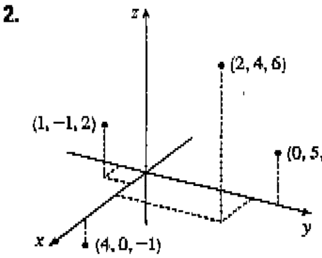
13 □ VECTORS AND THE GEOMETRY OF SPACE □ ET 12

13.1 Three-Dimensional Coordinate Systems ET 12.1

1. We start at the origin, which has coordinates $(0, 0, 0)$. First we move 4 units along the positive x-axis, affecting only the x-coordinate, bringing us to the point $(4, 0, 0)$. We then move 3 units straight downward, in the negative z-direction. Thus only the z-coordinate is affected, and we arrive at $(4, 0, -3)$.

2.

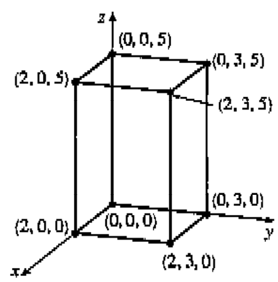


3. The distance from a point to the xz-plane is the absolute value of the y-coordinate of the point. $Q(-5, -1, 4)$ has the y-coordinate with the smallest absolute value, so Q is the point closest to the xz-plane. $R(0, 3, 8)$ must lie in the yz-plane since the distance from R to the yz-plane, given by the x-coordinate of R, is 0.

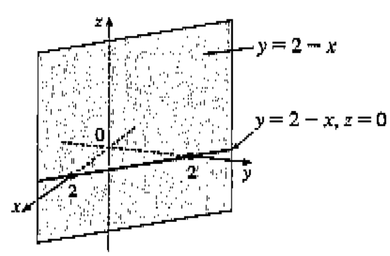
4. The projection of $(2, 3, 5)$ on the xy-plane is $(2, 3, 0)$; on the yz-plane, $(0, 3, 5)$; on the xz-plane, $(2, 0, 5)$.

The length of the diagonal of the box is the distance between the origin and $(2, 3, 5)$, given by

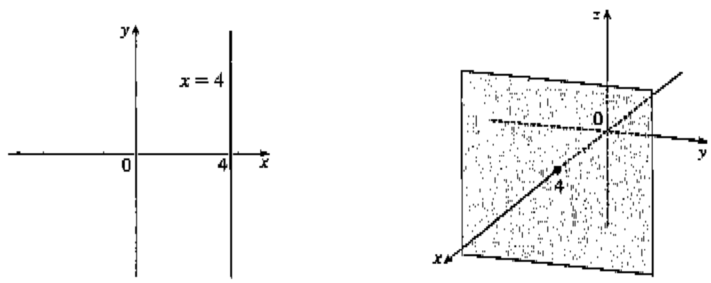
$$\sqrt{(2-0)^2 + (3-0)^2 + (5-0)^2} = \sqrt{38} \approx 6.16$$



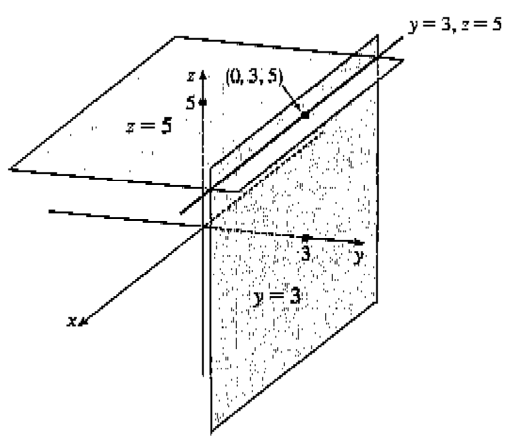
5. The equation $x + y = 2$ represents the set of all points in $\mathbb{R}^3$ whose x- and y-coordinates have a sum of 2, or equivalently where $y = 2 - x$. This is the set $\{(x, 2-x, z) \mid x \in \mathbb{R}, z \in \mathbb{R}\}$ which is a vertical plane that intersects the xy-plane in the line $y = 2 - x$, $z = 0$.



6. (a) In $\mathbb{R}^2$, the equation $x = 4$ represents a line parallel to the y-axis. In $\mathbb{R}^3$, the equation $x = 4$ represents the set $\{(x, y, z) \mid x = 4\}$, the set of all points whose x-coordinate is 4. This is the vertical plane that is parallel to the yz-plane and 4 units in front of it.



(b) In $\mathbb{R}^3$, the equation $y = 3$ represents a vertical plane that is parallel to the xz-plane and 3 units to the right of it. The equation $z = 5$ represents a horizontal plane parallel to the xy-plane and 5 units above it. The pair of equations $y = 3$, $z = 5$ represents the set of points that are simultaneously on both planes, or in other words, the line of intersection of the planes $y = 3$, $z = 5$. This line can also be described as the set $\{(x, 3, 5) \mid x \in \mathbb{R}\}$, which is the set of all points in $\mathbb{R}^3$ whose x-coordinate may vary but whose y- and z-coordinates are fixed at 3 and 5, respectively. Thus the line is parallel to the x-axis and intersects the yz-plane in the point $(0, 3, 5)$.



7. We first find the lengths of the sides of the triangle by using the distance formula between pairs of vertices:

$$|PQ| = \sqrt{[1-(-2)]^2 + (2-4)^2 + (-1-0)^2} = \sqrt{9+4+1} = \sqrt{14}$$

$$|QR| = \sqrt{(-1-1)^2 + (1-2)^2 + [2-(-1)]^2} = \sqrt{4+1+9} = \sqrt{14}$$

$$|PR| = \sqrt{[-1-(-2)]^2 + (1-4)^2 + (2-0)^2} = \sqrt{1+9+4} = \sqrt{14}$$

Since all three sides have the same length, PQR is an equilateral triangle.

8. We can find the lengths of the sides of the triangle by using the distance formula between pairs of vertices:

$$|AB| = \sqrt{(3-1)^2 + (4-2)^2 + [-2-(-3)]^2} = \sqrt{4+4+1} = 3$$

$$|BC| = \sqrt{(3-3)^2 + (-2-4)^2 + [1-(-2)]^2} = \sqrt{0+36+9} = \sqrt{45} = 3\sqrt{5}$$

$$|AC| = \sqrt{(3-1)^2 + (-2-2)^2 + [1-(-3)]^2} = \sqrt{4+16+16} = 6$$

Since the Pythagorean Theorem is satisfied by $|AB|^2 + |AC|^2 = |BC|^2$, ABC is a right triangle. ABC is not ososceles, as no two sides have the same length.

9. (a) First we find the distances between points:

$$|AB| = \sqrt{(7-5)^2 + (9-1)^2 + (-1-3)^2} = \sqrt{84} = 2\sqrt{21}$$

$$|BC| = \sqrt{(1-7)^2 + (-15-9)^2 + [11-(-1)]^2} = \sqrt{756} = 6\sqrt{21}$$

$$|AC| = \sqrt{(1-5)^2 + (-15-1)^2 + (11-3)^2} = \sqrt{336} = 4\sqrt{21}$$

In order for the points to lie on a straight line, the sum of the two shortest distances must be equal to the longest distance. Since $|AB| + |AC| = |BC|$, the three points lie on a straight line.

(b) The distances between points are

$$|KL| = \sqrt{(1-0)^2 + (2-3)^2 + [-2-(-4)]^2} = \sqrt{6}$$

$$|LM| = \sqrt{(3-1)^2 + (0-2)^2 + [1-(-2)]^2} = \sqrt{17}$$

$$|KM| = \sqrt{(3-0)^2 + (0-3)^2 + [1-(-4)]^2} = \sqrt{43}$$

Since $\sqrt{6} + \sqrt{17} \neq \sqrt{43}$, the three points do not lie on a straight line.

10. (a) The distance from a point to the xy-plane is the absolute value of the z-coordinate of the point. Thus, the distance is $|-5| = 5$.

(b) Similarly, the distance is the absolute value of the x-coordinate of the point: $|3| = 3$.

(c) The distance is the absolute value of the y-coordinate of the point: $|7| = 7$.

(d) The point on the x-axis closest to $(3, 7, -5)$ is the point $(3, 0, 0)$. (Approach the x-axis perpendicularly.) The distance from $(3, 7, -5)$ to the x-axis is the distance between these two points:
$\sqrt{(3-3)^2 + (7-0)^2 + (-5-0)^2} = \sqrt{74} \approx 8.60$.

(e) The point on the y-axis closest to $(3, 7, -5)$ is $(0, 7, 0)$. The distance between these points is
$\sqrt{(3-0)^2 + (7-7)^2 + (-5-0)^2} = \sqrt{34} \approx 5.83$.

(f) The point on the z-axis closest to $(3, 7, -5)$ is $(0, 0, -5)$. The distance between these points is
$\sqrt{(3-0)^2 + (7-0)^2 + [-5-(-5)]^2} = \sqrt{58} \approx 7.62$.

11. An equation of the sphere with center $(1, -4, 3)$ and radius 5 is $(x-1)^2 + [y-(-4)]^2 + (z-3)^2 = 5^2$ or $(x-1)^2 + (y+4)^2 + (z-3)^2 = 25$. The intersection of this sphere with the xz-plane is the set of points on the sphere whose y-coordinate is 0. Putting $y = 0$ into the equation, we have $(x-1)^2 + 4^2 + (z-3)^2 = 25$, $y = 0$ or $(x-1)^2 + (z-3)^2 = 9$, $y = 0$, which represents a circle in the xz-plane with center $(1, 0, 3)$ and radius 3.

12. An equation of the sphere with center $(6, 5, -2)$ and radius $\sqrt{7}$ is $(x-6)^2 + (y-5)^2 + [z-(-2)]^2 = \left(\sqrt{7}\right)^2$ or $(x-6)^2 + (y-5) + (z+2)^2 = 7$. The intersection of this sphere with the xy-plane is the set of points on the sphere whose z-coordinate is 0. Putting $z = 0$ into the equation, we have $(x-6)^2 + (y-5)^2 = 3$, $z = 0$ which represents a circle in the xy-plane with center $(6, 5, 0)$ and radius $\sqrt{3}$. To find the intersection with the xz-plane, we set $y = 0$: $(x-6)^2 + (z+2)^2 = -18$. Since no points satisfy this equation, the sphere does not intersect the xz-plane. (Also note that the distance from the center of the sphere to the xz-plane is greater than the radius of the sphere.) Similarly, the sphere does not intersect the yz-plane since substituting $x = 0$ into the equation gives $(y-5)^2 + (z+2)^2 = -29$.

13. The radius of the sphere is the distance between $(4, 3, -1)$ and $(3, 8, 1)$:
$r = \sqrt{(3-4)^2 + (8-3)^2 + [1-(-1)]^2} = \sqrt{30}$. Thus, an equation of the sphere is
$(x-3)^2 + (y-8)^2 + (z-1)^2 = 30$.

14. If the sphere passes through the origin, the radius of the sphere must be the distance from the origin to the point $(1, 2, 3)$: $r = \sqrt{(1-0)^2 + (2-0)^2 + (3-0)^2} = \sqrt{14}$. Then an equation of the sphere is
$(x-1)^2 + (y-2)^2 + (z-3)^2 = 14$.

15. Completing squares in the equation $x^2 + y^2 + z^2 - 6x + 4y - 2z = 11$ gives
$(x^2 - 6x + 9) + (y^2 + 4y + 4) + (z^2 - 2z + 1) = 11 + 9 + 4 + 1 \quad \Rightarrow$
$(x-3)^2 + (y+2)^2 + (z-1)^2 = 25$ which we recognize as an equation of a sphere with center $(3, -2, 1)$ and radius 5.

16. Completing squares in the equation gives
$(x^2 - 4x + 4) + (y^2 + 2y + 1) + z^2 = 0 + 4 + 1 \quad \Rightarrow \quad (x-2)^2 + (y+1)^2 + z^2 = 5$ which we recognize as an equation of a sphere with center $(2, -1, 0)$ and radius $\sqrt{5}$.

17. Completing squares in the equation gives $\left(x^2 - x + \frac{1}{4}\right) + \left(y^2 - y + \frac{1}{4}\right) + \left(z^2 - z + \frac{1}{4}\right) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \quad \Rightarrow$
$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 + \left(z - \frac{1}{2}\right)^2 = \frac{3}{4}$ which we recognize as an equation of a sphere with center $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ and radius $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$.

18. Completing squares in the equation gives $4\left(x^2 - 2x + 1\right) + 4\left(y^2 + 4y + 4\right) + 4z^2 = 1 + 4 + 16 \quad \Rightarrow$
$4(x-1)^2 + 4(y+2)^2 + 4z^2 = 21 \quad \Rightarrow \quad (x-1)^2 + (y+2)^2 + z^2 = \frac{21}{4}$, which we recognize as an equation of a sphere with center $(1, -2, 0)$ and radius $\sqrt{\frac{21}{4}} = \frac{\sqrt{21}}{2}$.

19. (a) If the midpoint of the line segment from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ is $Q = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$, then the distances $|P_1Q|$ and $|QP_2|$ are equal, and each is half of $|P_1P_2|$. We verify that this is the case:

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\begin{aligned}
|P_1Q| &= \sqrt{\left[\tfrac{1}{2}(x_1 + x_2) - x_1\right]^2 + \left[\tfrac{1}{2}(y_1 + y_2) - y_1\right]^2 + \left[\tfrac{1}{2}(z_1 + z_2) - z_1\right]^2} \\
&= \sqrt{\left(\tfrac{1}{2}x_2 - \tfrac{1}{2}x_1\right)^2 + \left(\tfrac{1}{2}y_2 - \tfrac{1}{2}y_1\right)^2 + \left(\tfrac{1}{2}z_2 - \tfrac{1}{2}z_1\right)^2} \\
&= \sqrt{\left(\tfrac{1}{2}\right)^2\left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right]} \\
&= \tfrac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\
&= \tfrac{1}{2}\,|P_1P_2|
\end{aligned}$$

$$\begin{aligned}
|QP_2| &= \sqrt{\left[x_2 - \tfrac{1}{2}(x_1 + x_2)\right]^2 + \left[y_2 - \tfrac{1}{2}(y_1 + y_2)\right]^2 + \left[z_2 - \tfrac{1}{2}(z_1 + z_2)\right]^2} \\
&= \sqrt{\left(\tfrac{1}{2}x_2 - \tfrac{1}{2}x_1\right)^2 + \left(\tfrac{1}{2}y_2 - \tfrac{1}{2}y_1\right)^2 + \left(\tfrac{1}{2}z_2 - \tfrac{1}{2}z_1\right)^2} \\
&= \sqrt{\left(\tfrac{1}{2}\right)^2\left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right]} \\
&= \tfrac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\
&= \tfrac{1}{2}\,|P_1P_2|
\end{aligned}$$

So Q is indeed the midpoint of P_1P_2.

(b) By part (a), the midpoints of sides AB, BC and CA are $P_1\left(-\frac{1}{2}, 1, 4\right)$, $P_2\left(1, \frac{1}{2}, 5\right)$ and $P_3\left(\frac{5}{2}, \frac{3}{2}, 4\right)$. (Recall that a median of a triangle is a line segment from a vertex to the midpoint of the opposite side.) Then the lengths of the medians are:

$$\begin{aligned}
|AP_2| &= \sqrt{0^2 + \left(\tfrac{1}{2} - 2\right)^2 + (5 - 3)^2} = \sqrt{\tfrac{9}{4} + 4} = \sqrt{\tfrac{25}{4}} = \tfrac{5}{2} \\
|BP_3| &= \sqrt{\left(\tfrac{5}{2} + 2\right)^2 + \left(\tfrac{3}{2}\right)^2 + (4 - 5)^2} = \sqrt{\tfrac{81}{4} + \tfrac{9}{4} + 1} = \sqrt{\tfrac{94}{4}} = \tfrac{1}{2}\sqrt{94} \\
|CP_1| &= \sqrt{\left(-\tfrac{1}{2} - 4\right)^2 + (1 - 1)^2 + (4 - 5)^2} = \sqrt{\tfrac{81}{4} + 1} = \tfrac{1}{2}\sqrt{85}
\end{aligned}$$

20. By Exercise 19(a), the midpoint of the diameter (and thus the center of the sphere) is $C(3, 2, 7)$. The radius is half the diameter, so $r = \frac{1}{2}\sqrt{(4-2)^2 + (3-1)^2 + (10-4)^2} = \frac{1}{2}\sqrt{44} = \sqrt{11}$. Therefore an equation of the sphere is $(x - 3)^2 + (y - 2)^2 + (z - 7)^2 = 11$.

21. (a) Since the sphere touches the xy-plane, its radius is the distance from its center, $(2, -3, 6)$, to the xy-plane, namely 6. Therefore $r = 6$ and an equation of the sphere is $(x - 2)^2 + (y + 3)^2 + (z - 6)^2 = 6^2 = 36$.

(b) The radius of this sphere is the distance from its center $(2, -3, 6)$ to the yz-plane, which is 2. Therefore, an equation is $(x - 2)^2 + (y + 3)^2 + (z - 6)^2 = 4$.

(c) Here the radius is the distance from the center $(2, -3, 6)$ to the xz-plane, which is 3. Therefore, an equation is $(x - 2)^2 + (y + 3)^2 + (z - 6)^2 = 9$.

22. The largest sphere contained in the first octant must have a radius equal to the minimum distance from the center $(5,4,9)$ to any of the three coordinate planes. The shortest such distance is to the xz-plane, a distance of 4. Thus an equation of the sphere is $(x-5)^2+(y-4)^2+(z-9)^2=16$.

23. The equation $y=-4$ represents a plane parallel to the xz-plane and 4 units to the left of it.

24. The equation $x=10$ represents a plane parallel to the yz-plane and 10 units in front of it.

25. The inequality $x>3$ represents a half-space consisting of all points in front of the plane $x=3$.

26. The inequality $y\geq 0$ represents a half-space consisting of all points on or to the right of the xz-plane.

27. The inequality $0\leq z\leq 6$ represents all points on or between the horizontal planes $z=0$ (the xy-plane) and $z=6$.

28. The equation $y=z$ represents a plane perpendicular to the yz-plane and intersecting the yz-plane in the line $y=z$, $x=0$.

29. The inequality $x^2+y^2+z^2>1$ is equivalent to $\sqrt{x^2+y^2+z^2}>1$, so the region consists of those points whose distance from the origin is greater than 1. This is the set of all points outside the sphere with radius 1 and center $(0,0,0)$.

30. The inequality $1\leq x^2+y^2+z^2\leq 25$ is equivalent to $1\leq\sqrt{x^2+y^2+z^2}\leq 5$, so the region consists of those points whose distance from the origin is at least 1 and at most 5. This is the set of all points on or between the concentric spheres with radii 1 and 5 and center $(0,0,0)$.

31. Completing the square in z gives $x^2+y^2+(z^2-2z+1)<3+1$ or $x^2+y^2+(z-1)^2<4$, which is equivalent to $\sqrt{x^2+y^2+(z-1)^2}<2$. Thus the region consists of those points whose distance from the point $(0,0,1)$ is less than 2. This is the set of all points inside the sphere with radius 2 and center $(0,0,1)$.

32. The equation $x^2+y^2=1$ represents the set of all points in $\mathbb{R}^3$ where $x^2+y^2=1$, a surface that intersects the xy-plane in the circle $x^2+y^2=1$, $z=0$. Since z can vary, the surface is a circular cylinder of radius 1. Thus, the equation represents the region consisting of all points on a circular cylinder of radius 1 with axis the z-axis.

33. Here $x^2+z^2\leq 9$ or equivalently $\sqrt{x^2+z^2}\leq 3$ which describes the set of all points in $\mathbb{R}^3$ whose distance from the y-axis is at most 3. Thus, the inequality represents the region consisting of all points on or inside a circular cylinder of radius 3 with axis the y-axis.

34. The equation $xyz=0$ is satisfied when any of x, y, or z is 0. Thus, the equation represents the region consisting of all points on the three coordinate planes $x=0$, $y=0$, and $z=0$.

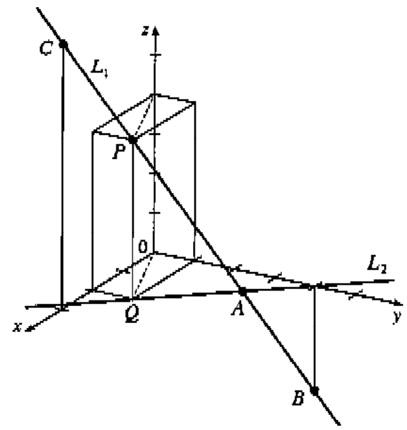
35. This describes all points with negative y-coordinates, that is, $y<0$.

36. Because the box lies in the first quadrant, each point must comprise only nonnegative coordinates. So inequalities describing the region are $0\leq x\leq 1$, $0\leq y\leq 2$, $0\leq z\leq 3$.

37. This describes a region all of whose points have a distance to the origin which is greater than r, but smaller than R. So inequalities describing the region are $r<\sqrt{x^2+y^2+z^2}<R$, or $r^2<x^2+y^2+z^2<R^2$.

38. The solid sphere itself is represented by $\sqrt{x^2+y^2+z^2}\leq 2$. Since we want only the upper hemisphere, we restrict the z-coordinate to nonnegative values. Then inequalities describing the region are $\sqrt{x^2+y^2+z^2}\leq 2$, $z\geq 0$, or $x^2+y^2+z^2\leq 4$, $z\geq 0$.

39. (a) To find the x- and y-coordinates of the point P, we project it onto L_2 and project the resulting point Q onto the x- and y-axes. To find the z-coordinate, we project P onto either the xz-plane or the yz-plane (using our knowledge of its x- or y-coordinate) and then project the resulting point onto the z-axis. (Or, we could draw a line parallel to QO from P to the z-axis.) The coordinates of P are $(2, 1, 4)$.



(b) A is the intersection of L_1 and L_2, B is directly below the y-intercept of L_2, and C is directly above the x-intercept of L_2.

40. Let $P = (x, y, z)$. Then $2\,|PB| = |PA| \;\Leftrightarrow\; 4\,|PB|^2 = |PA|^2 \;\Leftrightarrow$

$4\left((x-6)^2 + (y-2)^2 + (z+2)^2\right) = (x+1)^2 + (y-5)^2 + (z-3)^2 \;\Leftrightarrow$

$4(x^2 - 12x + 36) - x^2 - 2x + 4(y^2 - 4y + 4) - y^2 + 10y + 4(z^2 + 4z + 4) - z^2 + 6z = 35 \;\Leftrightarrow$

$3x^2 - 50x + 3y^2 - 6y + 3z^2 + 22z = 35 - 144 - 16 - 16 \;\Leftrightarrow\; x^2 - \frac{50}{3}x + y^2 - 2y + z^2 + \frac{22}{3}z = -\frac{141}{3}$.

By completing the square three times we get $\left(x - \frac{25}{3}\right)^2 + (y-1)^2 + \left(z + \frac{11}{3}\right)^2 = \frac{332}{9}$, which is an equation of a sphere with center $\left(\frac{25}{3}, 1, -\frac{11}{3}\right)$ and radius $\frac{\sqrt{332}}{3}$.

41. We need to find a set of points $\{P(x, y, z) \mid |AP| = |BP|\}$.

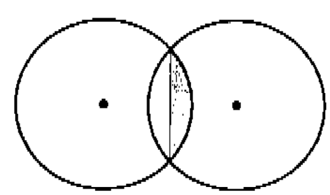
$\sqrt{(x+1)^2 + (y-5)^2 + (z-3)^2} = \sqrt{(x-6)^2 + (y-2)^2 + (z+2)^2} \;\Rightarrow$

$(x+1)^2 + (y-5) + (z-3)^2 = (x-6)^2 + (y-2)^2 + (z+2)^2 \;\Rightarrow$

$x^2 + 2x + 1 + y^2 - 10y + 25 + z^2 - 6z + 9 = x^2 - 12x + 36 + y^2 - 4y + 4 + z^2 + 4z + 4 \;\Rightarrow$

$14x - 6y - 10z = 9$. Thus the set of points is a plane perpendicular to the line segment joining A and B (since this plane must contain the perpendicular bisector of the line segment AB).

42. Completing the square three times in the first equation gives $(x+2)^2 + (y-1)^2 + (z+2)^2 = 2^2$, a sphere with center $(-2, 1, 2)$ and radius 2. The second equation is that of a sphere with center $(0, 0, 0)$ and radius 2. The distance between the centers of the spheres is $\sqrt{(-2-0)^2 + (1-0)^2 + (-2-0)^2} = \sqrt{4+1+4} = 3$. Since the spheres have the same radius, the volume inside both spheres is symmetrical about the plane containing the circle of intersection of the spheres. The distance from this plane to the center of the circles is $\frac{3}{2}$. So the region inside both spheres consists of two caps of spheres of height $h = 2 - \frac{3}{2} = \frac{1}{2}$. From Exercise 6.2.49 [ET 6.2.49], the volume of a cap of a sphere is $V = \frac{1}{3}\pi h^2(3r - h) = \frac{1}{3}\pi\left(\frac{1}{2}\right)^2\left(3 \cdot 2 - \frac{1}{2}\right) = \frac{11\pi}{24}$. So the total volume is $2 \cdot \frac{11\pi}{24} = \frac{11\pi}{12}$.



13.2 Vectors **ET 12.2**

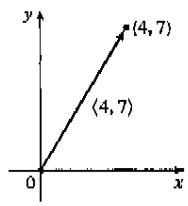
1. (a) The cost of a theater ticket is a scalar, because it has only magnitude.

(b) The current in a river is a vector, because it has both magnitude (the speed of the current) and direction at any given location.

(c) If we assume that the initial path is linear, the initial flight path from Houston to Dallas is a vector, because it has both magnitude (distance) and direction.

(d) The population of the world is a scalar, because it has only magnitude.

2. If the initial point of the vector $\langle 4, 7\rangle$ is placed at the origin, then $\langle 4, 7\rangle$ is the position vector of the point $(4, 7)$.



3. Vectors are equal when they share the same length and direction (but not necessarily location). Using the symmetry of the parallelogram as a guide, we see that $\overrightarrow{AB} = \overrightarrow{DC}$, $\overrightarrow{DA} = \overrightarrow{CB}$, $\overrightarrow{DE} = \overrightarrow{EB}$, and $\overrightarrow{EA} = \overrightarrow{CE}$.

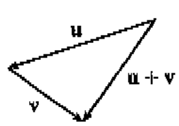
4. (a) The initial point of $\overrightarrow{QR}$ is positioned at the terminal point of $\overrightarrow{PQ}$, so by the Triangle Law the sum $\overrightarrow{PQ} + \overrightarrow{QR}$ is the vector with initial point P and terminal point R, namely $\overrightarrow{PR}$.

(b) By the Triangle Law, $\overrightarrow{RP} + \overrightarrow{PS}$ is the vector with initial point R and terminal point S, namely $\overrightarrow{RS}$.

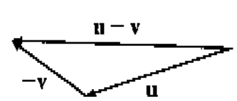
(c) First we consider $\overrightarrow{QS} - \overrightarrow{PS}$ as $\overrightarrow{QS} + \left(-\overrightarrow{PS}\right)$. Then since $-\overrightarrow{PS}$ has the same length as $\overrightarrow{PS}$ but points in the opposite direction, we have $-\overrightarrow{PS} = \overrightarrow{SP}$ and so $\overrightarrow{QS} - \overrightarrow{PS} = \overrightarrow{QS} + \overrightarrow{SP} = \overrightarrow{QP}$.

(d) We use the Triangle Law twice: $\overrightarrow{RS} + \overrightarrow{SP} + \overrightarrow{PQ} = \left(\overrightarrow{RS} + \overrightarrow{SP}\right) + \overrightarrow{PQ} = \overrightarrow{RP} + \overrightarrow{PQ} = \overrightarrow{RQ}$

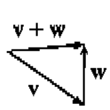
5. (a)



(b)



(c)



(d)