13.3 # 49 (b) A scratch proof.

$$B = T \times N$$

$$\frac{dB}{ds} = \frac{dT}{ds} \times N + T \times \frac{dN}{ds}$$

$$\frac{dB}{ds} \cdot T = T \cdot \left(\frac{dT}{ds} \times N + T \times \frac{dN}{ds} \right)$$

$$= T \cdot \left(\frac{d\vec{T}}{ds} \times N \right) + T \cdot \left(T \times \frac{d\vec{N}}{ds} \right)$$

$$= T \underbrace{\left(\frac{dT}{ds} \times \overbrace{\left(\frac{1}{k} \frac{dT}{ds} \right)}^{\text{by \#47}} \right)}_{\vec{0}} + \underbrace{(T \times T)}_{\vec{0}} \cdot \frac{dN}{ds}$$

$$= 0$$

since $\frac{d\vec{B}}{ds} \cdot \vec{T} = 0$, the

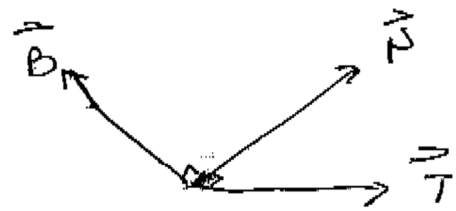
facts are orthogonal.

13.3 #49 (c)

$\frac{d\vec{B}}{ds} \perp \vec{B}$ (from a)

$\frac{d\vec{B}}{ds} \perp \vec{T}$ (from b)

$\frac{d\vec{B}}{ds} \neq \vec{0}$ (duh)



from (a) & (b), $\frac{d\vec{B}}{ds}$ is // to $\vec{N}$.

$\Rightarrow \frac{d\vec{B}}{ds} = f(s)\vec{N}$ for some $f(s)$

Define $\tau(s) = -f(s)$

$\Rightarrow \frac{d\vec{B}}{ds} = -\tau(s)\vec{N}$

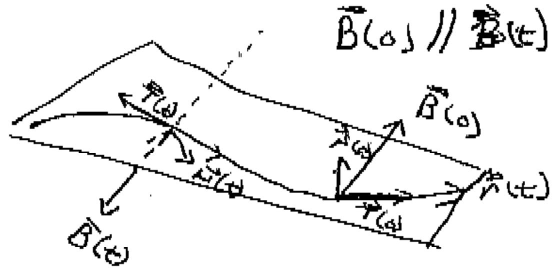
see next page for part (d).

~~(d) Suppose $\vec{r}(t)$ is a plane curve.~~
~~It has $\vec{T}(t)$ & $\vec{N}(t)$ & lives on a plane // to $\vec{T}$ & $\vec{N}$.~~
~~$\vec{r}(t) \cdot \vec{B}(t) = 0$ because~~
~~WTS $\frac{dB}{ds} = \vec{0}$ for the plane curve.~~

13.3 #49 (d)

If $\vec{r}(t)$ lives on a plane, then $\vec{T}$ & $\vec{N}$ are // to the plane $\forall t \in \mathbb{R}$ and $\vec{B}(t)$ is parallel to $\vec{B}(0)$ $\forall t \in \mathbb{R}$ (see picture)

$\vec{B}(0) \,//\, \vec{B}(t)$



$$\Rightarrow \quad \vec{B}(t) = \begin{cases} \vec{B}(0), & \text{when } \vec{r} \text{ is "concave up"} \\ -\vec{B}(0), & \text{when } \vec{r} \text{ is "concave down"} \end{cases}$$

But, this means $\vec{B}(t)$ is piecewise constant and so its derivative (irrespective of the variable of integration) is $\vec{0}$

$$\Rightarrow \quad \vec{0} = \frac{d\vec{B}}{ds} = -\tau(s) \underbrace{\vec{N}(s)}_{\text{non-zero unit vector}}$$

$\Rightarrow \quad \tau(s) = 0.$ (no twisting).