13.3 #40 If $\vec{r}(t) = \langle e^t, e^t \sin t, e^t \cos t \rangle$, Find $\vec{T}$, $\vec{N}$, and $\vec{B}$ at $(1,0,1)$, that is $t=0$.

Solution:

1st: find $\vec{T}(t)$

$$\vec{r}\,'(t) = \langle e^t, e^t \sin t + e^t \cos t, e^t \cos t - e^t \sin t \rangle$$

$$|\vec{r}\,'(t)| = \sqrt{(e^{2t} + e^{2t}(\sin^2 t + 2\sin t \cos t + \cos^2 t) + e^{2t}(\cos^2 t - 2\sin t \cos t + \sin^2 t)}$$

$$= \sqrt{3e^{2t}}$$

$$= \sqrt{3}\, e^t$$

so, $\vec{T}(t) = \left\langle \frac{1}{\sqrt{3}}, \frac{\sin t + \cos t}{\sqrt{3}}, \frac{\cos t - \sin t}{\sqrt{3}} \right\rangle$

2nd: Find $\vec{N}(t)$

$$\vec{T}\,'(t) = \left\langle 0, \frac{\cos t - \sin t}{\sqrt{3}}, \frac{-\sin t - \cos t}{\sqrt{3}} \right\rangle$$

$$|\vec{T}\,'(t)| = \sqrt{\left(0 + \frac{\cos^2 t - 2\sin t \cos t + \sin^2 t}{3} + \frac{\sin^2 t + 2\sin t \cos t + \cos^2 t}{3}\right)}$$

$$= \sqrt{2/3}$$

so $\vec{N}(t) = \left\langle 0, \frac{\cos t - \sin t}{\sqrt{2}}, -\frac{(\cos t + \sin t)}{\sqrt{2}} \right\rangle$

3rd: Find $\vec{T}(0)$ & $\vec{N}(0)$

$$\vec{T}(0) = \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$$

$$\vec{N}(0) = \left\langle 0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$$

4th: Find $\vec{B}(t)$

$$\vec{T} \times \vec{N} \text{ at } t=0 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{vmatrix}$$

$$= \left\langle -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$$

so $\vec{B}(0) = \left\langle -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$

5th: check

a) are all dot products zero ✓

b) are all vectors unit vectors ✓