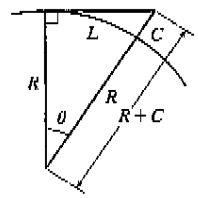
(c) We substitute $L = 1$, $g = 9.8$, and $k = \sin(10^\circ/2) \approx 0.08716$, and the inequality from part (b) becomes $2.01090 \le T \le 2.01093$, so $T \approx 2.0109$. The estimate $T \approx 2\pi\sqrt{L/g} \approx 2.0071$ differs by about 0.2%. If $\theta_0 = 42^\circ$, then $k \approx 0.35837$ and the inequality becomes $2.07153 \le T \le 2.08103$, so $T \approx 2.0763$. The one-term estimate is the same, and the discrepancy between the two estimates increases to about 3.4%.

35. (a) L is the length of the arc subtended by the angle θ, so $L = R\theta \Rightarrow \theta = L/R$. Now $\sec\theta = (R + C)/R \Rightarrow R\sec\theta = R + C \Rightarrow C = R\sec\theta - R = R\sec(L/R) - R$.



(b) From Exercise 11, $\sec x \approx T_4(x) = 1 + \frac{1}{2}x^2 + \frac{5}{24}x^4$. By part (a),

$$C \approx R\left[1 + \frac{1}{2}\left(\frac{L}{R}\right)^2 + \frac{5}{24}\left(\frac{L}{R}\right)^4\right] - R = R + \frac{1}{2}R\cdot\frac{L^2}{R^2} + \frac{5}{24}R\cdot\frac{L^4}{R^4} - R = \frac{L^2}{2R} + \frac{5L^4}{24R^3}.$$

(c) Taking $L = 100$ km and $R = 6370$ km, the formula in part (a) says that

$$C = R\sec(L/R) - R = 6370\sec(100/6370) - 6370 \approx 0.785\,009\,965\,44 \text{ km}$$

The formula in part (b) says that

$$C \approx \frac{L^2}{2R} + \frac{5L^4}{24R^3} = \frac{100^2}{2\cdot 6370} + \frac{5\cdot 100^4}{24\cdot 6370^3} \approx 0.785\,009\,957\,36 \text{ km}.$$

The difference between these two results is only 0.000 000 008 08 km, or 0.000 008 08 m!

36. $T_n(x) = f(a) + \dfrac{f'(a)}{1!}(x-a) + \dfrac{f''(a)}{2!}(x-a)^2 + \cdots + \dfrac{f^{(n)}(a)}{n!}(x-a)^n$. Let $0 \le m \le n$. Then

$$T_n^{(m)}(x) = m!\frac{f^{(m)}(a)}{m!}(x-a)^0 + (m+1)(m)\cdots(2)\frac{f^{(m+1)}(a)}{(m+1)!}(x-a)^1 + \cdots + n(n-1)\cdots(n-m+1)\frac{f^{(n)}(a)}{n!}(x-a)^{n-m}$$

For $x = a$, all terms in this sum except the first one are 0, so $T_n^{(m)}(a) = \dfrac{m!\,f^{(m)}(a)}{m!} = f^{(m)}(a)$.

37. Using $f(x) = T_n(x) + R_n(x)$ with $n = 1$ and $x = r$, we have $f(r) = T_1(r) + R_1(r)$, where T_1 is the first-degree Taylor polynomial of f at a. Because $a = x_n$, $f(r) = f(x_n) + f'(x_n)(r - x_n) + R_1(r)$. But r is a root of f, so $f(r) = 0$ and we have $0 = f(x_n) + f'(x_n)(r - x_n) + R_1(r)$. Taking the first two terms to the left side gives us $f'(x_n)(x_n - r) - f(x_n) = R_1(r)$. Dividing by $f'(x_n)$, we get $x_n - r - \dfrac{f(x_n)}{f'(x_n)} = \dfrac{R_1(r)}{f'(x_n)}$. By the formula for Newton's method, the left side of the preceding equation is $x_{n+1} - r$, so $|x_{n+1} - r| = \left|\dfrac{R_1(r)}{f'(x_n)}\right|$. Taylor's Inequality gives us $|R_1(r)| \le \dfrac{|f''(r)|}{2!}|r - x_n|^2$. Combining this inequality with the facts $|f''(x)| \le M$ and $|f'(x)| \ge K$ gives us $|x_{n+1} - r| \le \dfrac{M}{2K}|x_n - r|^2$.

33. (a) If the water is deep, then $2\pi d/L$ is large, and we know that $\tanh x \to 1$ as $x \to \infty$. So we can approximate $\tanh(2\pi d/L) \approx 1$, and so $v^2 \approx gL/(2\pi) \quad \Leftrightarrow \quad v \approx \sqrt{gL/(2\pi)}$.

(b) From the table, the first term in the Maclaurin series of $\tanh x$ is x, so if the water is shallow, we can approximate $\tanh \dfrac{2\pi d}{L} \approx \dfrac{2\pi d}{L}$, and so $v^2 \approx \dfrac{gL}{2\pi} \cdot \dfrac{2\pi d}{L} \quad \Leftrightarrow \quad v \approx \sqrt{gd}$.

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$\tanh x$	0
1	$\operatorname{sech}^2 x$	1
2	$-2\operatorname{sech}^2 x \tanh x$	0
3	$2\operatorname{sech}^2 x\,(3\tanh^2 x - 1)$	-2

(c) Since $\tanh x$ is an odd function, its Maclaurin series is alternating, so the error in the approximation $\tanh \dfrac{2\pi d}{L} \approx \dfrac{2\pi d}{L}$ is less than the first neglected term, which is $\dfrac{|f'''(0)|}{3!}\left(\dfrac{2\pi d}{L}\right)^3 = \dfrac{1}{3}\left(\dfrac{2\pi d}{L}\right)^3$.

If $L > 10d$, then $\dfrac{1}{3}\left(\dfrac{2\pi d}{L}\right)^3 < \dfrac{1}{3}\left(2\pi \cdot \dfrac{1}{10}\right)^3 = \dfrac{\pi^3}{375}$, so the error in the approximation $v^2 = gd$ is less than $\dfrac{gL}{2\pi} \cdot \dfrac{\pi^3}{375} \approx 0.0132gL$.

34. (a)
$$4\sqrt{\frac{L}{g}}\int_0^{\pi/2}\frac{dx}{\sqrt{1-k^2\sin^2 x}} = 4\sqrt{\frac{L}{g}}\int_0^{\pi/2}\left[1+(-k^2\sin^2 x)\right]^{-1/2}dx$$
$$= 4\sqrt{\frac{L}{g}}\int_0^{\pi/2}\left[1-\frac{1}{2}(-k^2\sin^2 x)+\frac{\frac{1}{2}\cdot\frac{3}{2}}{2!}(-k^2\sin^2 x)^2-\frac{\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}}{3!}(-k^2\sin^2 x)^3+\cdots\right]dx$$
$$= 4\sqrt{\frac{L}{g}}\int_0^{\pi/2}\left[1+\left(\frac{1}{2}\right)k^2\sin^2 x+\left(\frac{1\cdot 3}{2\cdot 4}\right)k^4\sin^4 x+\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)k^6\sin^6 x+\cdots\right]dx$$
$$= 4\sqrt{\frac{L}{g}}\left[\frac{\pi}{2}+\left(\frac{1}{2}\right)\left(\frac{1}{2}\cdot\frac{\pi}{2}\right)k^2+\left(\frac{1\cdot 3}{2\cdot 4}\right)\left(\frac{1\cdot 3}{2\cdot 4}\cdot\frac{\pi}{2}\right)k^4+\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cdot\frac{\pi}{2}\right)k^6+\cdots\right]$$

[split up the integral and use the result from Exercise 8.1.44 [ET 7.1.44]]

$$= 2\pi\sqrt{\frac{L}{g}}\left[1+\frac{1^2}{2^2}k^2+\frac{1^2\cdot 3^2}{2^2\cdot 4^2}k^4+\frac{1^2\cdot 3^2\cdot 5^2}{2^2\cdot 4^2\cdot 6^2}k^6+\cdots\right]$$

(b) The first of the two inequalities is true because all of the terms in the series are positive. For the second,

$$T = 2\pi\sqrt{\frac{L}{g}}\left[1+\frac{1^2}{2^2}k^2+\frac{1^2\cdot 3^2}{2^2\cdot 4^2}k^4+\frac{1^2\cdot 3^2\cdot 5^2}{2^2\cdot 4^2\cdot 6^2}k^6+\frac{1^2\cdot 3^2\cdot 5^2\cdot 7^2}{2^2\cdot 4^2\cdot 6^2\cdot 8^2}k^8+\cdots\right]$$
$$\le 2\pi\sqrt{\frac{L}{g}}\left[1+\frac{1}{4}k^2+\frac{1}{4}k^4+\frac{1}{4}k^6+\frac{1}{4}k^8+\cdots\right]$$

The terms in brackets (after the first) form a geometric series with $a = \frac{1}{4}k^2$ and $r = k^2 = \sin^2\left(\frac{1}{2}\theta_0\right) < 1$.
So $T \le 2\pi\sqrt{\dfrac{L}{g}}\left[1+\dfrac{k^2/4}{1-k^2}\right] = 2\pi\sqrt{\dfrac{L}{g}}\,\dfrac{4-3k^2}{4-4k^2}$.

(b) Using $\cos\phi \approx 1 - \frac{1}{2}\phi^2$ in (2) gives us

$$\ell_o = \sqrt{R^2 + (s_o + R)^2 - 2R(s_o + R)\left(1 - \tfrac{1}{2}\phi^2\right)}$$

$$= \sqrt{R^2 + s_o^2 + 2Rs_o + R^2 - 2Rs_o + Rs_o\phi^2 - 2R^2 + R^2\phi^2} = \sqrt{s_o^2 + Rs_o\phi^2 + R^2\phi^2}$$

Anticipating that we will use the binomial series expansion $(1+x)^k \approx 1 + kx$, we can write the last expression for ℓ_o as $s_o\sqrt{1 + \phi^2\left(\frac{R}{s_o} + \frac{R^2}{s_o^2}\right)}$ and similarly, $\ell_i = s_i\sqrt{1 - \phi^2\left(\frac{R}{s_i} - \frac{R^2}{s_i^2}\right)}$. Thus, from Equation 1,

$$\frac{n_1}{\ell_o} + \frac{n_2}{\ell_i} = \frac{1}{R}\left(\frac{n_2 s_i}{\ell_i} - \frac{n_1 s_o}{\ell_o}\right) \quad\Leftrightarrow\quad n_1\ell_o^{-1} + n_2\ell_i^{-1} = \frac{n_2}{R}\cdot\frac{s_i}{\ell_i} - \frac{n_1}{R}\cdot\frac{s_o}{\ell_o} \quad\Leftrightarrow$$

$$\frac{n_1}{s_o}\left[1 + \phi^2\left(\frac{R}{s_o} + \frac{R^2}{s_o^2}\right)\right]^{-1/2} + \frac{n_2}{s_i}\left[1 - \phi^2\left(\frac{R}{s_i} - \frac{R^2}{s_i^2}\right)\right]^{-1/2}$$

$$= \frac{n_2}{R}\left[1 - \phi^2\left(\frac{R}{s_i} - \frac{R^2}{s_i^2}\right)\right]^{-1/2} - \frac{n_1}{R}\left[1 + \phi^2\left(\frac{R}{s_o} + \frac{R^2}{s_o^2}\right)\right]^{-1/2}$$

Approximating the expressions for ℓ_o^{-1} and ℓ_i^{-1} by the first two terms in their binomial series, we get

$$\frac{n_1}{s_o}\left[1 - \tfrac{1}{2}\phi^2\left(\frac{R}{s_o} + \frac{R^2}{s_o^2}\right)\right] + \frac{n_2}{s_i}\left[1 + \tfrac{1}{2}\phi^2\left(\frac{R}{s_i} - \frac{R^2}{s_i^2}\right)\right]$$

$$= \frac{n_2}{R}\left[1 + \tfrac{1}{2}\phi^2\left(\frac{R}{s_i} - \frac{R^2}{s_i^2}\right)\right] - \frac{n_1}{R}\left[1 - \tfrac{1}{2}\phi^2\left(\frac{R}{s_o} + \frac{R^2}{s_o^2}\right)\right] \quad\Leftrightarrow$$

$$\frac{n_1}{s_o} - \frac{n_1\phi^2}{2s_o}\left(\frac{R}{s_o} + \frac{R^2}{s_o^2}\right) + \frac{n_2}{s_i} + \frac{n_2\phi^2}{2s_i}\left(\frac{R}{s_i} - \frac{R^2}{s_i^2}\right)$$

$$= \frac{n_2}{R} + \frac{n_2\phi^2}{2R}\left(\frac{R}{s_i} - \frac{R^2}{s_i^2}\right) - \frac{n_1}{R} + \frac{n_1\phi^2}{2R}\left(\frac{R}{s_o} + \frac{R^2}{s_o^2}\right) \quad\Leftrightarrow$$

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2}{R} - \frac{n_1}{R} + \frac{n_1\phi^2}{2s_o}\left(\frac{R}{s_o} + \frac{R^2}{s_o^2}\right) + \frac{n_1\phi^2}{2R}\left(\frac{R}{s_o} + \frac{R^2}{s_o^2}\right) + \frac{n_2\phi^2}{2R}\left(\frac{R}{s_i} - \frac{R^2}{s_i^2}\right) - \frac{n_2\phi^2}{2s_i}\left(\frac{R}{s_i} - \frac{R^2}{s_i^2}\right)$$

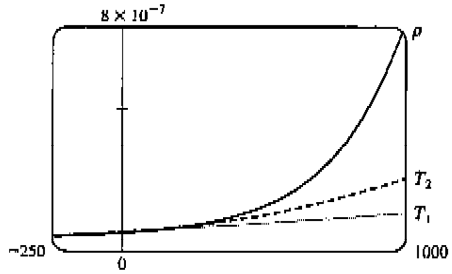
$$= \frac{n_2 - n_1}{R} + \frac{n_1\phi^2}{2}\left(\frac{R}{s_o} + \frac{R^2}{s_o^2}\right)\left(\frac{1}{s_o} + \frac{1}{R}\right) + \frac{n_2\phi^2}{2}\left(\frac{R}{s_i} - \frac{R^2}{s_i^2}\right)\left(\frac{1}{R} - \frac{1}{s_i}\right)$$

$$= \frac{n_2 - n_1}{R} + \frac{n_1\phi^2R^2}{2s_o}\left(\frac{1}{R} + \frac{1}{s_o}\right)\left(\frac{1}{R} + \frac{1}{s_o}\right) + \frac{n_2\phi^2R^2}{2s_i}\left(\frac{1}{R} - \frac{1}{s_i}\right)\left(\frac{1}{R} - \frac{1}{s_i}\right)$$

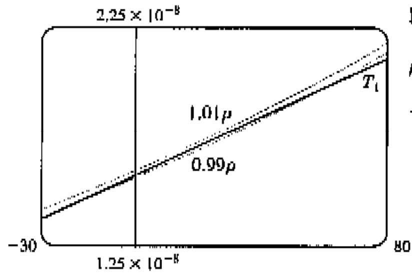
$$= \frac{n_2 - n_1}{R} + \phi^2R^2\left[\frac{n_1}{2s_o}\left(\frac{1}{R} + \frac{1}{s_o}\right)^2 + \frac{n_2}{2s_i}\left(\frac{1}{R} - \frac{1}{s_i}\right)^2\right]$$

From Figure 8, we see that $\sin\phi = h/R$. So if we approximate $\sin\phi$ with ϕ, we get $h = R\phi$ and $h^2 = \phi^2R^2$ and hence, Equation 4, as desired.

(b)



(c)



From the graph, it seems that $T_1(t)$ is within 1% of $\rho(t)$, that is, $0.99\rho(t) \le T_1(t) \le 1.01\rho(t)$, for $-14\ ^\circ\text{C} \le t \le 58\ ^\circ\text{C}$.

31. $E = \dfrac{q}{D^2} - \dfrac{q}{(D+d)^2} = \dfrac{q}{D^2} - \dfrac{q}{D^2(1+d/D)^2} = \dfrac{q}{D^2}\left[1 - \left(1 + \dfrac{d}{D}\right)^{-2}\right].$

We use the Binomial Series to expand $(1 + d/D)^{-2}$:

$$E = \frac{q}{D^2}\left[1 - \left(1 - 2\left(\frac{d}{D}\right) + \frac{2\cdot 3}{2!}\left(\frac{d}{D}\right)^2 - \frac{2\cdot 3\cdot 4}{3!}\left(\frac{d}{D}\right)^3 + \cdots\right)\right]$$

$$= \frac{q}{D^2}\left[2\left(\frac{d}{D}\right) - 3\left(\frac{d}{D}\right)^2 + 4\left(\frac{d}{D}\right)^3 - \cdots\right] \approx \frac{q}{D^2}\cdot 2\left(\frac{d}{D}\right) = 2qd\cdot\frac{1}{D^3}$$

when D is much larger than d; that is, when P is far away from the dipole.

32. (a) $\dfrac{n_1}{\ell_o} + \dfrac{n_2}{\ell_i} = \dfrac{1}{R}\left(\dfrac{n_2 s_i}{\ell_i} - \dfrac{n_1 s_o}{\ell_o}\right)$ (Equation 1) where

$$\ell_o = \sqrt{R^2 + (s_o + R)^2 - 2R(s_o + R)\cos\phi} \quad \text{and} \quad \ell_i = \sqrt{R^2 + (s_i - R)^2 + 2R(s_i - R)\cos\phi} \quad (2)$$

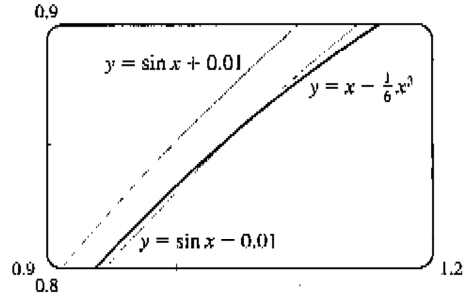
Using $\cos\phi \approx 1$ gives

$$\ell_o = \sqrt{R^2 + (s_o + R)^2 - 2R(s_o + R)} = \sqrt{R^2 + s_o^2 + 2Rs_o + R^2 - 2Rs_o - 2R^2} = \sqrt{s_o^2} = s_o$$

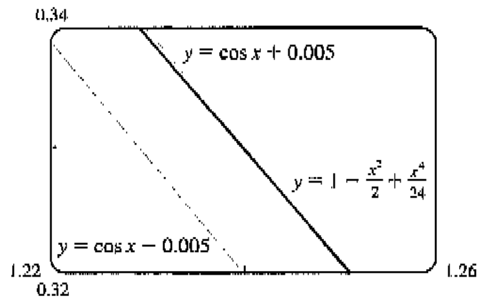
and similarly, $\ell_i = s_i$. Thus, Equation 1 becomes

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{1}{R}\left(\frac{n_2 s_i}{s_i} - \frac{n_1 s_o}{s_o}\right) \quad \Rightarrow \quad \frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

27. $\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots$. By the Alternating Series Estimation Theorem, the error in the approximation $\sin x = x - \frac{1}{3!}x^3$ is less than $\left|\frac{1}{5!}x^5\right| < 0.01 \Leftrightarrow \left|x^5\right| < 120(0.01) \Leftrightarrow$ $|x| < (1.2)^{1/5} \approx 1.037$. The curves $y = x - \frac{1}{6}x^3$ and $y = \sin x - 0.01$ intersect at $x \approx 1.043$, so the graph confirms our estimate. Since both the sine function and the given approximation are odd functions, we need to check the estimate only for $x > 0$. Thus, the desired range of values for x is $-1.037 < x < 1.037$.



28. $\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \cdots$. By the Alternating Series Estimation Theorem, the error is less than $\left|-\frac{1}{6!}x^6\right| < 0.005 \Leftrightarrow x^6 < 720(0.005) \Leftrightarrow$ $|x| < (3.6)^{1/6} \approx 1.238$. The curves $y = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$ and $y = \cos x + 0.005$ intersect at $x \approx 1.244$, so the graph confirms our estimate. Since both the cosine function and the given approximation are even functions, we need to check the estimate only for $x > 0$. Thus, the desired range of values for x is $-1.238 < x < 1.238$.



29. Let $s(t)$ be the position function of the car, and for convenience set $s(0) = 0$. The velocity of the car is $v(t) = s'(t)$ and the acceleration is $a(t) = s''(t)$, so the second degree Taylor polynomial is $T_2(t) = s(0) + v(0)t + \frac{a(0)}{2}t^2 = 20t + t^2$. We estimate the distance travelled during the next second to be $s(1) \approx T_2(1) = 20 + 1 = 21$ m. The function $T_2(t)$ would not be accurate over a full minute, since the car could not possibly maintain an acceleration of 2 m/s^2 for that long (if it did, its final speed would be 140 m/s $\approx$ 313 mi/h!)

30. (a)

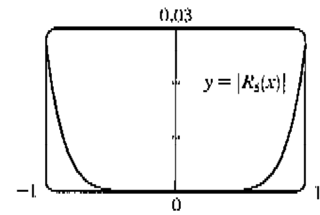
n	$\rho^{(n)}(t)$	$\rho^{(n)}(20)$
0	$\rho_{20}e^{\alpha(t-20)}$	ρ_{20}
1	$\alpha\rho_{20}e^{\alpha(t-20)}$	$\alpha\rho_{20}$
2	$\alpha^2\rho_{20}e^{\alpha(t-20)}$	$\alpha^2\rho_{20}$

The linear approximation is $T_1(t) = \rho(20) + \rho'(20)(t-20) = \rho_{20}\left[1 + \alpha(t-20)\right]$. The quadratic approximation is

$$T_2(t) = \rho(20) + \rho'(20)(t-20) + \frac{\rho''(20)}{2}(t-20)^2 = \rho_{20}\left[1 + \alpha(t-20) + \tfrac{1}{2}\alpha^2(t-20)^2\right]$$

(b) $|R_5(x)| \le \frac{M}{6!}|x|^6$, where $\left|f^{(6)}(x)\right| \le M$. For x in $[-1,1]$, we have $|x| \le 1$. Since $f^{(6)}(x)$ is an increasing odd function on $[-1,1]$, we see that $\left|f^{(6)}(x)\right| \le f^{(6)}(1) = 64\sinh 2 = 32(e^2 - e^{-2}) \approx 232.119$, so we can take $M = 232.12$ and get $|R_5(x)| \le \frac{232.12}{720} \cdot 1^6 \approx 0.3224$.

(c) From the graph of $|R_5(x)| = |\sinh 2x - T_5(x)|$, it seems that the error is less than 0.027 on $[-1,1]$.



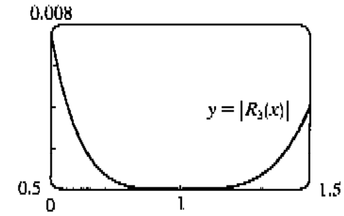
23. From Exercise 5, $\sin x = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) - \frac{1}{4}\left(x - \frac{\pi}{6}\right)^2 - \frac{\sqrt{3}}{12}\left(x - \frac{\pi}{6}\right)^3 + R_3(x)$, where $|R_3(x)| \le \frac{M}{4!}\left|x - \frac{\pi}{6}\right|^4$ with $\left|f^{(4)}(x)\right| = |\sin x| \le M = 1$. Now $x = 35° = (30° + 5°) = \left(\frac{\pi}{6} + \frac{\pi}{36}\right)$ radians, so the error is $\left|R_3\left(\frac{\pi}{36}\right)\right| \le \dfrac{\left(\frac{\pi}{36}\right)^4}{4!} < 0.000003$. Therefore, to five decimal places, $\sin 35° \approx \frac{1}{2} + \frac{\sqrt{3}}{2}\left(\frac{\pi}{36}\right) - \frac{1}{4}\left(\frac{\pi}{36}\right)^2 - \frac{\sqrt{3}}{12}\left(\frac{\pi}{36}\right)^3 \approx 0.57358$.

24. From Exercise 16, $\cos x = \frac{1}{2} - \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right) - \frac{1}{4}\left(x - \frac{\pi}{3}\right)^2 + \frac{\sqrt{3}}{12}\left(x - \frac{\pi}{3}\right)^3 + \frac{1}{48}\left(x - \frac{\pi}{3}\right)^4 + R_4(x)$. Now since $x = 69° = (60° + 9°) = \left(\frac{\pi}{3} + \frac{\pi}{20}\right)$ radians, the error is $|R_4(x)| \le \dfrac{\left(\frac{\pi}{20}\right)^5}{5!} < 8 \times 10^{-7}$. Therefore, to five decimal places, $\cos 69° \approx \frac{1}{2} - \frac{\sqrt{3}}{2}\left(\frac{\pi}{20}\right) - \frac{1}{4}\left(\frac{\pi}{20}\right)^2 + \frac{\sqrt{3}}{12}\left(\frac{\pi}{20}\right)^3 + \frac{1}{48}\left(\frac{\pi}{20}\right)^4 \approx 0.35837$.

25. All derivatives of e^x are e^x, so $|R_n(x)| \le \dfrac{e^x}{(n+1)!}|x|^{n+1}$, where $0 < x < 0.1$. Letting $x = 0.1$, $R_n(0.1) \le \dfrac{e^{0.1}}{(n+1)!}(0.1)^{n+1} < 0.00001$, and by trial and error we find that $n = 3$ satisfies this inequality since $R_3(0.1) < 0.0000046$. Thus, by adding the four terms of the Maclaurin series for e^x corresponding to $n = 0, 1, 2$, and 3, we can estimate $e^{0.1}$ to within 0.00001. (In fact, this sum is $1.1051\overline{6}$ and $e^{0.1} \approx 1.10517$.)

26. Example 6 in Section 12.9 [ET 11.9] gives the Maclaurin series for $\ln(1-x)$ as $-\sum_{n=1}^{\infty} \frac{x^n}{n}$ for $|x| < 1$. Thus, $\ln 1.4 = \ln[1 - (-0.4)] = -\sum_{n=1}^{\infty} \dfrac{(-0.4)^n}{n} = \sum_{n=1}^{\infty} (-1)^{n+1}\dfrac{(0.4)^n}{n}$. Since this is an alternating series, the error is less than the first neglected term by the Alternating Series Estimation Theorem, and we find that $|a_6| = (0.4)^6/6 \approx 0.0007 < 0.001$. So we need the first five (non-zero) terms of the Maclaurin series for the desired accuracy. (In fact, this sum is approximately 0.33698 and $\ln 1.4 \approx 0.33647$.)

(c) From the graph of $|R_3(x)| = |x \ln x - T_3(x)|$, it seems that the error is less than 0.0076 on $[0.5, 1.5]$.



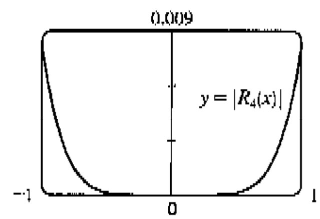
21.

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$x \sin x$	0
1	$\sin x + x \cos x$	0
2	$2 \cos x - x \sin x$	2
3	$-3 \sin x - x \cos x$	0
4	$-4 \cos x + x \sin x$	-4
5	$5 \sin x + x \cos x$	

(a) $f(x) = x \sin x \approx T_4(x) = \frac{2}{2!}(x-0)^2 + \frac{-4}{4!}(x-0)^4 = x^2 - \frac{1}{6}x^4$

(b) $|R_4(x)| \le \frac{M}{5!}|x|^5$, where $\left|f^{(5)}(x)\right| \le M$. Now $-1 \le x \le 1 \;\Rightarrow\; |x| \le 1$, and a graph of $f^{(5)}(x)$ shows that $\left|f^{(5)}(x)\right| \le 5$ for $-1 \le x \le 1$. Thus, we can take $M = 5$ and get $|R_4(x)| \le \frac{5}{5!} \cdot 1^5 = \dfrac{1}{24} = 0.041\overline{6}$.

(c) From the graph of $|R_4(x)| = |x \sin x - T_4(x)|$, it seems that the error is less than 0.0082 on $[-1, 1]$.

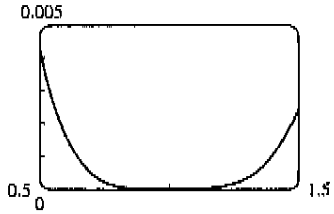


22.

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$\sinh 2x$	0
1	$2 \cosh 2x$	2
2	$4 \sinh 2x$	0
3	$8 \cosh 2x$	8
4	$16 \sinh 2x$	0
5	$32 \cosh 2x$	32
6	$64 \sinh 2x$	

(a) $f(x) = \sinh 2x \approx T_5(x) = 2x + \frac{8}{3!}x^3 + \frac{32}{5!}x^5 = 2x + \frac{4}{3}x^3 + \frac{4}{15}x^5$

(c)



From the graph of $|R_3(x)| = |\ln(1+2x) - T_3(x)|$, it seems that the error is less than 0.005 on $[0.5, 1.5]$.

19.

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	e^{x^2}	1
1	$e^{x^2}(2x)$	0
2	$e^{x^2}(2+4x^2)$	2
3	$e^{x^2}(12x+8x^3)$	0
4	$e^{x^2}(12+48x^2+16x^4)$	

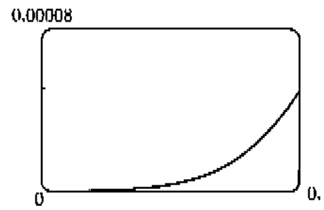
(a) $f(x) = e^{x^2} \approx T_3(x) = 1 + \frac{2}{2!}x^2 = 1 + x^2$

(b) $|R_3(x)| \le \dfrac{M}{4!}|x|^4$, where $\left|f^{(4)}(x)\right| \le M$.

Now $0 \le x \le 0.1 \quad \Rightarrow \quad x^4 \le (0.1)^4$, and letting $x = 0.1$ gives

$|R_3(x)| \le \dfrac{e^{0.01}(12 + 0.48 + 0.0016)}{24}(0.1)^4 \approx$ 0.00006.

(c)



From the graph of $|R_3(x)| = \left|e^{x^2} - (1+x^2)\right|$, it appears that the error is less than 0.000051 on $[0, 0.1]$.

20.

n	$f^{(n)}(x)$	$f^{(n)}(1)$
0	$x\ln x$	0
1	$\ln x + 1$	1
2	$1/x$	1
3	$-1/x^2$	-1
4	$2/x^3$	

(a) $f(x) = x\ln x \approx T_3(x) = (x-1) + \frac{1}{2}(x-1)^2 - \frac{1}{6}(x-1)^3$

(b) $|R_3(x)| \le \frac{M}{4!}|x-1|^4$, where $\left|f^{(4)}(x)\right| \le M$. Now $0.5 \le x \le 1.5 \quad \Rightarrow \quad |x-1| \le \frac{1}{2} \quad \Rightarrow \quad |x-1|^4 \le \frac{1}{16}$. Since $\left|f^{(4)}(x)\right|$ is decreasing on $[0.5, 1.5]$, we can take $M = \left|f^{(4)}(0.5)\right| = 2/(0.5)^3 = 16$, so $|R_3(x)| \le \frac{16}{24}(1/16) = \frac{1}{24} = 0.041\overline{6}$.