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0/22/06
Sec 12.3

20/20

1, 2, 4, 8, 10, 11, 14, 17, 22, 23, 29, 37, 41, 47, 52

#1. (a) $(\vec{a}\cdot\vec{b})\cdot\vec{c}$; meaningless, $(\vec{a}\cdot\vec{b})$ is scalar
Dot product is only used between vectors

(b) $(\vec{a}\cdot\vec{b})\vec{c}$; meaningful, $(\vec{a}\cdot\vec{b}) = d$ which is scalar.

(c) $|\vec{a}|(\vec{b}\cdot\vec{c})$; meaninful, $|\vec{a}|$ and $(\vec{b}\cdot\vec{c})$ are both scalar so combined by multiplication.

(d) $\vec{a}\cdot(\vec{b}+\vec{c})$; meaningful, $(\vec{b}+\vec{c})$ is vector, dot product can be used.

(e) $\vec{a}\cdot\vec{b}+\vec{c}$; meaningless, no parenthesis. $(\vec{a}\cdot\vec{b})$ is scalar. cannot add scalar and vector

(f) $|\vec{a}|\cdot(\vec{b}+\vec{c})$; meaningless, $|\vec{a}|$ is scalar, $(\vec{b}+\vec{c})$ is vector.

#2. $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta = 6\cdot\frac{1}{3}\cdot\cos\pi/4 = \sqrt{2}$

4/4

#4. $\vec{a} = \langle\frac{1}{2}, 4\rangle$, $\vec{b} = \langle -8, -3\rangle$
$\vec{a}\cdot\vec{b} = -4 + -12 = -16$ ✓

#8. $\vec{a} = 4\vec{j} - 3\vec{k}$, $\vec{b} = 2\vec{i} + 4\vec{j} + 6\vec{k}$
$\vec{a}\cdot\vec{b} = 0 + 16 - 18 = -2$

#10. $|\vec{a}| = 4$, $|\vec{b}| = 10$, $\theta = 120°$
$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta = 40\cos 120 = -20$

#11. $\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta = 1\cdot1\cdot\cos 60° = \frac{1}{2}$
$\vec{u}\cdot\vec{w} = (-\vec{u})\cdot\vec{w} = -(\vec{u}\cdot\vec{w}) = -(1\cdot1\cdot\cos 60°) = -\frac{1}{2}$

#14. $\vec{A} = \langle a, b, c\rangle$, $\vec{P} = \langle 2, 1.5, 1\rangle$
$\vec{A}\cdot\vec{P} = 2a + 1.5b + c$ total revenue of vendor

#17. $\vec{a} = \langle 1, 2, 3\rangle$, $\vec{b} = \langle 4, 0, -1\rangle$ $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\dfrac{4-3}{\sqrt{14}\times\sqrt{17}} = \dfrac{1}{\sqrt{238}} = \cos\theta$ $\theta = \cos^{-1}\left(\dfrac{1}{\sqrt{238}}\right) \approx 86.3°$

4/4 #22. $D(0,1,1)$ $E(-2,4,3)$, $F(1,2,-1)$

$\cos\alpha = \frac{\vec{DE}\cdot\vec{DF}}{|\vec{DE}||\vec{DF}|} = \frac{\langle -2,3,2\rangle\cdot\langle 1,1,-2\rangle}{\sqrt{4+9+4}\sqrt{1+1+4}} = \frac{-3}{\sqrt{17}\sqrt{6}}$ $\alpha = \cos^{-1}\left(-\frac{3}{\sqrt{102}}\right) \doteqdot 107°$ ✓

$\cos\beta = \frac{\vec{ED}\,\vec{EF}}{|\vec{ED}||\vec{EF}|} = \frac{\langle 2,-3,-2\rangle\cdot\langle 3,-2,-4\rangle}{\sqrt{4+9+4}\sqrt{9+4+16}} = \frac{20}{\sqrt{17}\sqrt{29}}$ $\beta = \cos^{-1}\left(\frac{20}{\sqrt{493}}\right) \doteqdot 26°$ ✓

$180° - (107° + 26°) = 47°$ ✓

4/4 #23. (a) $\vec{a} = \langle -5,3,7\rangle$, $\vec{b} = \langle 6,-8,2\rangle$

$\theta = \cos^{-1}\left(\frac{-30-24+14}{\sqrt{25+9+49}\sqrt{36+64+4}}\right) = \cos^{-1}\left(\frac{-40}{\sqrt{83}\sqrt{104}}\right) = 115.5°$ Neither ✓

$\vec{a}\cdot\vec{b} = -40 \neq 0$

(b) $\vec{a} = \langle 4,6\rangle$, $\vec{b} = \langle -3,2\rangle$

$\vec{a}\cdot\vec{b} = -12+12 = 0$ Orthogonal ✓

(c) $\vec{a} = -\vec{i} + 2\vec{j} + 5\vec{k}$, $b = 3\vec{i} + 4\vec{j} - \vec{k}$

$\vec{a}\cdot\vec{b} = -3 + 8 - 5 = 0$ Orthogonal ✓

(d) $\vec{a} = 2\vec{i} + 6\vec{j} - 4\vec{k}$, $\vec{b} = -3\vec{i} - 9\vec{j} + 6\vec{k}$

$-\frac{2}{3}(-3\vec{i} - 9\vec{j} + 6\vec{k}) = 2\vec{i} + 6\vec{j} - 4\vec{k}$

$-\frac{2}{3}\vec{b} = \vec{a}$ parallel ✓

#29. $\langle 3,4,5\rangle$

$\cos\alpha = \frac{3}{\sqrt{50}}$ $\cos\beta = \frac{4}{\sqrt{50}}$ $\cos\gamma = \frac{1}{\sqrt{2}}$

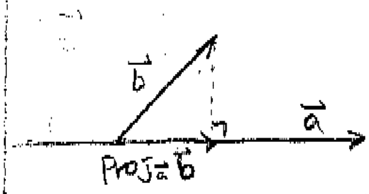
$\alpha = \cos^{-1}\left(\frac{3}{\sqrt{50}}\right) = 64.9°$ $\beta = \cos^{-1}\left(\frac{4}{\sqrt{50}}\right) = 55.6°$ $\gamma = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$

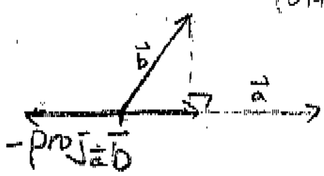
4/4 #37. $\vec{a} = \langle 4,2,0\rangle$ $\vec{b} = \langle 1,1,1\rangle$

$\text{Comp}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \frac{4+2}{\sqrt{20}} = \frac{6}{\sqrt{20}} = \frac{3}{\sqrt{5}}$ ✓

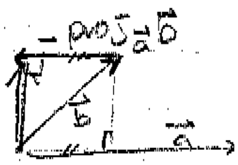
$\text{Proj}_{\vec{a}}\vec{b} = \frac{6}{\sqrt{20}}\left(\frac{\vec{a}}{\sqrt{20}}\right) = \frac{3}{10}\vec{a} = \left\langle \frac{6}{5}, \frac{3}{5}, 0\right\rangle$ ✓

#4. $\text{Orth}_{\vec a}\vec b = \vec b - \text{proj}_{\vec a}\vec b$ is orthogonal to $\vec a$.





$$(\text{orth}_{\vec a}\vec b)\cdot\vec a = (\vec b - \text{proj}_{\vec a}\vec b)\cdot\vec a$$
$$= \vec b\cdot\vec a - (\text{proj}_{\vec a}\vec b)\cdot\vec a$$
$$= \vec b\cdot\vec a - \frac{\vec a\cdot\vec b}{|\vec a|^2}\,\vec a\cdot\vec a$$
$$= \vec b\cdot\vec a - \frac{\vec a\cdot\vec b}{\cancel{|\vec a|^2}}\cancel{|\vec a|^2} = 0$$

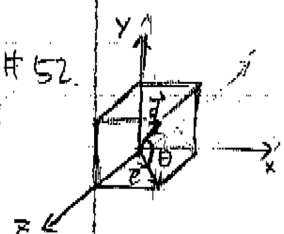


$\therefore\ \vec b - \text{proj}_{\vec a}\vec b = \perp$ to $\vec a$

4/4 #47

$W = |\vec F||\vec D|\cos\theta$
$= (25\text{ lb})(10\text{ ft})\cos 20° = 234.9\ \text{lb/ft}$ ✓

#52.



Assume side of cube is 1

$\vec d = \langle 1,1,1\rangle \quad \vec e = \langle 1,0,1\rangle$

$\cos\theta = \dfrac{1+0+1}{\sqrt3\sqrt2} = \dfrac{2}{\sqrt6} \qquad \theta = \cos^{-1}\left(\dfrac{2}{\sqrt6}\right) \doteqdot 35.2°$