LESSON 2.1 – ALGEBRAIC EXPRESSIONS

OVERVIEW

Here's what you'll learn in this lesson:

Simplifying Expressions

a. Constants and variables

b. Terms and coefficients

c. Combining like or similar terms

d. Parentheses

e. Evaluating expressions

f. Formulas: Substitution

If you wanted to know how many olives were in a container, you might begin by letting the unknown number of olives be denoted by a letter, like x. The letter x is called a variable, since you may decide to vary the number of olives in the box.

The study of algebra is concerned with variables. In this lesson you will learn about variables, how to use them in mathematical expressions, and how to simplify and evaluate these expressions.

EXPLAIN

SIMPLIFYING EXPRESSIONS

Summary

Definitions

An algebraic expression is a combination of numbers, letters, parentheses, brackets, and other grouping symbols such as +, –, ·, and ÷. The different elements of an algebraic expression are given special names to make it easier to refer to each part.

Look at the algebraic expression: $3x^4 - 8 - 7xy^2 + 2y$

Terms are the individual quantities: $\mathbf{3x^4 - 8 - 7xy^2 + 2y}$

Variables are the letters which stand for numbers: $3\mathbf{x}^4 - 8 - 7xy^2 + 2\mathbf{y}$

Coefficients are the numeric part of the terms: $\mathbf{3}x^4 \mathbf{- 8 - 7}xy^2 + \mathbf{2}y$

Constants are the terms without variables: $3x^4 \mathbf{- 8} - 7xy^2 + 2y$

Negative signs are included when writing terms, coefficients, and constants. In the expression $x^2 - 7$, the constant is –7, not 7.

Expressions enclosed in parentheses are considered a single term. The expression (y – 3) + (x + 1) has two terms: (y – 3) and (x + 1).

Simplifying Expressions

Simplifying an expression often makes it easier to work with.

To simplify an expression:

1. Use the distributive property to remove any parentheses.
2. Use the commutative property to write the like terms next to each other.
3. Combine the like terms.

Like terms are terms that have the same variables with the same exponent. For example, x, 3x, and –7x are all "like" terms.

For example, to simplify the expression: $2x(y + 3) - 4(1 - xy) + 7$

1. Distribute to remove the parentheses. $= 2xy + 6x - 4 + 4xy + 7$
2. Write the like terms next to each other. $= 2xy + 4xy + 6x - 4 + 7$
3. Combine the like terms. $= 6xy + 6x + 3$

Evaluating Expressions

Sometimes the variables in an expression are assigned specific values. When this happens you can replace the variables with the numbers and evaluate the expression.

To evaluate an expression:

1. Replace each variable with its assigned value.
2. Calculate the value of the expression.

For example, to evaluate the expression $3x^2y - 4y + 5$ when $x = 1$ and $y = 2$:

1. Replace x with 1.	$= 3(1)^2y - 4y + 5$
Replace y with 2.	$= 3(1)^2(2) - 4(2) + 5$
2. Calculate.	$= 6 - 8 + 5$
	$= 3$

When $x = 1$ and $y = 2$, $3x^2y - 4y + 5 = 3$.

Answers to Sample Problems

Sample Problems

1. Simplify the expression $3(y + 2x) - 5(1 - y) + 4$.

$3(y + 2x) - 5(1 - y) + 4$

☑ a. Distribute to remove parentheses. $= 3y + 6x - 5 + 5y + 4$

☐ b. Write like terms next to each other. $= 6x + \_\_\_ + 5y - \_\_\_ + 4$

☐ c. Combine like terms. $= \_\_\_ + \_\_\_ - \_\_\_$

b. 3y, 5

c. 6x, 8y, (in either order); 1

2. Evaluate the expression $2xy - 4y^2 + 3$ when $x = 3$ and $y = 2$.

$2xy - 4y^2 + 3$

☑ a. Replace x with 3. $= 2(3)y - 4y^2 + 3$

☐ b. Replace y with 2. $= 2(3)(\_\_\_) - 4(\_\_\_)^2 + 3$

☐ c. Calculate. $= \_\_\_ - \_\_\_ + \_\_\_$

$= \_\_\_$

b. 2, 2

c. 12, 16, 3

–1

HOMEWORK

Homework Problems

Circle the homework problems assigned to you by the computer, then complete them below.

Explain
Simplifying Expressions

1. What are the constants in the expression $11 + 4y - 6 + 2x - 1$?

2. Simplify the expression $2x - 5 + 4y + 3x - 7y + 4$.

3. Evaluate the expression $4x - 7$ when $x = -3$.

4. What are the terms in the expression $3xy - 5x + 8 - y - x^2y$?

5. Simplify the expression $5 + 3(x - 1)$.

6. Evaluate the expression $2x + 3y + 5$ when $x = 2$ and $y = 1$.

7. Simplify the expression $3(y - 4) + 4y(x + 2) + 5$.

8. Evaluate the expression $3xy - 2x + 1 - y$ when $x = -1$ and $y = 2$.

9. Melissa bought 3 gallons of white paint for $11.00 per gallon, 2 quarts of blue paint for $7.00 per quart, and 1 brush for $6.00. How much did she spend all together?

   Hint: The amount she spent can be expressed as: $3(11) + 2(7) + 1(6)$

10. Mr. Burton is in charge of the cookie sale for his daughter's Girl Scout troop. When the girls turned in their money, he collected 6 twenty-dollar bills, 8 ten-dollar bills, 17 five-dollar bills, and 25 one-dollar bills. How much money did he collect all together?

    Hint: The amount of money he collected can be expressed as: $6(20) + 8(10) + 17(5) + 25(1)$

11. Simplify the expression $7(2 - x) - 8 - 2(y - 3x) + 4y$.

12. Evaluate the expression $xy^2 - 4y + 2 - 3x$ when $x = 3$ and $y = -2$.

APPLY

Practice Problems

Here are some additional practice problems for you to try.

Simplifying Expressions

1. What are the terms in the expression $6x^3 + 5xy^2 - y + 25$?
2. What are the terms in the expression $3a^3 - 2a^2b + 7b^2 - 6$?
3. Simplify: $2(3y + 7) - 10$
4. Simplify: $8 - 4(a + 3)$
5. Simplify: $3 - 5(x - 7)$
6. Simplify: $7b + 10 + 3b - 17$
7. Simplify: $-4x - 15 + 9x - 12$
8. Simplify: $6a - 13 - 5a + 15$
9. Simplify: $2(y - 3) + 5(y + 4)$
10. Simplify: $5a(b - 7) - 2(3a + 4)$
11. Simplify: $4(x + 5) - 3x(y + 3)$
12. Simplify: $7(b^2 + 2b) - 3(b - 5)$
13. Simplify: $12(x - 3) - 7(2x^2 + 6x)$
14. Simplify: $11(a + 1) + 8(a^2 - 3a)$
15. Simplify: $10(y + 7) - 12 + 3(y^2 + 2y)$
16. Simplify: $15(2 - b) + 32 - 9(3b - b^2)$
17. Simplify: $15(x - 2) + 24 - 10(3x - x^2)$
18. Simplify: $4b(a + 5) - 7a - 2(3ab - b^2)$
19. Simplify: $7m(n - 6) + 10m + 3(n^2 - 8mn)$
20. Simplify: $5x(6 - y) + 5x + 4(y^2 - 2xy)$
21. Evaluate $7a - 3b + 9$ when $a = -3$ and $b = -4$.
22. Evaluate $8m + n - 17$ when $m = 5$ and $n = -1$.
23. Evaluate $3x + 4y - 5$ when $x = 6$ and $y = -2$.
24. Evaluate $3a^2 - 7a - 6b$ when $a = -3$ and $b = 11$.
25. Evaluate $10m + 2n - 8n^2$ when $m = 5$ and $n = -4$.
26. Evaluate $2x^2 - x - 2y$ when $x = 5$ and $y = 10$.
27. Evaluate $3x^3 - 6xy - 5xz + 4z - 1$ when $x = 2$, $y = -4$, and $z = 7$.
28. Evaluate $2a^3 - 7ab + 3ac - 10c + 8$ when $a = -2$, $b = 3$, and $c = 5$

EVALUATE

Practice Test

Take this practice test to be sure that you are prepared for the final quiz in Evaluate.

1. What are the coefficients in the expression $2x^2y - y + 7xy - 4y^3 + 12$?

2. Simplify the following expression by using the distributive property and combining like terms: $7(x + 3) + 2(9 - x)$.

3. Simplify the following expression by using the distributive property and combining like terms: $y(3 - y) + 5(x + y^2) - x(2 - 7y)$.

4. Evaluate the expression $2x^3 - 4x^2 + 7x - 6$ when $x = 2$.

5. Evaluate the expression $5x + 2xy - 5y^2$ when $x = 3$ and $y = -2$.

6. Simplify the following expression by using the distributive property and combining like terms: $y(6 + y) - 5(y^2 - 1) + 2$.

7. Evaluate the expression $4x^2y + y - 5xy^2 - 15$ when $x = 5$ and $y = 3$.

8. Simplify the following expression by using the distributive property and combining like terms: $x^2(3 + y) - 2(5x - x^2) + 6x^2y$.