad + ae + bc + bd + be$

You can multiply two polynomial each of which has more than one term.

To multiply a polynomial by a polynomial:

1. Distribute each term in the first polynomial to the second polynomial.
2. Distribute again to remove the parentheses.
3. Multiply each of the resulting terms.
4. Combine like terms.

For example, to find: $(x^2 + y)(3x^2 - 2y + xy)$

1. Distribute each term in the first polynomial to the second polynomial.
   $= x^2(3x^2 - 2y + xy) + y(3x^2 - 2y + xy)$
2. Distribute again. $= (x^2)(3x^2) - x^2(2y) + (x^2)(xy) + y(3x^2) - y(2y) + y(xy)$
3. Multiply. $= 3x^4 - 2x^2y + x^3y + 3x^2y - 2y^2 + xy^2$
   $= 3x^4 - 2x^2y + 3x^2y + x^3y - 2y^2 + xy^2$
4. Combine like terms. $= 3x^4 + x^2y + x^3y - 2y^2 + xy^2$

Dividing a Polynomial with More Than One Term by Another Polynomial with More Than One Term

To divide a polynomial (dividend) by another polynomial (divisor) where each has more than one term, use long division.

Before you can divide, both polynomials should be arranged so their terms are in descending order by degree. To arrange the terms of a polynomial in descending order:

1. Determine the degree of each term by looking at the exponent of the variable(s).
2. Arrange the terms so they are in descending order by degree.

For example, to rearrange the terms of $x^3 - x + x^4$ in descending order:

1. Determine the degree of each term.
   degree 3 → x^3, degree 1 → x^1, degree 4 → x^4: $x^3 - x^1 + x^4$
2. Arrange the terms in descending order by degree. $x^4 + x^3 - x^1$

Once the terms of the polynomials are correctly arranged, you are ready to divide.

To divide a polynomial by a polynomial where each has more than one term:

1. Arrange the terms of each polynomial in descending order. In the dividend, write missing terms as $0x^r$ where r is the exponent of the missing term.
2. Write the problem in long division form.
3. Divide the first term of the dividend by the divisor.
4. Multiply the divisor by the term you found in step (3).
5. Subtract the expression you found in step (4) from the dividend.
6. Continue dividing until the degree of the remainder is less than the degree of the divisor.
7. The answer is the expression that appears above the division sign plus the fraction $\frac{remainder}{divisor}$.
8. Check your division by multiplying the expression that appears above the division sign. Then add the remainder (not as a fraction).

For example, to find: $(x^2 + 3x^3 - 2) \div (x^2 + 2)$

1. Arrange the terms of the dividend in descending order. Include missing terms.

$$3x^3 + x^2 + 0x^1 - 2$$

2. Write the problem in long division form.

$$x^2 + 2 \overline{)3x^3 + x^2 + 0x^1 - 2}$$

3. Divide the first term of the dividend by the divisor.
4. Multiply the divisor by the term in (3).
5. Subtract the expression you found in step (4) from the dividend.

$$\begin{array}{r} 3x \qquad\qquad\qquad \\ x^2 + 2 \overline{)3x^3 + x^2 + 0x^1 - 2} \\ -\underline{(3x^3 \qquad + 6x)} \\ x^2 - 6x - 2 \end{array}$$

6. Continue dividing until the degree of the remainder is less than the degree of the divisor.

$$\begin{array}{r} 3x + 1 \qquad\qquad \\ x^2 + 2 \overline{)3x^3 + x^2 + 0x^1 - 2} \\ -\underline{(3x^3 \qquad + 6x)} \\ x^2 - 6x - 2 \\ -\underline{(x^2 \qquad + 2)} \\ -6x - 4 \end{array}$$

7. Write your answer.

$$3x + 1 + \frac{-6x - 4}{x^2 + 2}$$

8. Check your answer by multiplying.

$$\begin{aligned} &(3x + 1)(x^2 + 2) + (-6x - 4) \\ &= 3x^3 + 6x + x^2 + 2 - 6x - 4 \\ &= 3x^3 + x^2 - 2 \end{aligned}$$

Answers to Sample Problems

c. $10tu^2$, $-2t^2u$, $-8u^3$ (in any order)

d. $3t^2u - t^3 + 6tu^2 - 8u^3$ (in any order)

c. 7

5

$21x - 21$

16

d. $5x^2$, $5x$, 7

e. $5x$, 7, 16

Here's one way to do the check:

$(5x^2 + 5x + 7)(3x - 3) + 16$
$= 15x^3 - 15x^2 + 15x^2 - 15x + 21x - 21 + 16$
$= 15x^3 + 6x - 5$

Sample Problems

1. Find: $(t + 2u)(5tu - t^2 - 4u^2)$

☑ a. Distribute each term in the first polynomial to each term in the second polynomial. $= t(5tu - t^2 - 4u^2) + 2u(5tu - t^2 - 4u^2)$

☑ b. Distribute again to remove parentheses. $= t \cdot 5tu - t \cdot t^2 - t \cdot 4u^2 + 2u \cdot 5tu - 2u \cdot t^2 - 2u \cdot 4u^2$

☐ c. Multiply each of the resulting terms. $= 5t^2u - t^3 - 4tu^2 +$ ____ $+$ ____ $+$ ____

☐ d. Combine like terms. $=$ ________________

2. Find: $(6x + 15x^3 - 5) \div (3x - 3)$

☑ a. Arrange the terms of the dividend in descending order. Include "missing" terms. $15x^3 + 0x^2 + 6x - 5$

☑ b. Write the division in long division form.

$$\begin{array}{r} 5x^2 + 5x + \underline{\quad} \\ 3x - 3\,\overline{)\,15x^3 + 0x^2 + 6x - 5} \\ -\underline{(15x^3 - 15x^2)} \\ 15x^2 + 6x - 5 \\ -\underline{(15x^2 - 15x)} \\ 21x - \underline{\quad} \\ -(\underline{\qquad}) \\ \underline{\quad} \end{array}$$

☐ c. Divide.

☐ d. Write the quotient. ____ $+$ ____ $+$ ____ $+ \frac{16}{3x - 3}$

☐ e. Check your division by multiplying the quotient by the divisor. $(5x^2 +$ ____ $+$ ____ $)(3x - 3) +$ ____

EXPLORE

Sample Problems

Answers to Sample Problems

On the computer, you found the products of two binomial factors. In particular, you found patterns such as a perfect square trinomial and the difference of two squares to help you multiply binomials without using the FOIL method. Below are some additional problems using these patterns.

1. Find $(x + y)^2 + (x - y)^2$.

   - ☐ a. First find $(x + y)^2$. This follows the pattern $(a + b)^2 = a^2 + 2ab + b^2$. $(x + y)^2 =$ ____________

     a. $x^2 + 2xy + y^2$

   - ☐ b. Then find $(x - y)^2$. This follows the pattern $(a - b)^2 = a^2 - 2ab + b^2$. $(x - y)^2 =$ ____________

     b. $x^2 - 2xy + y^2$

   - ☐ c. Now combine terms.

     $(x + y)^2 + (x - y)^2 = (x^2 + 2xy + y^2) + (x^2 - 2xy + y^2)$

     $=$ ____________

     c. $2x^2 + 2y^2$ *or* $2(x^2 + y^2)$

2. Find $(x + y)^2 - (x - y)^2$ first by using the perfect square trinomial patterns and then by using the pattern of a difference of two squares.

   - ☐ a. First find $(x + y)^2$. This follows the pattern $(a + b)^2 = a^2 + 2ab + b^2$. $(x + y)^2 =$ ____________

     a. $x^2 + 2xy + y^2$

   - ☐ b. Then find $(x - y)^2$. This follows the pattern $(a - b)^2 = a^2 - 2ab + b^2$. $(x - y)^2 =$ ____________

     b. $x^2 - 2xy + y^2$

   - ☐ c. Now simplify.

     $(x + y)^2 - (x - y)^2 = (x^2 + 2xy + y^2) - (x^2 - 2xy + y^2)$

     $=$ ____________

     c. $4xy$

   - ☐ d. Now solve the same problem using the pattern of a difference of two squares.

     Hint: Use the pattern $a^2 - b^2 = (a + b)(a - b)$. For this example, $a = (x + y)$ and $b = (x - y)$.

     $(x + y)^2 - (x - y)^2 = [(x + y) + (x - y)][(x + y) - (x - y)]$

     $= \quad 2x \quad \cdot$ _____

     $=$ _______

     d. $2y$

     $4xy$

Answers to Sample Problems

a. $10a^3b^2, 10a^2b^3, 5a^1b^4$

c. $3x, 2y, 3x, 2y, 3x, 2y$

d. *216, 216, 96*

Refer to this diagram of Pascal's triangle as you complete the additional exploration problems below.

$(a+b)^0$	1
$(a+b)^1$	$1a + 1b$
$(a+b)^2$	$1a^2 + 2ab + 1b^2$
$(a+b)^3$	$1a^3 + 3a^2b + 3ab^2 + 1b^3$
$(a+b)^4$	$1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

3. Expand $(a+b)^5$.

☐ a. Find the exponents and coefficients for each term.

$1a^5 + 5a^4b^1 + \_\_\_a^{\_\_}b^{\_\_} + \_\_\_a^{\_\_}b^{\_\_} + \_\_\_a^{\_\_}b^{\_\_} + 1b^5$

Hint: Use the coefficients for the expansion of $(a+b)^4$ to help you find the coefficient for each term in $(a+b)^5$.

$1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

$1a^5 + 5a^4b + \_\_a^{-}b^{-} + \_\_a^{-}b^{-} + \_\_a^{-}b^{-} + 1b^5$

4. Use Pascal's triangle to find: $(3x+2y)^4$

☑ a. Determine which row of Pascal's triangle will help.

$(3x+2y)^4$

So use the row that expands $(a+b)^4$

☑ b. Write down the expansion for $(a+b)^4$.

$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

☐ c. Replace a with $3x$ and b with $2y$.

$1(3x)^4 + 4(\_\_\_)^3(\_\_\_) + 6(\_\_\_)^2(\_\_\_)^2 + 4(\_\_\_)(\_\_\_)^3 + 1(2y)^4$

☐ d. Simplify.

$= 81x^4 + \_\_\_x^3y + \_\_\_x^2y^2 + \_\_\_xy^3 + 16y^4$

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain
Multiplying Binomials

1. Given $(2p + 3)(p - p^2)$, find the:

   First terms: _____ and _____

   Outer terms: _____ and _____

   Inner terms: _____ and _____

   Last terms: _____ and _____

2. Which pattern could you use to find each of the products (a) - (f) below? Write the appropriate pattern number next to each polynomial.

   a. ___ $(2x + 5y)^2$
   b. ___ $(2x + 5y)(2x - 5y)$
   c. ___ $(3t - 2)^2$
   d. ___ $(3t + 2)^2$
   e. ___ $(3t - 2)(3t - 2)$
   f. ___ $(2x^2 - 5y^3)^2$

   I. $(a + b)^2 = a^2 + 2ba + b^2$
   II. $(a - b)^2 = a^2 - 2ba + b^2$
   III. $(a + b)(a - b) = a^2 - b^2$

3. Given $(2s^3 + 5)^2$ and the pattern $(a + b)^2 = a^2 + 2ba + b^2$:

   a. What would you replace a with in the pattern? ______

   b. What would you replace b with? ______

4. Use a pattern to find: $(3s + 5)^2$

5. Use the FOIL method to find: $(4x - 2y)(3x + 6)$

6. Use a pattern to find: $(3t + 4u)(3t - 4u)$

7. Use patterns to find these products:

   a. $(3x^2 - 2)(3x^2 + 2)$
   b. $(3x^2 - 2)(3x^2 - 2)$
   c. $(3x^2 + 2)(3x^2 + 2)$

8. Find: $(5x^3 + 3y^2)^2$

9. A fish tank broke at the pet store where Angelina works, and part of the store was flooded. Since Angelina lost her measuring tape, she used a stick and her handspan to figure out the approximate size of the flooded area. If s equals the length of the stick and h equals the width of her handspan, these are the measurements:

   length of flooded space = $13s + 2h$

   width of flooded space = $13s - 2h$

   area of flooded space = $(13s + 2h)(13s - 2h)$

   Simplify the equation for the area by multiplying the binomials.

10. The owner of the pet store where Angelina works wants to replace the tile covering the entire floor, not just the flooded area. If the length of the entire floor is $250s - 3h$ and the width is $98s + h$, what is the area of the floor in terms of s and h?
    Hint: area = length · width.

11. Find: $(13x^2y^2 - 10x^3)(7x^2y^2 - 6x^3)$

12. Find:

    a. $\left(\frac{1}{2}x^3 - \frac{2}{3}y^5\right)\left(\frac{1}{2}x^3 - \frac{2}{3}y^5\right)$

    b. $\left(\frac{1}{2}x^3 - \frac{2}{3}y^5\right)\left(\frac{1}{2}x^3 + \frac{2}{3}y^5\right)$

    c. $\left(\frac{1}{2}x^3 + \frac{2}{3}y^5\right)\left(\frac{1}{2}x^3 + \frac{2}{3}y^5\right)$

Multiplying and Dividing

13. Find: $(x + 2)(3x + 4xy + 1)$

14. Find: $(p^2 + 2r + 2)(3r^4 - 2p^4)$

15. Find: $(x + y + 1)(x - y)$

16. Find: $(2t + u)(t + 2u - 1)$

17. Angelina is cleaning the windows of the guinea pig case at the pet store where she works. The surface area of the outside of the windows can be described as follows:

surface area = $2(x + 3)(x - 2) + 2(x - 2)(x - 3)$

Simplify this equation by multiplying the polynomials.

18. The pet store where Angelina works sells an exercise arena for guinea pigs, consisting of two spheres connected by a tube. The volume of the exercise arena can be described by this equation:

volume = $4\pi r^3 + 3\pi(r^2 + 2r + 4)(r + 2) + \pi r(r - 5)(r - 5)$

Simplify the equation by multiplying the polynomials.

19. Find: $(12x^3 - 2x^2 - 7x)\left(4x^2 - \frac{10}{3}x - \frac{1}{3} + \frac{7}{12x^3 - 2x^2 - 7x}\right)$

20. Find: $\left(\frac{1}{3}t^2 + \frac{2}{3}v^3\right)\left(\frac{1}{3}t^2 + \frac{2}{3}v^3\right)\left(\frac{1}{3}t^2 + \frac{2}{3}v^3\right)$

21. Find: $(3x^2 + 2x - 1) \div (x + 3)$

22. Here is how Tony answered a question on his algebra test. $(12x^3 - 17x^2 + 3) \div (3x - 2) = 4x^2 - 3x - 2$ remainder -1.

Is his answer right or wrong? Why? Circle the most appropriate response.

His answer is right.

His answer is wrong. When doing the long division, he sometimes added negative terms rather than subtracting them. The right answer is $4x^2 + 2x - 1$.

His answer is wrong. He did not include missing terms in the quotient. The right answer is $0x^3 + 4x^2 - 3x - 2$ remainder -1.

His answer is wrong. He did not put the remainder over the dividend. The right answer is $4x^2 - 3x - 2 + \frac{-1}{3x - 2}$.

23. Find: $(15x^3 + x^2 + 5) \div (x + 3)$

24. Find: $(4y^3 + 5y + 3) \div (2y + 1)$

Explore

25. Find: $(3a - 1)(3a + 1)$

26. Use the table below to find a general form for multiplying two polynomials: $(ax^2 + bx + c)(dx - e)$

terms	dx	$-e$
ax^2		
bx		
c		

$(ax^2 + bx + c)(dx - e) =$

27. Use the table below to find the general form for a difference of two squares: $(a + b)(a - b)$. Then use this pattern to find $(2x + 3y)(2x - 3y)$.

terms	a	$-b$
a		
b		

$(a + b)(a - b) =$

$(2x + 3y)(2x - 3y) =$

28. Find: $(x^2 + 3y)^2$

29. Use the table below to find the general form for a perfect square trinomial: $(a - b)(a - b)$. Then use the pattern to find $(2t^3 - 4u^2)(2t^3 - 4u^2)$.

terms	a	$-b$
a		
$-b$		

$(a - b)(a - b) =$

$(2t^3 - 4u^2)(2t^3 - 4u^2) =$

30. Use the table below to find the general form for a perfect square trinomial: $(a + b)(a + b)$. Then use this general form to find $(x^2 + 3y)(x^2 + 3y)$.

terms	a	b
a		
b		

$(a + b)(a + b) =$

$(x^2 + 3y)(x^2 + 3y) =$

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Multiplying Binomials

1. Find: $(a + 2)(a + 5)$
2. Find: $(m - 3)(m - 7)$
3. Find: $(x - 4)(x - 11)$
4. Find: $(3b + 2)(b - 6)$
5. Find: $(5y - 8)(y + 3)$
6. Find: $(6t + 1)(t - 7)$
7. Find: $(4a + 3b)(2a + 5b)$
8. Find: $(3m - 4n)(7m + 2n)$
9. Find: $(6y + 5x)(3y - x)$
10. Find: $(p + 9)(p + 9)$
11. Find: $(x + 3)(x + 3)$
12. Find: $(3z + 2)(3z + 2)$
13. Find: $(5q + 3)(5q + 3)$
14. Find: $(4x + 1)(4x + 1)$
15. Find: $(z - 5)(z - 5)$
16. Find: $(m - 11)(m - 11)$
17. Find: $(t - 6)(t - 6)$
18. Find: $(3x - 2y)(3x - 2y)$
19. Find: $(4a - 7c)(4a - 7c)$
20. Find: $(5r - 8s)(5r - 8s)$
21. Find: $(5m + n)(5m - n)$
22. Find: $(a + 7b)(a - 7b)$
23. Find: $(2x + y)(2x - y)$
24. Find: $(3y + 8)(3y - 8)$
25. Find: $(5x + 3)(5x - 3)$
26. Find: $(m + 12n)(m - 12n)$
27. Find: $(2a + 7b)(2a - 7b)$
28. Find: $(x + 7y)(x - 7y)$

Multiplying and Dividing

29. Find: $(4a - 3b)(2a - 7b)$
30. Find: $(3x + 5)(y + 8)$
31. Find: $(6m - 5n)(3m + 4n)$
32. Find: $(8y + 3z)(2y - 9z)$
33. Find: $(7x - 4)(2y + 3)$
34. Find: $(a + 2b)(a^2 + 6a - 3b)$
35. Find: $(3mn - n)(m^2 - 3n + 4m)$
36. Find: $(2xy - y)(x^2 + 5y - 6x)$
37. Find: $(3ab + 4b)(7a^2 + 3b - 4a)$
38. Find: $(7uv - 3v)(2u^2 - 5v + 8u)$
39. Find: $(5xy + 2y)(2x^2 - 6y + 3x)$
40. Find: $(3a^2 - 4b^2)(2a^3 + 5a^2b - 11ab - b)$
41. Find: $(5m^2n + 3n)(4m^3 - 3m^2n + 8mn^2 - 3n^2)$
42. Find: $(7x^2y + 2y)(3x^3 - 6x^2y + 8xy + y)$
43. Find: $(x^3 + x^2 - 13x + 14) \div (x - 2)$
44. Find: $(x^3 + 11x^2 + 22x - 24) \div (x + 4)$
45. Find: $(x^3 + 10x^2 + 23x + 6) \div (x + 3)$
46. Find: $(x^3 + 7x^2 - 36) \div (x + 6)$
47. Find: $(x^3 - 26x + 5) \div (x - 5)$
48. Find: $(3x^3 + 17x^2 - 58x + 40) \div (3x - 4)$
49. Find: $(4x^3 + 4x^2 - 13x + 5) \div (2x + 5)$
50. Find: $(2x^3 + 7x^2 - x - 2) \div (2x + 1)$
51. Find: $(4x^3 + 7x^2 - 14x + 6) \div (4x - 1)$
52. Find: $(2x^3 - 9x^2 + 12x - 8) \div (2x + 3)$
53. Find: $(3x^3 + 14x^2 + 11x - 8) \div (3x + 2)$
54. Find: $(6x^3 - 7x^2 - 34x + 35) \div (2x - 5)$
55. Find: $(10x^3 - 26x^2 - 7x + 2) \div (5x + 2)$
56. Find: $(8x^3 - 18x^2 + 25x - 12) \div (4x - 3)$

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. Use the FOIL method to find: $(2x^2 + 3xy)(3x^3y - 2)$

2. Use a pattern to find: $(2x - 3y)^2$

3. Find: $(2x + 3y)^2$

4. Use a pattern to find: $(2x - 3y)(2x + 3y)$

5. Find: $(3x - 2)(5x^2 + 8x - 2)$

6. Find: $(3p^2 + 4r^4 - 5)(3r^4 - 6p^2 + 2)$

7. Find: $(6t^2 + 5t + 1) \div (2t + 1)$

8. Find: $(8x^3 + 6x - 2) \div (4x + 2)$

9a. Find: $(a^3 - a^5)(a + a^2)$

b. What is the degree of the resulting polynomial?

10. Find: $(5y^4 - 2y^2 + y)(3y^2 - y + 2)$

11. Use the table in Figure 6.3.1 to find:
$(2x^3 - 3x + 7)(5x^4 + 8)$

	$2x^3$	$-3x$	7
$5x^4$			
8			

Figure 6.3.1

12. Use the table in Figure 6.3.2 to find:
$(5x^4 - 7x^3 + 7x^2 - 8x)(x^2 + 1)$

x^2				$-8x^3$
1	$5x^4$		$7x^2$	

Figure 6.3.2

TOPIC 6 CUMULATIVE ACTIVITIES

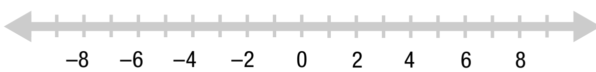
CUMULATIVE REVIEW PROBLEMS

These problems combine all of the material you have covered so far in this course. You may want to test your understanding of this material before you move on to the next topic. Or you may wish to do these problems to review for a test.

1. Find:

   a. $2^7 \cdot 2^9$ b. $\frac{x^{12}}{x^5}$ c. $(a^5b^2)^4$

2. Solve $-3 \leq 6 + 2y < 4$ for y, then graph its solution on the number line below.



3. Find the equation of the line through the point (3, 7) with slope $-\frac{2}{7}$:

   a. in point-slope form.   c. in standard form.

   b. in slope-intercept form.

4. Solve this system:

$$\begin{aligned} x + 2y &= 5 \\ x - 2y &= -13 \end{aligned}$$

5. The difference of two numbers is –32. The sum of three times the smaller number and twice the larger number is 134. What are the two numbers?

6. Circle the true statements.

   The GCF of two numbers that have no factors in common is 1.

   $\frac{2}{9} - \frac{1}{5} = \frac{1}{4}$

   The LCM of 4 and 8 is 4.

   $3^2(4 + 2) = 9(4 + 2)$

   $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

7. Write the equation of the line through the point (20, –9) with slope $-\frac{8}{5}$:

   a. in point-slope form.   c. in standard form.

   b. in slope-intercept form.

8. Graph the system of inequalities below to find its solution.

$$\begin{aligned} 2x + y &\geq 3 \\ x - y &< 4 \end{aligned}$$

9. Find:

   a. 3^0 b. -3^0 c. $(-3)^0$

10. Graph the inequality $4x + y \leq 6$.

11. Find: $15x^3y^8z^5 \div 10xy^4z^{11}$

12. Lisa emptied a vending machine and got a total of 279 quarters and dimes worth $57.30. How many quarters did she get?

13. Find the slope and y-intercept of the line $4x - y = 7$.

14. Evaluate the expression $3x^2 - 4xy + 2y$ when $x = 3$ and $y = -5$.

15. Solve $-6 < 4 + 2x < -2$ for x.

16. Find:

    a. $-5x^0 + y^2$ c. $b^4 \cdot b^2 \cdot b \cdot b^6$

    b. $\left(\frac{a^3 \cdot b^7 \cdot c}{b^4 \cdot c^2}\right)^3$

17. Circle the true statements.

    $4(3 - 5) = 4 \cdot 3 - 5$

    $\frac{26}{117} = \frac{2}{9}$

    The LCM of 72 and 108 is 36.

    The GCF of 72 and 108 is 36.

    $\frac{4}{7} - \frac{2}{3} = -\frac{2}{4}$

18. Write the equation of the line through the point (5, 2) with slope $-\frac{7}{3}$:

    a. in point-slope form.

    b. in slope-intercept form.

    c. in standard form.

19. Graph the line $y = 6$.

20. Find: $(11p^2 - 3pr - 6r)(3p - 9r)$

21. Solve this system:

$$4x - y = 9$$
$$6x + 5y = -6$$

22. Find the equation of the line that is **parallel** to the line $x + 3y = 4$ and passes through the point (2, 2).

23. Find the equation of the line that is **perpendicular** to the line $x + 3y = 4$ and passes through the point (2, 2).

24. Graph the system of inequalities below to find its solution.

$$y < 2x + 3$$
$$4x - y \geq 1$$

25. Find the slope of the line perpendicular to the line through the points (8, 9) and (6, –4).

26. Find: $(x^3 + 5x^2 + x - 10) \div (x + 2)$

27. Solve $2y + 5 = 4(\frac{1}{2}y + 2)$ for y.

28. Find: $3xy(x^2y - 4)$

Use Figure 6.1 to answer questions 29, 30, and 31.

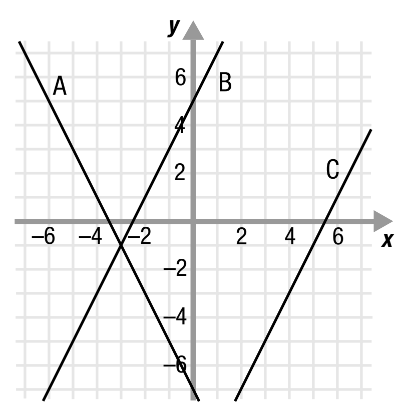


Figure 6.1

29. Which two lines form a system that has a solution of (–3, –1)?

30. Which two lines form a system that has no solution?

31. Which two lines form a system that has a solution that is not shown on the grid?

32. Find:

    a. $(x^2yz^3)^4$

    b. $\frac{x^5y^3}{xy^6}$

    c. $(x^5)^9$

33. Evaluate the expression $-7a^4 + 3ab^2 + b - 4$ when $a = -2$ and $b = 5$.

34. Find the slope of the line through the points (9, –4) and (2, 7).

35. Graph the system of inequalities below to find its solution.

$$9x - 4y < 20$$
$$9x - 4y \leq 8$$

36. Graph the inequality $\frac{5}{2}x - y \geq 2$.

37. Last year Manuel split \$2565 between his savings account, which paid 5% in interest, and his checking account, which paid 3.5% in interest. If he earned a total of \$113.49 in interest, how did he split his money between the two accounts?

38. Graph the inequality $\frac{2}{3}x - \frac{1}{3}y \geq 2$.

39. Next to each polynomial below, write whether it is a monomial, a binomial, or a trinomial.

    a. $2 + x$

    b. $8ab^3 - 9abc + 1$

    c. $3x^7yz^5$

    d. $a^5b^2c^3d - 103a^7cd^4$

    e. 10

    f. $a + 7b - 4c$

40. Solve $-8(1 - \frac{1}{2}x) = 4(x - 2)$ for x.

41. Find: $(4a^2b + 3a - 9b) + (7a + 2b - 8a^2b)$

42. The perimeter of a square is the same as the perimeter of a regular hexagon. If each side of the square is 7 feet longer than each side of the hexagon, what is the perimeter of each figure?