11.10

Key

8. $f(x) = xe^x$

$f^{(0)}(0) = xe^x = 0$

$f^{(1)}(0) = (x+1)e^x = 1$

$f^{(2)}(0) = (x+2)e^x = 2$

$f^{(3)}(0) = (x+3)e^x = 3$

$$xe^x = \sum_{n=0}^{\infty} \frac{n}{n!}(x)^n = \sum_{n=1}^{\infty} \frac{x^n}{(n-1)!}$$

$$\lim_{n\to\infty}\left|\frac{x^{n+1}}{\cancel{n!}} \cdot \frac{\cancel{(n-1)!}}{x^n}\right| = |x| \lim_{n\to\infty} \frac{1}{n} = 0 < 1 \quad \text{so } R = \infty$$

14. $f(x) = \ln(x) \qquad a = 2$

$f^{(0)}(2) = \ln x = \ln 2$

$f^{(1)}(2) = x^{-1} = 1/2$

$f^{(2)}(2) = -x^{-2} = -1/4$

$f^{(3)}(2) = 2x^{-3} = 2/8$

$f^{(4)}(2) = -6x^{4} = -3/8$

$$f^{(n)}(2) = \frac{(-1)^{n-1}(n-1)!}{2^n}$$

$$\ln(x) = \ln 2 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-2)^n}{n2^n}$$

$$\lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{(n+1)2^{n+1}} \cdot \frac{n2^n}{(x-2)^n}\right| =$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{|x-2|}{2} \lim_{n\to\infty} \frac{n}{n+1} = \frac{|x-2|}{2} < 1$$

$$\Rightarrow \frac{|x-2|}{2} < 1 \Rightarrow |x-2| < 2$$

$$-2 < x-2 < 2$$

$$0 < x < 4$$

$$\text{RoC} = 2$$

18. $f(x) = x^{-2} \qquad a = 1$

$f^{(0)}(1) = x^{-2} = 1$

$f^{(1)}(1) = -2x^{-3} = -2$

$f^{(2)}(1) = 6x^{-4} = 6$

$f^{(3)}(1) = -24x^{-5} = -24$

$$x^{-2} = 1 - \frac{2(x-1)}{1!} + \frac{6(x-1)}{2!} - \frac{24(x-1)}{3!} + \ldots$$

$$\sum_{n=0} (-1)^n (n+1)(x-1)^n$$

$$\lim_{n\to\infty}\left|\frac{((n+1)+1)(x-1)^{n+1}}{(n+1)(x-1)^{n}}\right| = \lim_{n\to\infty}\left|\frac{(n+2)(x-1)}{(n+1)}\right| = (x-1)\lim_{n\to\infty}\frac{(n+2)}{(n+1)}$$

$$= |x-1| < 1 \Rightarrow -1 < x-1 < 1$$

$$\Rightarrow 0 < x < 2$$

$$RoC = 1$$

28. $f(x) = x\cos 2x$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos 2x = \sum_{n=0}^{\infty}(-1)^n\frac{2x^{2n}}{(2n)!} = \sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n}}{(2n)!}x^{2n}$$

$$x\cos 2x = x\sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n}}{(2n)!}x^{2n} = \sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n}}{(2n)!}x^{2n+1}$$

$$\lim_{n\to\infty}\left|\frac{2^{2n+2}x^{2n+3}}{(2n+2)!}\cdot\frac{(2n)!}{2^{2n}x^{2n+1}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{4x^2}{(2n+2)(2n+1)}\right| = 0 < 1 \qquad R = \infty$$

thats a one