

Amin
 Solution key
 11.9

3: $f(x) = \frac{1}{1+x}$

$$\frac{1}{1+x} = \frac{1}{1-(-x)} \Rightarrow \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n x^n$$

$$| -x | < 1 = |x| < 1 \quad \text{interval of convergence } (-1, 1)$$

9: $f(x) = \frac{x}{9+x^2}$

$$\frac{x}{9+x^2} = \frac{1}{9(1+\frac{x^2}{9})} (x) = \frac{x}{9} \left(\frac{1}{1+(\frac{x}{3})^2} \right) = \frac{x}{9} \left(\frac{1}{1-(-\frac{x}{3})^2} \right)$$

$$\frac{x}{9} \sum_{n=0}^{\infty} \left[\left(-\frac{x}{3}\right)^2 \right]^n = \frac{x}{9} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{9^n} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{9^{n+1}}$$

$$\frac{|-x^2|}{9} < 1 = |x^2| < 9 = |x| < 3 \quad \text{ROC} = (-3, 3)$$

15: $f(x) = \ln(5-x)$

$$= - \int \frac{1}{5-x} dx = - \int \frac{1}{5(1-\frac{x}{5})} dx = - \frac{1}{5} \int \left(\frac{x}{5}\right)^n dx$$

$$- \frac{1}{5} \left[\frac{x^{n+1}}{5^{n+1}} \right] + C \Rightarrow C - \frac{1}{5} \sum_{n=0}^{\infty} \frac{x^{n+1}}{5^{n+1}} = C - \sum_{n=0}^{\infty} \frac{x^n}{n5^n}$$

$$x=0 \Rightarrow C = \ln 5$$

$$\left| \frac{x}{5} \right| < 1 \Rightarrow |x| < 5 \quad \text{ROC} = 5$$

$$23: \int \frac{z}{1-t^8} dt = \int \frac{1}{1-t^8} (t) dt$$

$$= t \sum_{n=0}^{\infty} (t^8)^n = \sum_{n=0}^{\infty} t^{8n+1}$$

$$\Rightarrow \int \frac{t}{1-t^8} dt = \int \frac{t^{8n+2}}{8n+2}$$

$$|t^8| < 1 = |x| < 1 \quad R > 1$$