13.2 Answer Key

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10. $\vec{r}(t) = \langle \cos 3t, t, \sin 3t \rangle$

$r'(t) = \langle -3\sin 3t, 1, 3\cos 3t \rangle$

12. $\vec{r}(t) = \sin^{-1} t\,\vec{i} + \sqrt{1-t^2}\,\vec{j} + \vec{k}$

$= \sin^{-1} t\,\vec{i} + (1-t^2)^{1/2}\,\vec{j} + \vec{k}$

$\vec{r}\,'(t) = \frac{1}{\sqrt{1-t^2}}\,\vec{i} + \left(\frac{-2t}{2\sqrt{1-t^2}}\right)\vec{j}$

$= \frac{1}{\sqrt{1-t^2}}\,\vec{i} - \frac{t}{\sqrt{1-t^2}}\,\vec{j}$

22. $\vec{r}(t) = \langle e^{2t}, e^{-2t}, te^{2t} \rangle$

$r'(t) = \langle 2e^{2t}, -2e^{-2t}, (2t+1)e^{2t} \rangle$

$\vec{T}(0) = \frac{r'(0)}{|r'(0)|}$

$= \frac{\langle 2, -2, 1 \rangle}{\sqrt{9}}$

$= \langle \frac{2}{3}, \frac{-2}{3}, \frac{1}{3} \rangle$

$r''(t) = \langle 4e^{2t}, 4e^{-2t}, 2(2t+1)e^{2t} + 2e^{2t} \rangle$

$= \langle 4e^{2t}, 4e^{-2t}, (4t+4)e^{2t} \rangle$

$= 4\langle e^{2t}, e^{-2t}, (t+1)e^{2t} \rangle$

$r''(0) = 4\langle 1, 1, 1 \rangle = \langle 4, 4, 4 \rangle$

$r'(t) \cdot r''(t) = (2e^{2t})(4e^{2t}) + (-2e^{-2t})(4e^{-2t}) + (2t+1)e^{2t}(4t+4)e^{2t}$

$= 8e^{4t} + (-8e^{-4t}) + (8t^2+12t+4)e^{4t}$

$= (8t^2+12t+12)e^{4t} - 8e^{-4t}$

36. $\int_1^4 \left(\sqrt{t}\,\vec{i} + te^{-t}\,\vec{j} + \frac{1}{t^2}\,\vec{k}\right)dt$

$= \int_1^4 \sqrt{t}\,dt\,\vec{i} + \int_1^4 te^{-t}\,dt\,\vec{j} + \int_1^4 \frac{1}{t^2}\,dt\,\vec{k}$

$= \frac{2}{3}t^{3/2}\Big|_1^4\vec{i} + \left(-te^{-t} - e^{-t}\right)\Big|_1^4\vec{j} + \left(-\frac{1}{t}\right)\Big|_1^4\vec{k}$

$= \left(\frac{16}{3} - \frac{2}{3}\right)\vec{i} + \left[(-4e^{-4} - e^{-4}) - (-e^{-1} - e^{-1})\right]\vec{j} + \left(-\frac{1}{4} + 1\right)\vec{k}$

$= \frac{14}{3}\,\vec{i} + \left(-5e^{-4} + 2e^{-1}\right)\vec{j} + \frac{3}{4}\,\vec{k}$

$= \frac{14}{3}\,\vec{i} + e^{-1}\left(2 - 5e^{-3}\right)\vec{j} + \frac{3}{4}\,\vec{k}$

t	e^{-t}
1	$-e^{-t}$
0	e^{-t}