	12.4 answer key	Chris McGlenn

(2) Given: $a = \langle 5, 1, 4 \rangle$ Find: $a \times b$ + prove orthogonal

Soln: $b = \langle -1, 0, 2 \rangle$

$$\vec{a} \times \vec{b} = [(1\cdot 2) - (4)(-1)]\hat{i} - [(5\cdot 2) - (4)(-1)]\hat{j} + [\cancel{(5\cdot 0)} - (1)(-1)]\hat{k}$$

$$= 2\hat{i} - 14\hat{j} + 1\hat{k} = \langle 2, -14, 1 \rangle = \vec{c}$$

this vector is orthogonal to both of the original vectors given

$$a \cdot c = [(5\cdot 2) + (1)(-14) + (4\cdot 1)] \qquad b \cdot c = [(-1)(2) + \cancel{(0)} + (1)(2)]$$

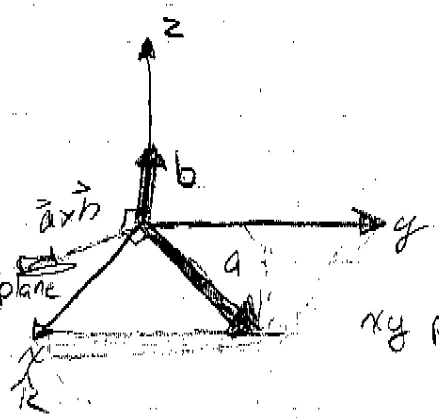
$$= [10 - 14 + 4] = 0 \qquad = [-2 + 0 + 2] = 0$$

Thus both vectors are orthogonal

(12) Given: Find: $a \times b$, value of dir

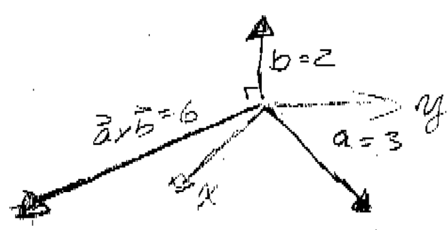
$|a| = 3$

$|b| = 2$



(a) $3 \cdot 2 \sin\frac{\pi}{2} = 6 = \text{mag}$

a lies on xy plane
b points towards the direction $\hat{k}$



b) thus $\vec{a}$ + $\vec{b}$ are orthogonal
and the vector $\vec{a} \times \vec{b}$ will orthogonal
to both b + a pointing towards us
(coming out of the page) per right hand rule.

$a \times b$ will have
- posative $\hat{i}$ value
- negative $\hat{j}$ "
- zero $\hat{k}$ "

$(a_1 b_2 - a_2 b_1)\hat{k}$ by rhrule

we know new vector is
$\perp$ to xy plane + z axis

by rhrule zero $\hat{k}$

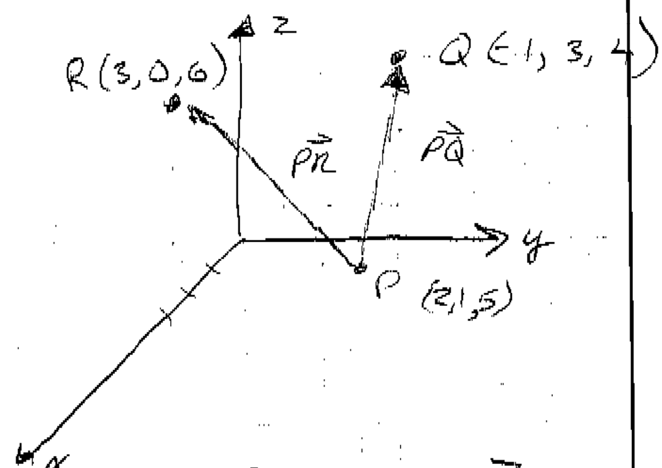
(26) Given: $P(2,1,5)$ $Q(-1,3,4)$ $R(3,0,6)$

Find! (a) a vector orthoganal to the plane through points P, Q, R

(b) Find the area of triangle PQR

Soln: (a) vector $\vec{PQ}$ & $\vec{PR}$ will both lie on the plane

$$\vec{PQ} = \langle (-1-2), (3-1), (4-5) \rangle$$
$$= \langle -3, 2, -1 \rangle$$
$$\vec{PR} = \langle (3-2), (0-1), (6-5) \rangle$$
$$= \langle 1, -1, 1 \rangle$$



now that we have 2 vectors that define our plane & it goes thru all points we $\vec{PQ} \times \vec{PR}$ to find an orthoganal vector.

$$\vec{PQ} \times \vec{PR} = \begin{bmatrix} -3, & 2, & -1 \\ 1, & -1, & 1 \end{bmatrix} = [(2 \cdot 1) - (-1)(-1)]\hat{\imath} - [(-3)(1) - (-1)(1)]\hat{\jmath} + [(-3)(-1) - (2)(1)]\hat{k}$$

$$= \boxed{\langle 1, 2, 1 \rangle \text{ or } 1\hat{\imath} + 2\hat{\jmath} + 1\hat{k}}$$

(b) to find the area. we use example 3 ⇒ 4th edition

the area PQR is $\frac{1}{2}$ the area of the parallelogram with adjacent sides PQ + PR

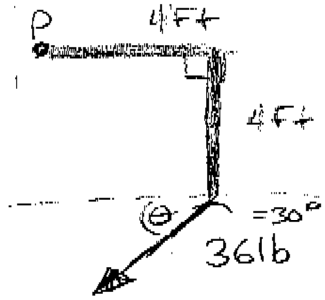
$$A = \frac{1}{2} |a \times b| = \frac{1}{2}\sqrt{1^2 + 2^2 + 1^2} \quad \frac{1}{2}\sqrt{6}$$

$$\boxed{A = \frac{\sqrt{6}}{2}}$$

36 Given: 36 lb Force

$\Theta = 30^\circ$

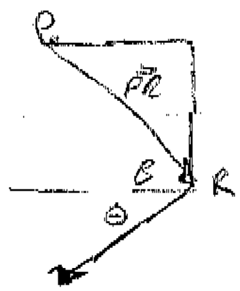
$\beta = 45^\circ$



Find: magnitude of Torque about point P.

Soln: $\tau = r \times F = r\,F \sin\gamma$ where $\gamma =$ the angle between moment arm.

So we need the magnitude of r



$|\vec{PR}| = (4^2+4^2)^{1/2} = \sqrt{32} = 4\cdot\sqrt{2} \approx 5.656$ mag of $\vec{PR}$

now we need γ

$\gamma = \beta + \Theta = 30^\circ + 45^\circ = 75^\circ = \gamma$

so $\tau = r \times F = |\vec{PR}| \cdot |F| \sin\gamma$

$= (4\sqrt{2})\,ft\,(36\,lbf) \sin 75^\circ$

$$\boxed{\tau_P = 196.7 \text{ ft} \cdot \text{lb}}$$