Gebreab, Habteab

Key 11.6

4. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^n}{n^4} \Rightarrow \lim_{n\to\infty} \frac{2^n}{n^4} = \infty$ so diverges by test for divergence

8. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+1}$ converges by by alternating series test.

(i) Decreasing (ii) $\lim_{n\to\infty} \frac{n}{n^2+1} = 0$

But $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$

$\therefore$ It is conditional convergent.

38. (a) $|a_n| < r^n$ for $n \geq N$. and $\sum_{n=1}^{\infty} r^n$ converges $\Rightarrow \sum_{n=N}^{\infty} |a_n|$ converges by comparis and so does $\sum_{n=1}^{\infty} |a_n|$. so $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

(b) $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$, then there is an $\exists N$ such that $\sqrt[n]{|a_n|} > 1$ $\forall n > N$, so $|a_n| > 1$ $n \geq N$. Thus, $\lim_{n\to\infty} a_n \neq 0$, so $\sum_{n=1}^{\infty} a_n$ diverges by test for divergence

40. let $\sum b_n$ is an absolutely convergent, since $|b_n^+| \leq |b_n|$ & $|b_n^-| < |b_n|$ we conclude that by comparison $\sum b_n^+$ & $\sum b_n^-$ must be absolutely convergent.

$\sum b_n$ will have partial sum S_n that oscillate in value back and forth across r. Since $\lim_{n\to\infty} a_n = 0$ and since the size of the oscillation $|S_n - r|$ is always less than $|a_n|$ because of the way $\sum b_n$ was constructed, we have $\sum b_n = \lim_{n\to\infty} S_n = r$