12.5 Key 20/20

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1) a) true
b) false
c) true
d) false
e) false
f) true
g) false
h) true
i) true
j) false
k) true

3) $r_0 = \langle -2, 4, 10\rangle$, $v = \langle 3, 1, -8\rangle$
$r = (-2+3t)i + (4+t)j + (10-8t)k$
$x = -2+3t$, $y = 4+t$, $z = 10-8t$

4) $r_0 = \langle 0, 0, 0\rangle$ $v = 2i - j + 3k$
$r = (2t)i + (-t)j + (3t)k$
$x = 2t$, $y = -t$, $z = 3t$

9) $(0, 1/2, 1)$ and $(2, 1, -3)$
$v = \langle 2, 1/2, -4\rangle$
$x = 2+2t$, $y = 1 + \frac{1}{2}t$, $z = -3 - 4t$
or
$$\frac{x-2}{2} = 2y - 2 = \frac{z+3}{-4}$$

11) $(1, -1, 1)$, $x+2 = \frac{1}{2}y = z-3$
$v = \langle 1, 2, 1\rangle$
$r_0 = \langle 1, -1, 1\rangle$
$v = (1+t)i + (-1+2t)j + (1+t)k$
$x = 1+t$, $y = -1+2t$, $z = 1+t$
or
$$x - 1 = \frac{y+1}{2} = z - 1$$

15) a) $(0, 2, -1)$, $x = 1 + 2t$, $y = 3t$, $z = 5 - 7t$

$V = \langle 2, 3, -7 \rangle$

$r_0 = \langle 0, 2, -1 \rangle$

$r = (2t)i + (2 + 3t)j + (-1 - 7t)k$

$\frac{x}{2} = \frac{y-2}{3} = \frac{z+1}{-7}$

b) $x = -2/7$, $y = 11/7$ $(-2/7, 11/7, 0)$ intersects at $(4/3, 0, 11/3)$

16) a) $(5, 1, 0)$ $2x - y + z = 1$

$v = \langle 2, -1, 1 \rangle$

$r_0 = \langle 5, 1, 0 \rangle$

$r = (5 + 2t)i + (1 - 1t)j + (t)k$

$x = 5 + 2t$, $y = 1 - t$, $z = t$

b) $(7, 0, 1)$ point of intersection $y = 0$, $z = 0$ $(5, 1, 0)$

17) $(2, -1, 4)$, $(4, 6, 1)$

$r(t) = 2i - j + 4k + (2t)i + 7tj - 3t\,k$ $0 \le t \le 1$

19) L_1: $x = -6t$, $y = 1 + 9t$, $z = -3t$

L_2: $x = 1 + 2s$, $y = 4 - 3s$, $z = s$

$v_1 = \langle -6, 9, -3 \rangle$ $v_2 = \langle 2, -3, 1 \rangle$

$v_1 = -3v_2$ lines are parallel

23) $(6, 3, 2)$ $v = \langle -2, 1, 5 \rangle$

$-2(x - 6) + (y - 3) + 5(z - 2) = 0$

27) $(0, 0, 0)$ $n = \langle 2, -1, 3 \rangle$

$2x - y + 3z = 0$

30) $x = 3+2t,\ y = t,\ z = 8-t \qquad 2x+4y+8z = 17$

$(3,0,8) \qquad n = \langle 2,4,8 \rangle$

$2(x-3) + 4(y) + 8(z-8) = 0$

38) $x - z = 1 \quad y + 2z = 3, \quad x + y - 2z = 1 \qquad (1,3,0)$

$n_1 = \langle 1,0,-1 \rangle, \quad n_2 = \langle 0,1,2 \rangle$

$n_1 \times n_2 = \langle 1,-2,1 \rangle = v_1$

$v_2 = \langle 1,1,-2 \rangle$

$v_1 \times v_2 = \langle 3,3,3 \rangle$

$3(x-1) + 3(y-3) + 3z = 0$

41) $x = y-1 = 2z; \quad 4x - y + 3z = 8$

$x = t,\ y = 1+t,\ z = \frac{1}{2}t$

$4(t) - (1+t) + 3(\frac{1}{2}t) = 8$

$4t - 1 - t + \frac{3}{2}t = 8$

$3t + \frac{3}{2}t - 1 = 8 \qquad x = 2, \quad y = 3, \quad z = 1$

$\frac{9}{2}t = 9$

$t = 2$

intersection at $(2,3,1)$

51) $x + y - z = 2, \quad 3x - 4y + 5z = 6$

$n_1 = \langle 1,1,-1 \rangle, \ n_2 = \langle 3,-4,5 \rangle$

$n_1 \times n_2 = \langle (5-4), -(5+3), (-4-3) \rangle = \langle 1,-8,-7 \rangle$

$x + y = 2, \ 3x - 4y = 6$

$x = 2 - y \quad 6 - 3y - 4y = 6 \qquad (2,0,0)$

$-3y - 4y = 0$

$-7y = 0 \qquad$ a) $x - 2 = \frac{y}{-8} = \frac{z}{-7}$

$y = 0$

b) $\frac{n_1 \cdot n_2}{|n_1||n_2|} = \frac{3-4-5}{\sqrt{3}\sqrt{50}} = \frac{\sqrt{6}}{5} \qquad \theta \approx 119^\circ$

54) $2x+5z+3=0$, $x-3y+z+2=0$

$n_1 = \langle 2, 0, 3\rangle$, $n_2 = \langle 1, -3, 1\rangle$

$n_1 \times n_2 = 3\langle 5, 1, -2\rangle$

$2x+5z=-3$ $\quad$ $x+z=-2$ $\quad$ $y=0$

$x=-2-z$

$-4-2z+5z=-3$

$3z=1$

$z=1/3$ $\quad$ $x=-7/3$ $\quad$ $(-7/3, 0, 1/3)$

$x = -7/3+5t$, $y=t$, $z=1/3-2t$

61) $P_1: 4x-2y+6z=3$ $\quad$ $P_2: 4x-2y-2z=6$

$P_3: -6x+3y-9z=5$ $\quad$ $P_4: z=2x-y-3$

$n_1 = \langle 4, -2, 6\rangle$, $n_2 = \langle 4, -2, -2\rangle$

$n_3 = \langle -6, 3, -9\rangle$, $n_4 = \langle 2, -1, -3\rangle$

$n_1 = -2/3\, n_3$ $\quad$ n_1 and n_3 are parallel so are P_1 and P_3

$n_2 = 2n_4$ $\quad$ n_2 and n_4 are pravallel so are P_2 and P_4

P_1 and P_3 is are parallel

P_2 and P_4 shave $(0,0,-3)$ so two plane are atendical

62) $L_1: x=1+t$, $y=t$, $z=2-5t$

$L_2: x+1=y-2=1-z$

$L_3: x=1+t$, $y=4+t$, $z=1-t$

$L_4: r=\langle 2,1,-3\rangle + t\langle 2,2,-10\rangle$

$v_1 = \langle 1,1,-5\rangle$, $v_2 = \langle 1,1,-1\rangle$, $v_3 = \langle 1,1,-1\rangle$, $v_4 = \langle 2,2,-10\rangle$

v_2 and v_3 are same . $v_4 = 2v_1$ L_4 and L_1 are parallel. L_3 $(1,4,1)$ and L_2 doesn't have the point. L_1 and L_4 are identical them point $(2, 1, -3)$

73) $ax+by+cz+d=0$

$$a\left(x+\frac{d}{a}\right)+b(y-0)+c(z-0)=0 \qquad (-d/a, 0, 0)$$

scalar equation of the plane

$$a(x-0)+b(y-d/b)+c(z-0)=0 \qquad (0, -d/b, 0)$$

$$a(x-0)+b(y-0)+c(z-d/c)=0 \qquad (0, 0, -d/c)$$

is the scalar equation of a plane through the point with normal vector $\langle a, b, c\rangle$.