30. All six graphs have different traces in the planes $x = 0$ and $y = 0$, so we investigate these for each function.

(a) $f(x, y) = |x| + |y|$. The trace in $x = 0$ is $z = |y|$, and in $y = 0$ is $z = |x|$, so it must be graph VI.

(b) $f(x, y) = |xy|$. The trace in $x = 0$ is $z = 0$, and in $y = 0$ is $z = 0$, so it must be graph V.

(c) $f(x, y) = \dfrac{1}{1 + x^2 + y^2}$. The trace in $x = 0$ is $z = \dfrac{1}{1 + y^2}$, and in $y = 0$ is $z = \dfrac{1}{1 + x^2}$. In addition, we can see that f is close to 0 for large values of x and y, so this is graph I.

(d) $f(x, y) = (x^2 - y^2)^2$. The trace in $x = 0$ is $z = y^4$, and in $y = 0$ is $z = x^4$. Both graph II and graph IV seem plausible; notice the trace in $z = 0$ is $0 = (x^2 - y^2)^2 \Rightarrow y = \pm x$, so it must be graph IV.

(e) $f(x, y) = (x - y)^2$. The trace in $x = 0$ is $z = y^2$, and in $y = 0$ is $z = x^2$. Both graph II and graph IV seem plausible; notice the trace in $z = 0$ is $0 = (x - y)^2 \Rightarrow y = x$, so it must be graph II.

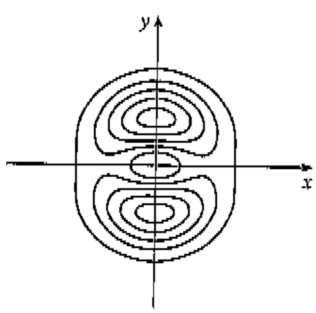
(f) $f(x, y) = \sin(|x| + |y|)$. The trace in $x = 0$ is $z = \sin|y|$, and in $y = 0$ is $z = \sin|x|$. In addition, notice that the oscillating nature of the graph is characteristic of trigonometric functions. So this is graph III.

31. The point $(-3, 3)$ lies between the level curves with z-values 50 and 60. Since the point is a little closer to the level curve with $z = 60$, we estimate that $f(-3, 3) \approx 56$. The point $(3, -2)$ appears to be just about halfway between the level curves with z-values 30 and 40, so we estimate $f(3, -2) \approx 35$. The graph rises as we approach the origin, gradually from above, steeply from below.

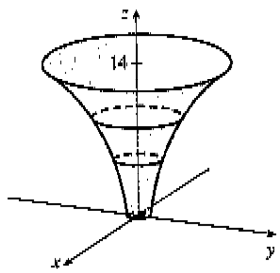
32. If we start at the origin and move along the x-axis, for example, the z-values of a cone centered at the origin increase at a constant rate, so we would expect its level curves to be equally spaced. A paraboloid with vertex the origin, on the other hand, has z-values which change slowly near the origin and more quickly as we move farther away. Thus, we would expect its level curves near the origin to be spaced more widely apart than those farther from the origin. Therefore contour map I must correspond to the paraboloid, and contour map II the cone.

33. Near A, the level curves are very close together, indicating that the terrain is quite steep. At B, the level curves are much farther apart, so we would expect the terrain to be much less steep than near A, perhaps almost flat.

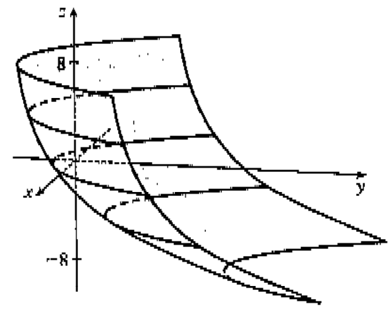
34.



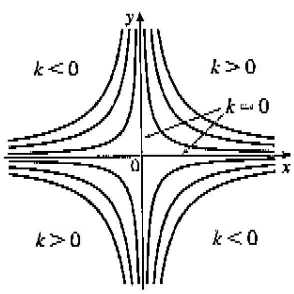
35.



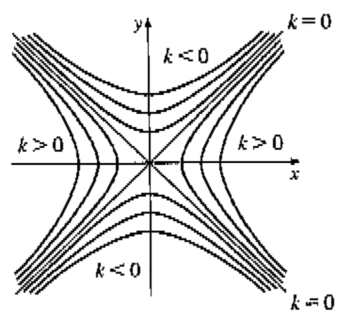
36.



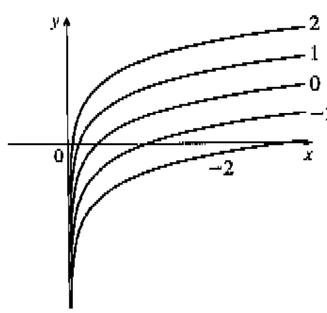
37. The level curves are $xy = k$. For $k = 0$ the curves are the coordinate axis; if $k > 0$, they are hyperbolas in the first and third quadrants; if $k < 0$, they are hyperbolas in the second and fourth quadrants.



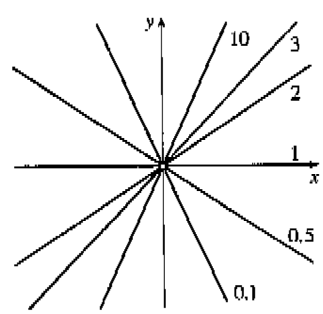
38. The level curves are $k = x^2 - y^2$. When $k = 0$, these are the lines $y = \pm x$. When $k > 0$, the curves are hyperbolas with axis the x-axis and when $k < 0$, they are hyperbolas with axis the y-axis.



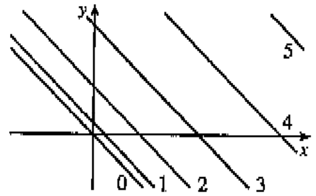
39. The level curves are $y - \ln x = k$ or $y = \ln x + k$.



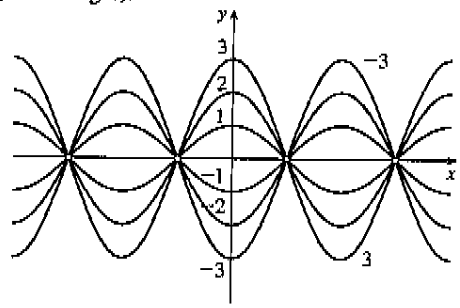
40. The level curves are $e^{y/x} = k$ or equivalently $y = x \ln k$ ($x \neq 0$), a family of lines with slope $\ln k$ ($k > 0$) without the origin.



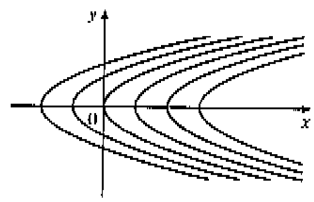
41. $k = \sqrt{x+y}$ or for $x + y \geq 0$, $k^2 = x + y$, or $y = -x + k^2$.
Note: $k \geq 0$ since $k = \sqrt{x+y}$.



42. $k = y \sec x$ or $y = k \cos x$, $x \neq \frac{\pi}{2} + n\pi$ (n an integer).

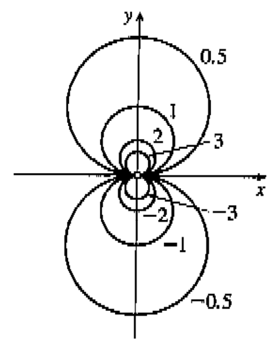


43. $k = x - y^2$, or $x - k = y^2$, a family of parabolas with vertex $(k, 0)$.

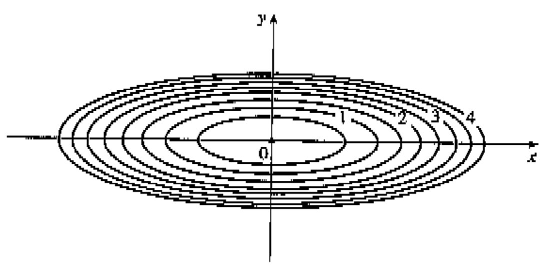


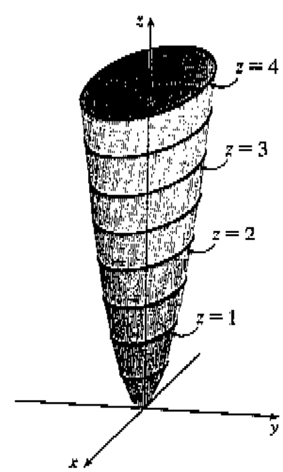
44. For $k \neq 0$ and $(x, y) \neq (0, 0)$, $k = \dfrac{y}{x^2 + y^2}$ $\Leftrightarrow$

$x^2 + y^2 - \dfrac{y}{k} = 0$ $\Leftrightarrow$ $x^2 + \left(y - \frac{1}{2k}\right)^2 = \dfrac{1}{4k^2}$, a family of circles with center $\left(0, \frac{1}{2k}\right)$ and radius $\frac{1}{2k}$ (without the origin). If $k = 0$, the level curve is the x-axis.



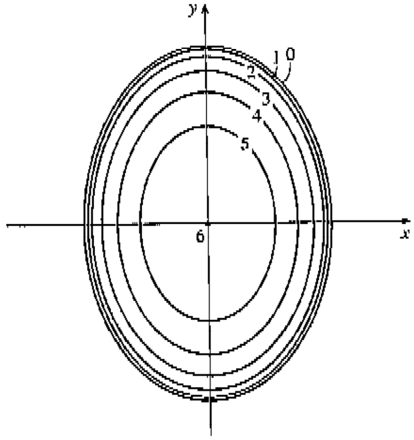
45. The contour map consists of the level curves $k = x^2 + 9y^2$, a family of ellipses with major axis the x-axis. (Or, if $k = 0$, the origin.) The graph of $f(x, y)$ is the surface $z = x^2 + 9y^2$, an elliptic paraboloid.



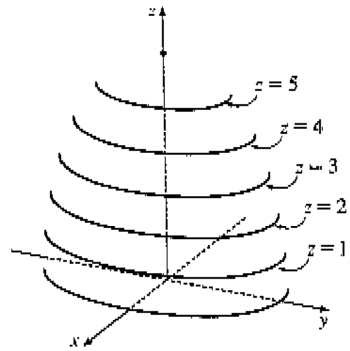


If we visualize lifting each ellipse $k = x^2 + 9y^2$ of the contour map to the plane $z = k$, we have horizontal traces that indicate the shape of the graph of f.

46.

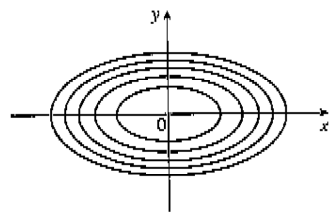


The contour map consists of the level curves $k = \sqrt{36 - 9x^2 - 4y^2}$ $\Rightarrow$ $9x^2 + 4y^2 = 36 - k^2$, $k \geq 0$, a family of ellipses with major axis the y-axis. (Or, if $k = 6$, the origin.)



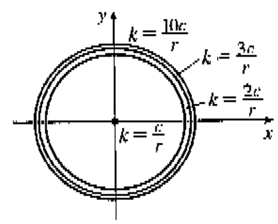
The graph of $f(x, y)$ is the surface $z = \sqrt{36 - 9x^2 - 4y^2}$, or equivalently the upper half of the ellipsoid $9x^2 + 4y^2 + z^2 = 36$. If we visualize lifting each ellipse $k = \sqrt{36 - 9x^2 - 4y^2}$ of the contour map to the plane $z = k$, we have horizontal traces that indicate the shape of the graph of f.

47. The isothermals are given by $k = 100/(1 + x^2 + 2y^2)$ or $x^2 + 2y^2 = (100 - k)/k$ $(0 < k \leq 100)$, a family of ellipses.

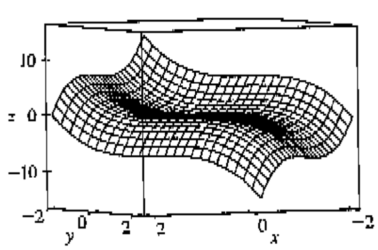


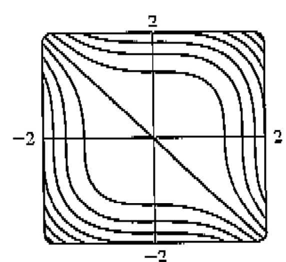
48. The equipotential curves are $k = \dfrac{c}{\sqrt{r^2 - x^2 - y^2}}$ or $x^2 + y^2 = r^2 - \left(\dfrac{c}{k}\right)^2$, a family of circles $(k \geq c/r)$.

Note: As $k \to \infty$, the radius of the circle approaches r.



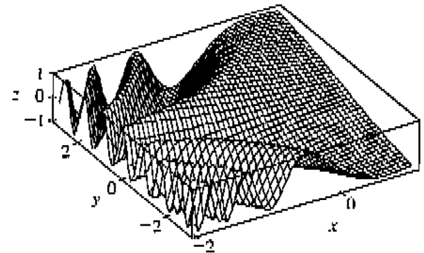
49. $f(x, y) = x^3 + y^3$

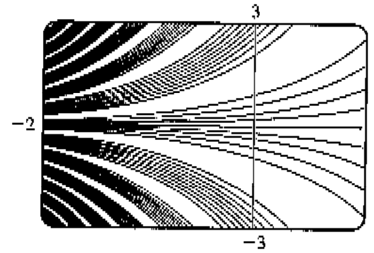




Note that the function is 0 along the line $y = -x$.

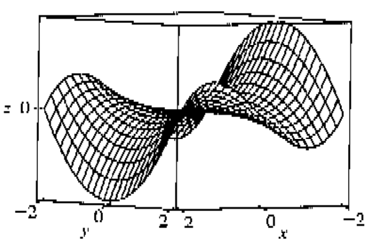
50. $f(x,y) = \sin(ye^{-x})$

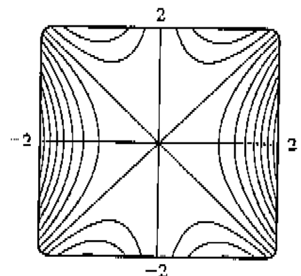




Cross-sections parallel to the yz-plane (such as the left-front trace in the first graph above) are sine-like curves. The periods of these curves decrease as x decreases.

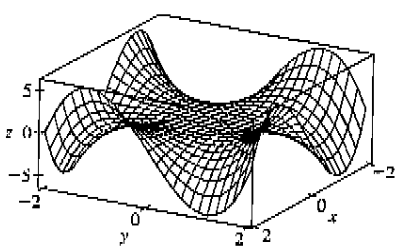
51. $f(x,y) = xy^2 - x^3$

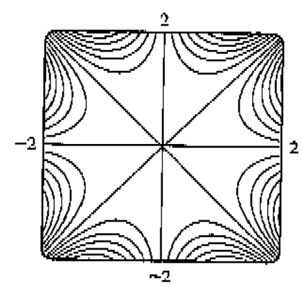




The traces parallel to the yz-plane (such as the left-front trace in the graph above) are parabolas; those parallel to the xz-plane (such as the right-front trace) are cubic curves. The surface is called a monkey saddle because a monkey sitting on the surface near the origin has places for both legs and tail to rest.

52. $f(x,y) = xy^3 - yx^3$





The traces parallel to either the yz-plane or the xz-plane are cubic curves.

53. (a) B *Reasons:* This function is constant on any circle centered at the origin, a description which matches
(b) III only B and III.

54. (a) C *Reasons:* This function is the same if x is interchanged with y, so its graph is symmetric about the
(b) II plane $x = y$. Also, $z(0,0) = 0$ and the values of z approach 0 as we use points farther from the origin. These conditions are satisfied only by C and II.

55. (a) F *Reasons:* z increases without bound as we use points closer to the origin, a condition satisfied only
(b) V by F and V.

56. (a) A *Reasons:* Along the lines $y = \pm\frac{1}{\sqrt{3}}x$ and $x = 0$, this function is 0.
(b) VI

57. (a) D *Reasons:* This function is periodic in both x and y, with period 2π in each variable.
(b) IV

58. (a) E *Reasons:* This function is periodic along the x-axis, and increases as $|y|$ increases.
(b) I

59. $k = x + 3y + 5z$ is a family of parallel planes with normal vector $\langle 1, 3, 5\rangle$.

60. $k = x^2 + 3y^2 + 5z^2$ is a family of ellipsoids for $k > 0$ and the origin for $k = 0$.

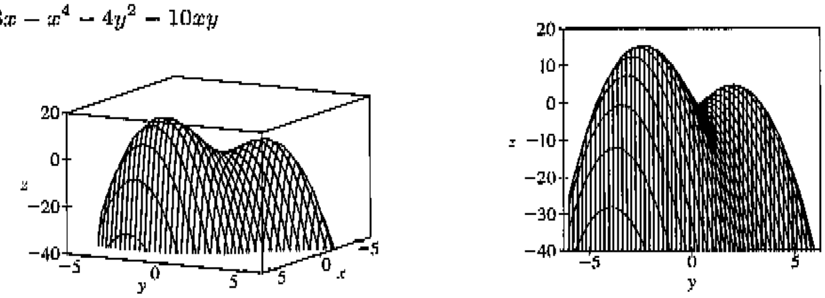
61. $k = x^2 - y^2 + z^2$ are the equations of the level surfaces. For $k = 0$, the surface is a right circular cone with vertex the origin and axis the y-axis. For $k > 0$, we have a family of hyperboloids of one sheet with axis the y-axis. For $k < 0$, we have a family of hyperboloids of two sheets with axis the y-axis.

62. $k = x^2 - y^2$ is a family of hyperbolic cylinders. The cross section of this family in the xy-plane has the same graph as the level curves in Exercise 38.

63. (a) The graph of g is the graph of f shifted upward 2 units.
(b) The graph of g is the graph of f stretched vertically by a factor of 2.
(c) The graph of g is the graph of f reflected about the xy-plane.
(d) The graph of $g(x, y) = -f(x, y) + 2$ is the graph of f reflected about the xy-plane and then shifted upward 2 units.

64. (a) The graph of g is the graph of f shifted 2 units in the positive x-direction.
(b) The graph of g is the graph of f shifted 2 units in the negative y-direction.
(c) The graph of g is the graph of f shifted 3 units in the negative x-direction and 4 units in the positive y-direction.

65. $f(x, y) = 3x - x^4 - 4y^2 - 10xy$



Three-dimensional view Front view

It does appear that the function has a maximum value, at the higher of the two "hilltops." From the front view graph, the maximum value appears to be approximately 15. Both hilltops could be considered local maximum points, as