# $LESSON \ 2.2 - SOLVING \ LINEAR \ EQUATIONS$





## Here's what you'll learn in this lesson:

#### Solving Equations I

- a. Recognizing a linear equation
- b. The addition and subtraction principles for solving a linear equation
- c. The multiplication and division principles for solving a linear equation
- d. Combining the principles

#### Solving Equations II

- a. Equations with fractions as coefficients
- b. Equations with no solutions or infinitely many solutions
- c. Formulas: Solving for a particular unknown

Suppose a friend hands you 25 olives and a container which has an unknown number of olives in it. She tells you she has just handed you a total of 67 olives.

Using this information you can figure out the number of olives in the container by setting up and solving an equation.

Solving equations for an unknown, like the number of olives in the container, is one of the most important ideas in algebra. In this lesson you will study a particular type of equation, the linear equation.



## SOLVING EQUATIONS I

## Summary

#### Definitions

An equation consists of two expressions separated by an equals sign. One kind of equation is a linear equation in one variable. A linear equation in *x* can be written in the form ax + b = c, where *x* is a variable and *a*, *b*, and *c* are any real numbers,  $a \neq 0$ . A number which is substituted for the variable in a linear equation and makes that equation true is called a solution of the equation.

#### **Checking Solutions**

To check if a number is a solution of an equation, substitute the number for the variable in the equation. If the two sides are equal then the number is a solution of the equation.

For example, to see if y = 6 is a solution of y - 4 = 1,

Substitute 6 for y and ask:	ls 6 -	- 4 = 1?
Now ask:	ls	2 = 1? No, so $y = 6$ is not a solution of $y - 4 = 1$ .

Now see if y = 5 a solution of y - 4 = 1.

Substitute 5 for y and ask:	ls 5 –	4 = 1?
Now ask:	ls	1 = 1? Yes, so $y = 5$ is a solution
		of $y - 4 = 1$ .

### Solving Equations

Some equations are simple enough that you can find the solution just by looking at the equation. Other equations are more complicated and you need a systematic approach to find the solution. When this is the case you find the solution by isolating the variable on one side of the equation. You can isolate the variable by using the principles listed on the following page.

The variable in a linear equation can be any letter, not just x. For example, 3w + 5 = 2, -9y + 4 = 7, and 6z - 1 = 8 are all linear equations.

Principle	Rule	Example
Addition Principle	You can add the same number to both sides of an equation without changing the solution.	Solve: $x - 2 = 5$ Add 2: $x - 2 + 2 = 5 + 2$ Simplify: $x = 7$
Subtraction Principle	You can subtract the same number from both sides of an equation without changing the solution.	Solve: $x + 5 = 8$ Subtract 5: $x + 5 - 5 = 8 - 5$ Simplify: $x = 3$
Multiplication Principle	You can multiply both sides of an equation by the same nonzero number without changing the solution.	Solve: Multiply by 4: Simplify: $\frac{1}{4}x = 2$ $4 \cdot \frac{1}{4}x = 4 \cdot 2$ x = 8
Division Principle	You can divide both sides of an equation by the same nonzero number without changing the solution.	Solve: $3x = 12$ Divide by 3: $\frac{3x}{3} = \frac{12}{3}$ Simplify: $x = 4$

Sometimes you need to use more than one principle to solve an equation.

For example, to solve the equation 2x - 7 = 5, you need to apply both the Addition Principle and the Division Principle:

	2x - 7 = 5
1. Add 7.	2x - 7 + 7 = 5 + 7
	2 <i>x</i> = 12
2. Divide by 2.	$\frac{2x}{2} = \frac{12}{2}$
	<i>x</i> = 6

## Sample Problems

Answers to Sample Problems

#### 1. Is x = -7 a solution of the equation 3x + 25 = 4?

		3x + 25 = 4	
□ a.	Replace $x$ with $-7$ .	ls 3() + 25 = 4?	a. –7
□ b.	Simplify.		b. —21 4
□ C.	Is $x = -7$ a solution of the equation? (Yes or No)		c. Yes

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#### 2. Solve for x: 5x - 14 = 21.

#### 5x - 14 = 21

□ a.	Isolate the x-term on	5 <i>x</i> -14 + = 21 +	а.	14, 14
	one side.	5 <i>x</i> =		35
□ b.	Isolate <i>x</i> .	$\frac{5x}{5} = $	b.	$\frac{35}{5}$ or 7
□ C.	Simplify.	X =	C.	7

## SOLVING EQUATIONS II

### Summary

#### **Equations with Fractions**

An equation can have fractional coefficients. To make it easier to solve such an equation you can eliminate the fractions by multiplying both sides by the least common denominator (LCD) of the fractions. Then you can solve the equation as you would any equation with integer coefficients.

For example, here is one way to solve  $\frac{3}{4}x = 9$  for *x*:

	$\frac{3}{4}X = 9$
1. Multiply both sides by 4 to clear the fraction.	$4 \cdot \frac{3}{4}x = 4 \cdot 9$
2. Cancel and multiply.	$\overset{1}{\mathscr{A}} \cdot \frac{3}{\mathscr{A}_1} x = 4 \cdot 9$
	3x = 36
3. Divide both sides by 3.	$\frac{3x}{3} = \frac{36}{3}$
	<i>x</i> = 12

An equation can contain fractions and parentheses.

Here is one way to solve the equation $\frac{1}{7}(x-3) = 3$ for <i>x</i> :		
	$\frac{1}{7}(x-3) = 3$	
1. Multiply both sides by 7 to clear the fraction.	$7 \cdot \frac{1}{7}(x-3) = 7 \cdot 3$	
2. Cancel and multiply.	$\frac{1}{7} \cdot \frac{1}{7} (x-3) = 7 \cdot 3$	
	x - 3 = 21	
3. Add 3 to both sides.	x - 3 + 3 = 21 + 3	
	<i>x</i> = 24	
There are often several ways to solve an equation	l.	
Here is another way to solve $\frac{1}{7}(x-3) = 3$ for x:		
	$\frac{1}{7}(x-3) = 3$	
1. Distribute the fraction.	$\frac{1}{7}X - \frac{3}{7} = 3$	
2. Multiply both sides by 7 to clear $(1, 2)$		
$7\left(\frac{1}{7}X - \frac{3}{7}\right) = 7(3)$	1	
	$(1^1) - (3^1) -$	
3. Distribute. 7	$7\left(\frac{1}{7}X\right) - 7\left(\frac{3}{7}\right) = 7(3)$	

To clear fractions, multiply both sides by the LCD of the fractions. This will get rid of the fractions without making the coefficients bigger than necessary. You can also clear fractions by multiplying both sides by any multiple of the LCD, but then you will get bigger coefficients.

- 4. Cancel and multiply. x 3 = 21
- 5. Add 3 to both sides. x 3 + 3 = 21 + 3

*x* = 24

To solve some equations, you need to distribute a negative number.

Here is one way to solve the equation -5(x + 2) = 35 for *x*:

-5(x + 2) = 351. On the left side, distribute the -5. 2. Add 10 to both sides. 3. Divide both sides by -5. -5x - 10 = 35 -5x - 10 + 10 = 35 + 10 -5x = 45  $\frac{-5x}{-5} = \frac{45}{-5}$ x = -9

Some equations have parentheses on both sides.

Here is one way to solve 2(x-2) = -6(x-10) for *x*:

2(x-2) = -6(x-10)1. On the left side, distribute the 2. 2x - 2(2) = -6(x - 10)2x - 4 = -6(x - 10)2x - 4 = -6x - (-6)(10)2. On the right side, distribute the -6. 2x - 4 = -6x + 602x - 4 + 6x = -6x + 60 + 6x3. Add 6x to both sides. 8x - 4 = 608x - 4 + 4 = 60 + 44. Add 4 to both sides. 8x = 64 $\frac{8x}{8} = \frac{64}{8}$ 5. Divide both sides by 8. X = 8

As an example with fractional coefficients, here is one way to solve  $\frac{1}{6}(2x + 4) = \frac{2}{9}(x + 5)$  for *x*:

$$\frac{1}{6}(2x+4) = \frac{2}{9}(x+5)$$

1. Multiply by 18 to clear the fractions.  $18 \cdot \frac{1}{6}(2x+4) = 18 \cdot \frac{2}{9}(x+5)$ 

 $3(2x + 4) = 2 \cdot 2(x + 5)$ 

Why do you multiply by 18? 18 is the LCD of  $\frac{1}{6}$  and  $\frac{2}{9}$ .

2. Distribute.	3(2x + 4) = 4(x + 5)
3. Subtract 12.	6x + 12 = 4x + 20 $6x + 12 - 12 = 4x + 20 - 12$
4. Subtract 4x.	6x = 4x + 8 $6x - 4x = 4x + 8 - 4x$
E Divida by 2	2x = 8 $2x = 8$
5. Divide by 2.	$\frac{-2}{2} = \frac{-2}{2}$ $x = 4$

#### **Equations with No Solutions**

Until now, all of the equations you have worked with have had a single solution. Once you found the solution you could substitute it for the variable in the equation to get a true statement. However, some equations do not have a solution. This means that there is no number that you can substitute for the variable in the equation to get a true statement.

For example, to solve 2(1 + x) = 2x - 3 for x:

	2(1 + x) = 2x - 3
1. Distribute:	2 + 2x = 2x - 3
2. Subtract 2 <i>x</i> :	2 + 2x - 2x = 2x - 3 - 2x
3. Simplify:	2 = -3 No!

Since 2 = -3 is never true, this equation has no solution.

#### Identities

Just as there are equations with no solutions, there are also equations for which every value of x is a solution. These equations are called identities. Any number you substitute for the variable in an identity will give you a true statement.

For example, to solve 3(x-4) + x = 4x - 12 for x:

	3(x-4) + x = 4x - 12
1. Distribute.	3x - 12 + x = 4x - 12
2. Combine like terms.	4x - 12 = 4x - 12

Because the same expression is on both sides of the equals sign, the equation is an identity so every value of x is a solution. Try it: pick any number for x and substitute it into the original equation. You will always get a true statement.

When you try to solve an equation that has no solution, you will get a nonsense statement such as 2 = -9 or  $\frac{3}{5} = 7$ .

You can recognize identities because when simplified, one side is the same as the other. Formulas

A formula is an equation which relates two or more variables.

Some examples of formulas are:

$$F = m \cdot a$$
$$d = r \cdot t$$
$$C = 2\pi r$$

Sometimes you need to solve a formula for one of the variables in terms of the others.

For example, to solve  $A = \frac{1}{2}b \cdot h$  for *h*:

- 1. Multiply by 2 to clear the fraction.
- 2. Divide by b.

As another example, here is one way to solve the equation  $6x - \frac{2}{5}y = 2$  for y:

$$6x - \frac{2}{5}y = 2$$

- 1. Subtract 6*x* from both sides to get *y* by itself on one side of the equation.
- $6x \frac{2}{5}y 6x = 2 6x$  $-\frac{2}{5}y = 2 6x$

5 ·

-2y = 10 - 30x

 $A = \frac{1}{2}b \cdot h$ 

 $2 \cdot A = 2 \cdot \frac{1}{2}b \cdot h$ 

 $2A = b \cdot h$ 

 $\frac{2A}{b} = \frac{b \cdot h}{b}$ 

 $\frac{2A}{h} = h$ 

- 2. Multiply both sides by 5 to clear the fraction.
- $\left(-\frac{2}{5}y\right) = 5 \cdot (2-6x)$
- 3. Distribute.
- 4. Divide both sides by -2.

Force = mass  $\cdot$  acceleration distance = rate  $\cdot$  time Circumference =  $2\pi \cdot$  radius

Don't be put off by all the letters! Just follow the process you've learned.



Answers to Sample Problems	Sample Problems	
	1. Solve for $x: \frac{2}{2}x - 1 = 9 - x$	
	5	$\frac{2}{3}x - 1 = 9 - x$
a. 3, 3	<ul> <li>a. Clear the fraction by multiplying by the LCD.</li> </ul>	$\underline{\qquad}\cdot\left(\frac{2}{3}x-1\right) = \underline{\qquad}\cdot(9-x)$ $2x-3 = 27-3x$
	□ b. Add 3.	2x - 3 + 3 = 27 - 3x + 3
b. 2x, 30		$_{} = _{} - 3x$
c. 3x; 30, 3x (in either order) 30	$\Box$ c. Add 3 <i>x</i> .	$2x + \_\_ = \ 3x + \_\_$ $5x = \_\_$
d. $\frac{30}{5}$ or 6	$\Box$ d. Divide by 5.	$\frac{5x}{5} = $
6		X =
	2. Solve for $y: \frac{1}{2}(5y-2) = \frac{3}{4}(y-6)$	
		$\frac{1}{2}(5y-2) = \frac{3}{4}(y-6)$
	<ul> <li>a. Clear the fractions by multiplying by the LCD.</li> </ul>	$4 \cdot \frac{1}{2}(5y-2) = 4 \cdot \frac{3}{4}(y-6)$
a. 2,3		(5y - 2) = (y - 6)
	🗹 b. Distribute.	10y - 4 = 3y - 18
c. 14	$\Box$ c. Add 4.	10y - 4 + 4 = 3y - 18 + 4 $10y = 3y - \$
d. 3y; 14, 3y (in either order) —14	$\Box$ d. Subtract 3 <i>y</i> .	$10y - \_ = 3y - \ \_$
e. $-\frac{14}{7}$ or $-2$	$\Box$ e. Divide by 7.	$\frac{7y}{7} = $
-2		<i>y</i> =
	3. Solve the formula $r \cdot t = d$ for $t$ .	
		$r \cdot t = d$
a. $\frac{d}{r}$	$\Box$ a. Divide both sides by <i>r</i> .	$\frac{r \cdot t}{r} = $
$\frac{d}{r}$		t =



## Sample Problems

On the computer you used the Solver to analyze and solve linear equations. Below are some additional exploration problems.

- 1. Apply the distributive property to remove the parentheses on both sides of the equation -3(2x-5) = 2(7-x), then solve for *x*.
  - -3(2x-5) = 2(7-x) a. Distribute. -6x + 15 = 14 - 2x b. Subtract 15.  $-6x + 15 - \underline{\qquad} = 14 - 2x - \underline{\qquad} \\ -6x = \underline{\qquad} -6x = \underline{\qquad} -6x = \underline{\qquad} +6x = \underline{\qquad} +7x =$ 
    - $\Box$  c. Add 2x.
        $-6x + \_ = \_ -\_ + \_$  c. 2x; -1, 2x or -2x, 1; 2x -1 

        $\Box$  d. Divide by -4.
        $-4x = \_$  d. -1 

        $x = \_$   $x = \_$   $\frac{1}{2}$

2. Find the least common multiple of the denominators of the fractions in the equation  $\frac{4}{9}x = \frac{7}{15}(x + 1)$ , then use it to solve the equation.

$$\frac{4}{9}x = \frac{7}{15}(x+1)$$

---  $\cdot \frac{4}{9}x = -- \cdot \frac{7}{15}(x+1)$ 

20x = 21(x + 1)

X = \_\_\_\_\_

 a. Multiply by the LCM of the denominators.

 $\Box$  b. Distribute.  $20x = \_\_+\_$ 

 $\Box$  c. Subtract 21x.
  $20x - \_ = \_ + \_ - \_$ 
 $= \_$   $= \_$ 
 $\Box$  d. Divide by -1.

b. 21x, 21 (in either order)

c. 21x; 21x, 21 (in either order); 21x -x, 21

d. 
$$\frac{21}{-1}$$
 or -21

a. 45, 45

b. 15, 15

-1, 2x or -2x, 1

Answers to Sample Problems



## **Homework Problems**

Circle the homework problems assigned to you by the computer, then complete them below.

# Explain Solving Equations I

- 1. Solve for x: x + 15 = 37
- 2. Is y = 77 a solution of the equation y 23 = 54?
- 3. Solve for t: 9t = 108
- 4. Solve for w: -7 = w + 29
- 5. Solve for  $v: \frac{1}{3}v = 2$
- 6. Solve for x: 2x + 3 = 17
- 7. Solve for  $y: -1 = \frac{1}{4}y + 2$
- 8. Is s = 4 a solution of the equation 5s 4 = 11?
- 9. Francisco bought eight bottles of juice for \$12.00. How much did a single bottle of juice cost?
- 10. Vanessa took the \$50 she got for birthday money and went to buy fish. If she got six angel fish and had \$14 left over, how much did one angel fish cost?
- 11. Solve for z: 4z + 13 = 1
- 12. Solve for  $x: -3 = \frac{1}{7}x 6$

#### **Solving Equations II**

- 13. Solve for *y*:  $\frac{2}{3}y = 2$
- 14. Solve for *x*:  $\frac{1}{3}(x+8) = 7$

- 15. Solve for *x*: x + 1 = x 3
- 16. Solve for  $x: \frac{2}{5}(x-3) = \frac{3}{5}x$
- 17. Solve for  $z: -\frac{2}{3}(2z+3) = \frac{1}{2}(1-z)$
- 18. Solve for *w*: 4(w + 1) 3w = w + 4
- 19. The formula to find the circumference of a circle is  $C = 2\pi r$ , where *C* is the circumference of a circle and *r* is the radius. Solve the formula  $C = 2\pi r$  for *r*.
- 20. Solve for *y*:  $\frac{1}{2}y + 2 = \frac{1}{6}(3y 9)$
- 21. Solve for x: -3(2x + 1) = 7(2 x)
- 22. The math score on a college entrance exam can be written as S = 200 + 20R - 5W, where *S* is the score, *R* is the number of right answers, and *W* is the number of wrong answers. Dana's score on the test was 525 and he answered 19 questions correctly. How many questions did he answer incorrectly?
- 23. Solve for *z*:  $\frac{1}{3}(4z-3) = 4x-5$
- 24. A formula which relates the measure of the interior angles of a regular polygon to the number of sides of the polygon is 360 + an = 180n, where *n* is the number of sides and *a* is the measure of the interior angle. Solve this equation for *a*.



- 25. Apply the distributive property to remove the parentheses on both sides of the equation 9(x + 5) = 6(2x + 7), then solve for *x*.
- 26. Solve for *x*:  $\frac{3x}{7} + 2 = 8$
- 27. Find the least common multiple of the denominators of the fractions in the equation  $\frac{5}{6}y = \frac{3}{14}(4y + 3)$ , then use it to solve the equation.
- 28. Apply the distributive property to remove the parentheses on both sides of the equation -2(5 3x) = 4(x 7), then solve for *x*.

29. Solve for 
$$z: -7 = \frac{2}{3}z - 5$$

30. Find the least common multiple of the denominators of the fractions in the equation  $\frac{5}{12}(7 + x) = \frac{7}{18}(x + 8)$ , then use it to solve the equation.



## **Practice Problems**

Here are some additional practice problems for you to try.

#### Solving Equations I

- 1. Is x = 3 a solution of x 7 = 4?
- 2. Is y = -5 a solution of y + 3 = -2?
- 3. Solve for a: a + 5 = 23
- 4. Solve for x: x + 6 = 19
- 5. Solve for b: b 10 = 14
- 6. Solve for m: m 9 = 24
- 7. Solve for z: z 7 = 12
- 8. Solve for x: 15 x = 8
- 9. Solve for x: 24 x = 16
- 10. Solve for t: 21 t = 11
- 11. Solve for r: 3r + 2 = 17
- 12. Solve for s: 7s + 12 = 26
- 13. Solve for a: 5a + 3 = 23
- 14. Solve for m: 5m 9 = 41
- 15. Solve for p: 6p 11 = 13
- 16. Solve for k: 8k 5 = 19
- 17. Solve for b: 4b 5 = -21
- 18. Solve for b: 9b + 3 = -42
- 19. Solve for n: 3n 12 = -33
- 20. Solve for h: 12 + 5h = -38
- 21. Solve for q: 14 + 7q = -42
- 22. Solve for *v*: 16 + 4v = -20
- 23. Solve for c: 22 4c = 42

- 24. Solve for d: 56 5d = 31
- 25. Solve for x: 16 3x = 22
- 26. Solve for k: -10 6k = 26
- 27. Solve for f: -25 9f = 11

#### Solving Equations II

28. Solve for y: -7 - 3y = 829. Solve for h: 10h - 9 = 6h + 330. Solve for y: 12y - 13 = 7y + 1231. Solve for t: 3(t-6) = -8(1-t)32. Solve for u: -6(2u - 3) = 5(u - 10)33. Solve for c: -7(2c + 5) = 3(c - 6)34. Solve for x: 4(x + 3) = -5(3x - 10)35. Solve for  $p: \frac{1}{4}(p-5) = 3$ 36. Solve for  $r: \frac{1}{8}(r+3) = 6$ 37. Solve for  $y: -\frac{2}{3}(4 - y) = 6$ 38. Solve for *z*:  $\frac{3}{4}(z+3) = 9$ 39. Solve for  $c: \frac{1}{2}(c+8) = \frac{1}{4}c$ 40. Solve for  $b: -\frac{1}{3}(4-b) = \frac{1}{7}b$ 41. Solve for  $a: \frac{1}{5}a + 8 = -\frac{3}{5}(a - 15)$ 42. Solve for  $m: 12 - \frac{3}{10}m = \frac{7}{10}(m + 20)$ 43. Solve for  $n: \frac{1}{8}n + 6 = -\frac{5}{8}(n - 16)$ 44. Solve for  $b: -\frac{1}{3}(15 - 6b) = 2b - 5$ 45. Solve for  $r: 5r + 2 = \frac{1}{7}(35r + 14)$ 46. Solve for  $p: \frac{1}{2}(6p + 12) = 3p + 6$ 

47. Solve for  $t: -8\left(\frac{1}{4}t - 4\right) = 12 - 2t$ 48. Solve for  $y: 3(5 + \frac{1}{6}y) = 8 + \frac{1}{2}y$ 49. Solve for  $x: 6(3 + \frac{1}{2}x) = 3x + 7$ 50. Solve for  $d: \frac{4}{3}d + 16 = \frac{4}{3}(d + 12)$ 51. Solve for  $z: \frac{5}{4}z - 10 = -\frac{5}{4}(8 - z)$ 52. Solve for  $w: \frac{3}{2}w + 12 = \frac{3}{2}(w + 8)$ 53. Solve for z: 4z - 3y = 854. Solve for c: 5b - 2c = 1055. Solve for  $x: 3y - \frac{1}{3}x = 4$ 56. Solve for  $t: \frac{1}{2}t + 3v = 5$ 

- 57. The formula for the area of a triangle is  $A = \frac{1}{2} \cdot b \cdot h$ , where *A* is the area of the triangle, *b* is the length of its base, and *h* is its height. Solve this formula for *b*.
- 58. The formula for the area of a trapezoid is  $A = \frac{1}{2}h(a + b)$ , where *A* is the area of the trapezoid, *a* and *b* are the lengths of its two bases, and *h* is its height. Solve this formula for *a*.
- 59. The formula for the volume of a pyramid with a rectangular base is  $V = \frac{1}{3}$  *lwh*, where *V* is the volume of the pyramid, *I* is the length of its base, *w* is the width of its base and *h* is the height of the pyramid. Solve this formula for *w*.
- 60. The formula for the volume of a cylinder is  $V = \pi r^2 h$ , where *V* is the volume, *r* is the radius of the base, and *h* is the height of the cylinder. Solve this formula for *h*.



## **Practice Test**

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

- 1. Solve for x: x + 16 = 5
- 2. To isolate *z* in the equation  $-\frac{1}{2}z = 6$ , by what number do you multiply both sides of the equation?
- 3. Solve for y: -2y = 18
- 4. Solve for x: 3x 4 = 11
- 5. Solve for x: 3(2x + 4) = 2(3x + 6)
- 6. Solve for y: 2(y-10) = 10 + 2y
- 7. To solve the equation 8x 2 = 6 2x, you might begin by adding 2x to both sides of the equation. What would be the resulting equation?

- 8. Solve for *z*:  $\frac{1}{4}(z+3) = 1$
- 9. What is the resulting equation when you use the distributive property to remove parentheses from the equation 5(3x-2) = 2(x+3)?
- 10. Solve for  $x: -\frac{2}{3}(1-4x) = \frac{2}{9}(5x+4)$
- 11. Solve for y: 8x y = 5
- 12. Solve for x: 8x y = 5