Answer Key, 11.8 # 2, 12, 30, 34

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Section : 11.8

2. (a) What is the radius of convergence of given power series $\sum_{n=0}^{\infty} c_n (x-a)^n$

- By the theorem 3 in 11.8, the ROC is 0 if the series converges only when $x = a$.
- By the theorem 3 in 11.8, the ROC is ∞ if the series converges for all x
- positive number ROC such that series converges if $|x-a| < R$ and diverges if $|x-a| > R$.
- Ratio Test can be used to find the radius of convergence.

(b) Interval that consists of all values of x for which the series converges.

Interval of convergence can be :

- A single point $\{a_n\}$
- $\mathbb{R} \to (-\infty, \infty)$
- An interval with endpoints $a-R$ & $a+R$
- Test the series for convergence at each endpoint to determine the interval of convergence.

12. $\sum_{n=1}^{\infty} \frac{x^n}{5^n n^5}$ $\quad a_n = \frac{x^n}{5^n n^5}$ $\quad$ Ratio test

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{x^{n+1}}{5^{n+1}(n+1)^5} \cdot \frac{5^n n^5}{x^n} \right| = \lim_{n\to\infty} \left| \frac{x}{5} \cdot \left(\frac{n}{n+1}\right)^5 \right|$$

$$= \lim_{n\to\infty} \left| \frac{x}{5} \left(\frac{1}{1+\frac{1}{n}}\right)^5 \right| = \left| \frac{x}{5} \right| < 1 \qquad -5 \le x \le 5$$

$$|x| \le 5$$

ROC : 5

IOC : $[-5, 5]$

30. Suppose that $\sum_{n=0}^{\infty} C_n x^n$ converges when $x = -4$ and diverges when $x = 6$.

(a) $\sum_{n=0}^{\infty} C_n$ Converges $x = 1$, $\sum_{n=0}^{\infty} C_n$ is convergent.

(b) $\sum_{n=0}^{\infty} C_n 8^n$ diverge $x = 8$, $\sum_{n=0}^{\infty} C_n 8^n$ diverges

(c) $\sum_{n=0}^{\infty} C_n (-3)^n$ Converges $x = -3$, $\sum_{n=0}^{\infty} C_n (-3)^n$ Converges

(d) $\sum_{n=0}^{\infty} C_n (-9)^n$, diverges when $x = -9$, divergent series.

34.

(a) $A(x) = 1 + \sum_{n=1}^{\infty} a_n$, where $\dfrac{x^{3n}}{2 \cdot 3 \cdot 5 \cdot 6 \cdots (3n-1)(3n)}$

$\lim_{n\to\infty} \left| \dfrac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \dfrac{|x|^3}{(3n+2)(3n+3)} = 0 < 1$, therefore

Converges for all x. Domain: $\mathbb{R}$, $(-\infty, \infty)$

(b)