Answer Key
(via Lance)
13.4

1. a.) $v_{avg} = \frac{\Delta r(t)}{\Delta t}$

$[0,1] = 1.8i - 3.8j - .7k$

$[.5,1] = \frac{r(1) - r(.5)}{1 - .5} = 2i - 2.4j - .6k$

$[1,2] = \frac{r(2) - r(1)}{2 - 1} = 2.8i + .8j - .4k$

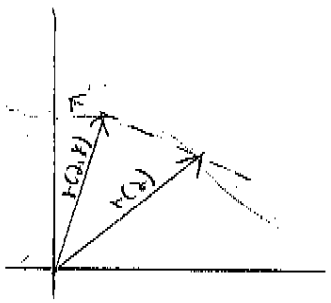
b.) $v(1) = \frac{1}{2}[(2.8i + .8j - .4k) + (2i - 2.4j - .6k)]$

$= \frac{1}{2}[4.8i - 1.6j - k]$

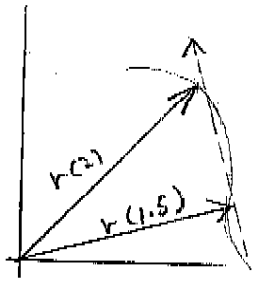
$= 2.4i - .8j - .5k$

$|v(1)| = \sqrt{(2.4)^2 + (.8)^2 + (.5)^2} = 2.58$

2. a.)

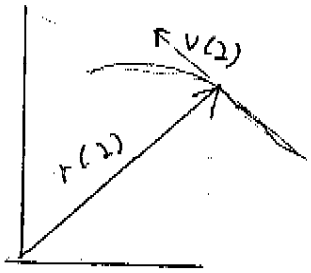


b.)



c.) $\lim_{h \to 0} \dfrac{r(2+h) - r(2)}{h}$

d.)



$|v(2)| \approx \frac{1}{2}(2.8 + 2.7) \approx 2.75$

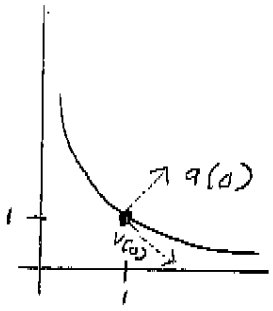
5. $r'(t) = v(t) = (e^t)\,i - (e^{-t})\,j$

$v(0) = i - j$

$r''(t) = a(t) = (e^t)\,i + (e^t)\,j$

$a(0) = i + j$

$|v(t)| = \sqrt{e^{2t} + e^{-2t}}$



9. $v(t) = \langle 2t, 3t^2, 2t \rangle$

$a(t) = \langle 2, 6t, 2 \rangle$

$|v(t)| = \sqrt{4t^2 + 9t^4 + 4t^2} = \sqrt{8t^2 + 9t^4}$

16. $v(t) = \int -10k \, dt = -10tk + C$

$C = i + j - k$

$v(t) = i + j - (1+10t)k$

$r(t) = \int i + j - (1+10t)k \, dt$

$= ti + tj - (t+5t^2)k + C_2$

$C_2 = 2 + 3j$

$r(t) = i(t+2) + j(t+3) - k(t+5t^2)$

21. $a = \frac{F}{m} \qquad \frac{20}{4} = 5k$

$v(t) = \int a(t)\,dt = \int 5k\,dt = 5tk + C$

$v(t) = i - j + 5tk$

$r(t) = \int (i - j + 5tk)\,dt = ti - tj + \frac{5t^2}{2}k$

$|v(t)| = \sqrt{1 + 1 + (5t)^2} = \sqrt{2 + 25t^2}$

22. $\vec{v}(t) = C$ C is constant

$\vec{a}(t) = 0$

$\vec{v} \cdot \vec{a} = 0$

therefore, $\vec{v}$ & $\vec{a}$ are orthogonal

23. a.) $d = \frac{(500)^2 \sin 60°}{9.8} = 20\text{ km}$ or $2e4m$

b.) $y = 500 \sin 30° t - \frac{1}{2}(9.8)t^2$

$= 250t - 4.9t^2 = 0$ $\qquad \frac{51}{2} = 25.5 = t$

$\frac{-250 \pm 250}{-9.81} = 51$ $\qquad y(25.5) \approx 3.2\text{ km}$

c.) $v(t) = 250\sqrt{3}\,i + (250 - 9.8t)j$

$|v(51)| = \sqrt{(250\sqrt{3})^2 + (250 - 9.8(51))^2} = 500\text{ m/s}$

28. $x = V_0 \cos \alpha t + \ldots$

$V_0 \cos \alpha t = 400$ $\quad 115 \cos 50° t = 400$

$t = 5.4$

$V(5.4) = 115 \sin 50° (5.4) - \frac{1}{2}(5.4)^2(32) + 3$

$= 11.2 \text{ ft} > 10 \text{ ft}$

homerun

33. $r'(t) = -\sin t\, i + \cos t\, j + k$

$|r'(t)| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}$

$r''(t) = -\cos t\, i - \sin t\, j$

$r'(t) \times r''(t) = \sin t\, i - \cos t\, j + k$

$a_T = \dfrac{\sin t \cos t - \sin t \cos t}{\sqrt{2}} = 0$

$a_N = \dfrac{\sqrt{\sin^2 t + \cos^2 t + 1}}{\sqrt{2}} = 1$

38. $L(t) = m r(t) \times v(t)$

$L'(t) = m\left(r'(t) \times v(t) + r(t) \times v'(t)\right)$

$= m\left(0 + r(t) \times a(t)\right)$

$= \tau(t)$